Coupling structural optimization and trajectory optimization methods in additive manufacturing

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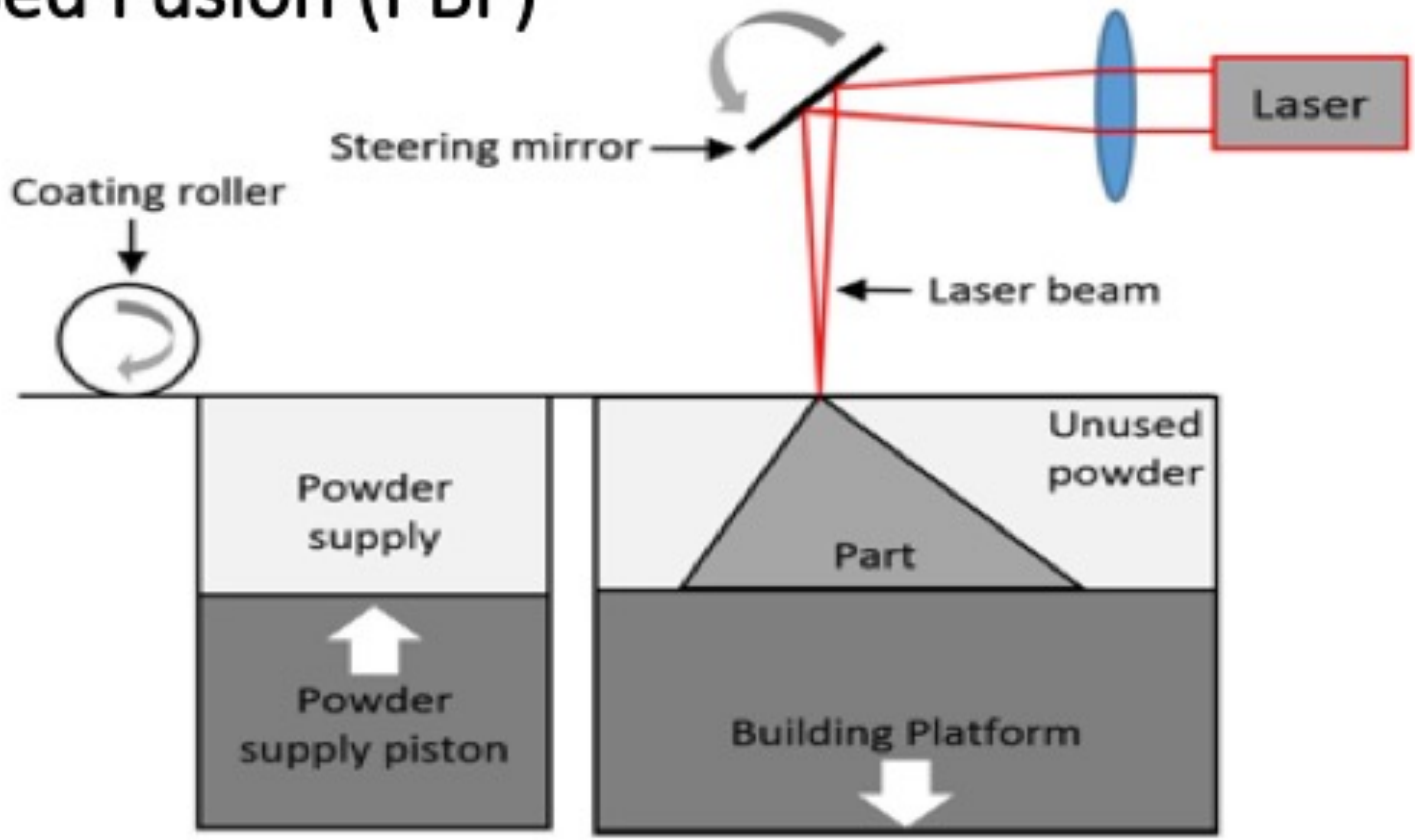
Powder Bed Fusion (PBF)

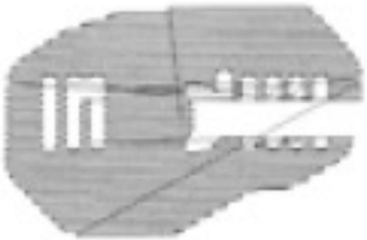


Bikas, H., P. Stavropoulos, and G. Chryssolouris, *Additive Manufacturing Methods and Modelling Approaches: A Critical Review* In: The International Journal of Advanced Manufacturing Technology 83.1-4 (2016), pp. 389–405.

https://www.youtube.com/watch?v=yiUUZxp7bLQ

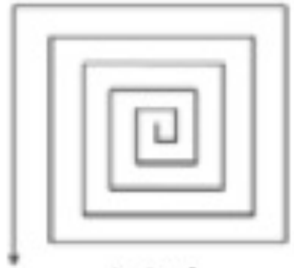
Notion of “good path”[a]

Parallel

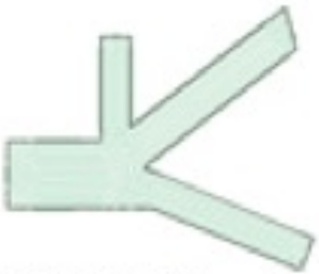


Zigzag

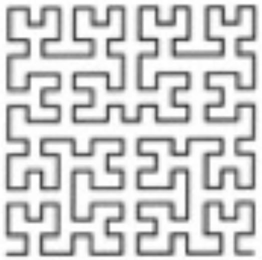
Contour



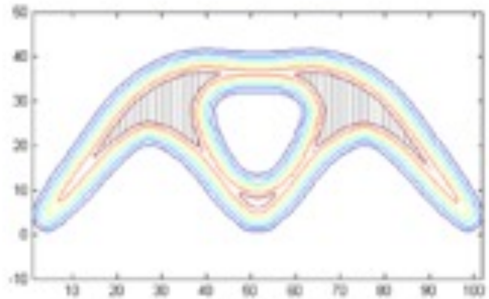
Spiral



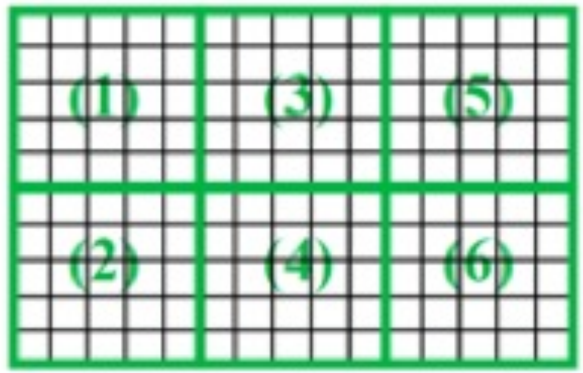
Medial Axis Transform



Fractal



Hybrid



(a) D. Ding, Z. Pan, D. Cuiuri, H. Li, and S. van Duin, *Advanced design for additive manufacturing: 3d slicing and 2d path planning*, New trends in 3d printing, (2016), pp. 1–23.

Objectives of this work

How to use shape optimization to facilitate the generation of "good" scanning path?

Objectives:

- Optimization of the path "from scratch"
- Concurrent optimization between part shape and scanning path

Bibliography:

- T.M. ALAM, Some optimal control problem of partial differential equations and applications to the selective laser melting process (SLM), PhD thesis, Université Polytechnique Hauts-de-France, 2020
- Q. Chen, J. Liu, X. Liang, and A. C. To, A level-set based continuous scanning path optimization method for reducing residual stress and deformation in metal additive manufacturing, Computer Methods in Applied Mechanics and Engineering, 360 (2020), p. 112719.
- M. Boissier, G. Allaire, C. Tournier, Scanning path optimization using shape optimization tools, Structural and Multidisciplinary Optimization, 61:6, pp. 2437-2466, 2020

Overview

- Modelling assumptions
- Scanning path optimization
- Part optimization
- Concurrent optimization

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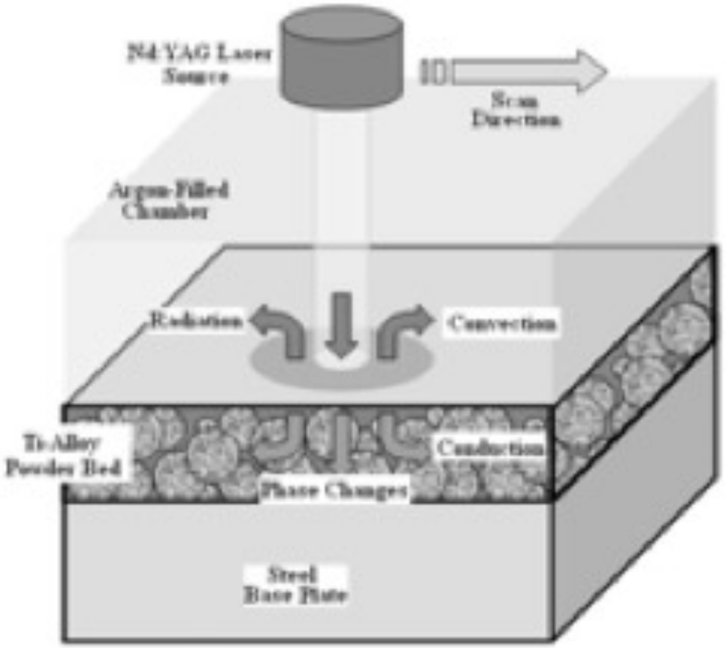
Modelling the scanning process [(a)]



Microscale modelling:

- accurate model for the change of state and melting pool
- 4 states considered: powder, solid, liquid and gaseous

Macroscale modelling:

- simplified model without accurate computation of the change of state and melting pool
- 2 states considered: powder, solid

STAKES AT A MACROSCOPIC SCALE

- **thermo-mechanics:** thermal expansion, residual stresses, solidification of a layer
- **kinematics:** minimal execution time

(*a*) T. DebRoy, H. Wei, J. Zuback, T. Mukherjee, J. Elmer, J. Milewski, A. M. Beese, A. Wilson-Heid, A. De, and W. Zhang, *Additive manufacturing of metallic components–process, structure and properties*, Progress in Materials Science, 92 (2018), pp. 112–224.

Steady model

The whole source is switched on at once

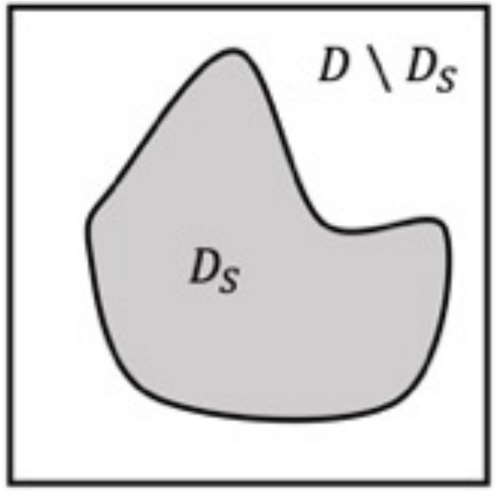
Dirac function of the path Γ: $q = P\chi_\Gamma$

Temperature equation:

$$\begin{cases} -\nabla(\lambda\nabla y) + \beta(y - y_{ini}) = P\chi_\Gamma, & x \in D \\ \lambda\partial_n y = 0, & x \in \partial D \end{cases}$$

Optimization problem:

$$\min L_F = |\Gamma| \quad s.t. \begin{cases} \forall x \in D_S, & y(x) \geq y_\phi \\ \forall x \in D \setminus D_S, & y(x) \leq y_{M,out} \\ \forall x \in D_S, & y(x) \leq y_{M,in} \end{cases}$$

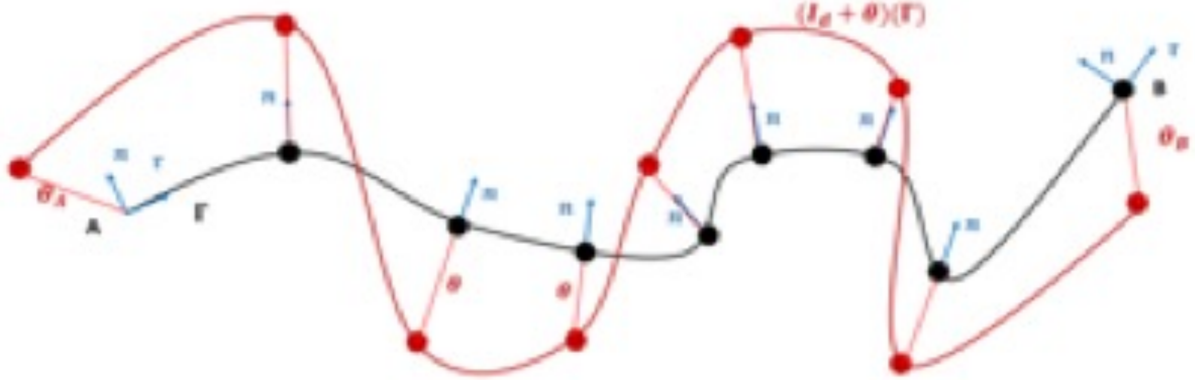


Overview

- Modelling assumptions
- Scanning path optimization
- Object's shape optimization
- Concurrent optimization

Optimization algorithm

Gradient computation: shape differentiation theory (Differentiate and then discretize)



Γ regular curve with chosen orientation, tangent τ, normal $\mathbf{n}$, curvature κ and endpoints A and B.

Shape derivative of $J(\Gamma) = \int_\Gamma f(s)ds$: $\quad DJ(\Gamma)(\theta) = \int_\Gamma (\partial_n f + \kappa f)\theta \cdot n ds + f(B)\theta(B) \cdot \tau(B) - f(A)\theta(A) \cdot (A)$

Gradient descent:

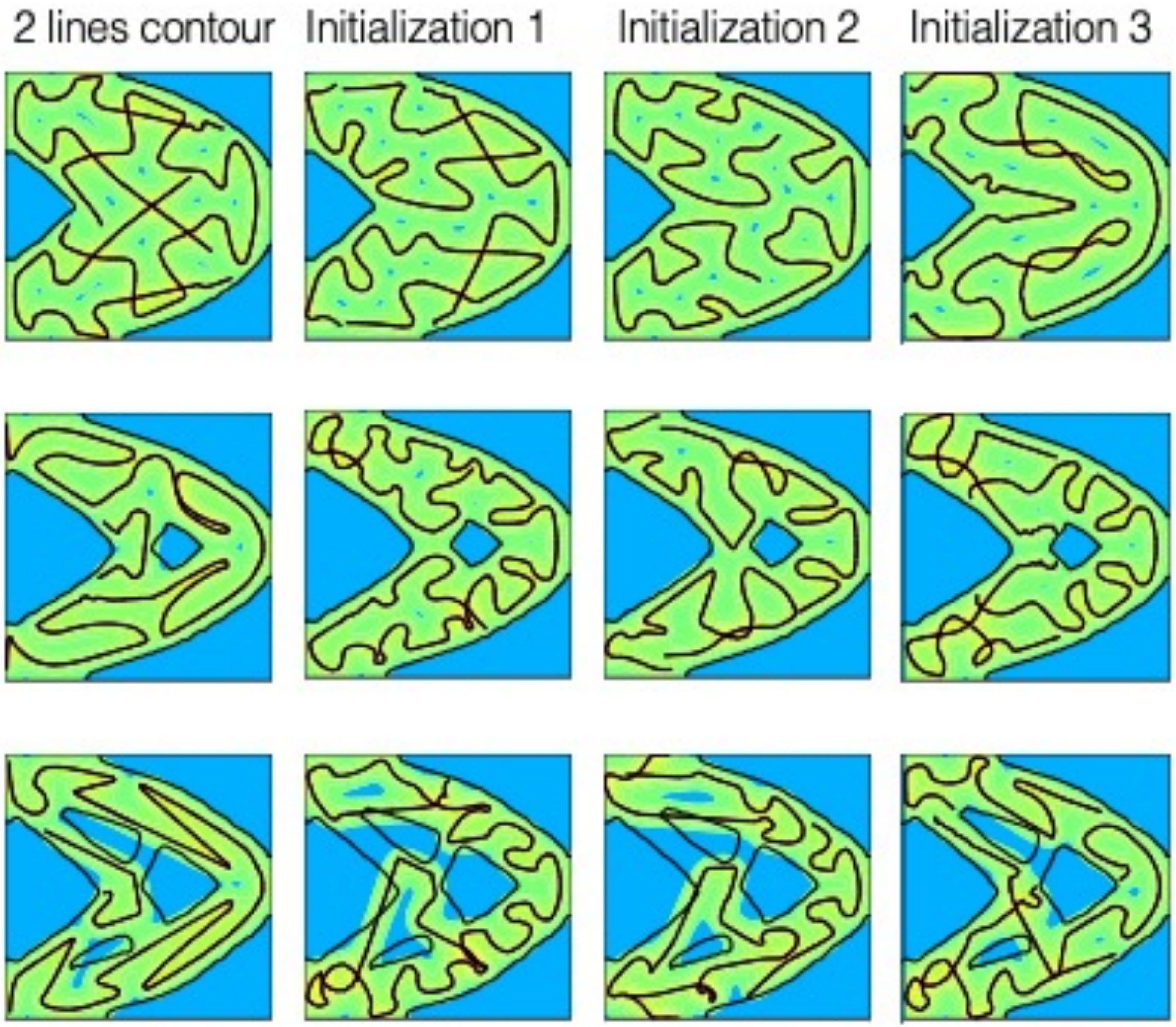
$$J(\Gamma^{n+1}) = J(\Gamma^n) + DJ(\Gamma^n)(\theta) + o(\theta)$$

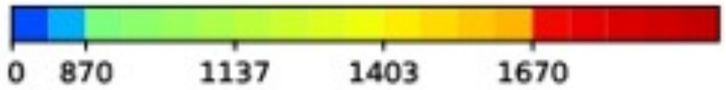
θ chosen such that $J(\Gamma^{n+1}) \leq J(\Gamma^n)$

Combined with an Augmented Lagrangian method to deal with the constraints

A. Henrot and M. Pierre, *Shape variation and optimization*, EMS Tracts in Mathematics, European Mathematical Society (EMS), Zürich, 2018.

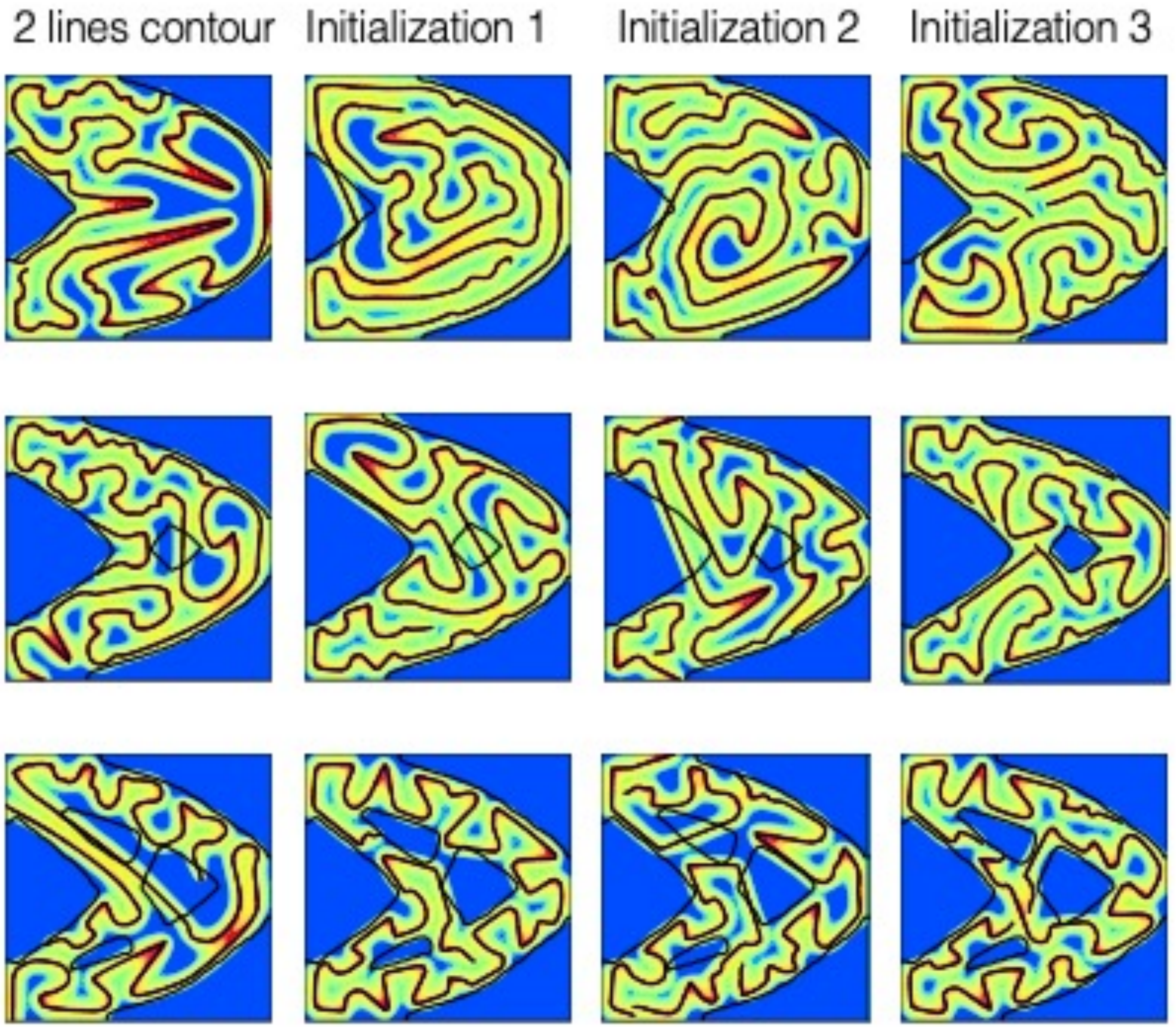
Different initializations – Aluminum ($\lambda = 130 W m^{-1} K^{-1}$)

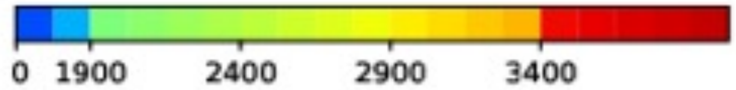




- Results really dependent on the initialization
- Correct adaptation to the shape if allowed by the conductivity => **shape thickness**

Different initializations – Titanium ($\lambda = 15 W m^{-1} K^{-1}$)





- Low conductivity complicates the optimization
- Results really dependent on the initialization
- Correct adaptation to the shape if allowed by the conductivity => **shape thickness**

Overview

- Modelling assumptions
- Scanning path optimization
- Part optimization
- Concurrent optimization

Part optimization

$$\min_{\Omega} Cply(\Omega) = \int_{\Omega} Ae(u) : e(u)\, dx \quad \text{such that} \quad \int_{\Omega} dx = V_{target}$$

$$\begin{cases} -div\big(A\epsilon(u)\big) = 0, & x \in \Omega, \\ A\epsilon(u) \cdot n = g, & x \in \partial\Omega_N, \\ A\epsilon(u) \cdot n = 0, & x \in \partial\Omega_F, \\ u = 0, & x \in \partial\Omega_D. \end{cases}$$

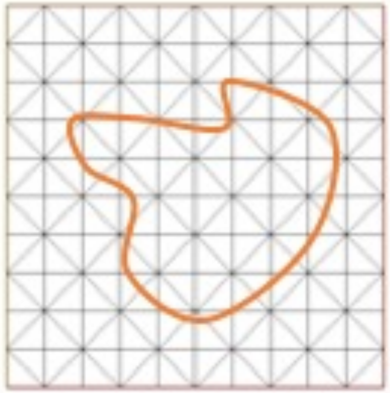
A, Hooke tensor $\qquad \epsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T)$

Gradient computation: shape differentiation theory (Differentiate and then discretize)

Combined with an Augmented Lagrangian method and a dichotomy to deal with the volume constraint

Numerical representation: level set [(a)]

$$\begin{cases} \psi(x) < 0, & x \in \Omega, \\ \psi(x) = 0, & x \in \partial\Omega, \\ \psi(x) > 0, & x \in \mathrm{D} \setminus \bar{\Omega}. \end{cases}$$



G. Allaire, F. Jouve, and A.-M. Toader, *Structural optimization using sensitivity analysis and a level-set method*, J. Comput. Phys., 194 (2004), pp. 363–393.

Results – cantilever

$V_{tar} = 1.1V^0$

Overview

- Modelling assumptions
- Scanning path optimization
- Partoptimization
- Concurrent optimization

Coupling part and path optimizations

$$\min_{\Omega,\Gamma} J(\Omega,\Gamma) = \underbrace{\int_\Omega Ae(u):e(u)\,dx}_{Cply} + \underbrace{\int_\Gamma ds}_{L_F}$$

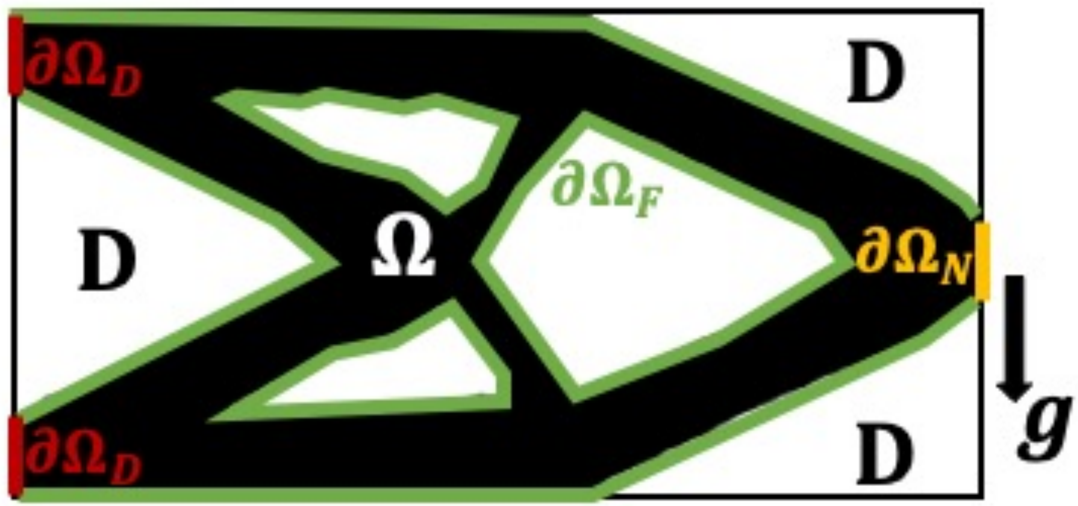
$$\begin{cases} \int_\Omega dx = V_{target} \\ C(\Omega,\Gamma) = C_\phi(\Omega,\Gamma) + C_{M,in}(\Omega,\Gamma) + C_{M,out}(\Omega,\Gamma) = 0 \end{cases}$$

$$C_\phi(\Omega,\Gamma) = \int_\Omega \left[(y_\phi - y)^+\right]^2 dx, \quad C_{M,in}(\Omega,\Gamma) = \int_\Omega \left[(y - y_{M,in})^+\right]^2 dx, \quad C_{M,out}(\Omega,\Gamma) = \int_{D\setminus\Omega} \left[(y - y_{M,out})^+\right]^2 dx$$

$$\begin{cases} -div(A\epsilon(u)) = 0, & x \in \Omega, \\ A\epsilon(u)\cdot n = g, & x \in \partial\Omega_N, \\ A\epsilon(u)\cdot n = 0, & x \in \partial\Omega_F, \\ u = 0, & x \in \partial\Omega_D. \end{cases}$$

A, Hooke tensor $\qquad \epsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T)$

$$\begin{cases} -\nabla(\lambda\nabla y) + \beta(y - y_{ini}) = P\chi_\Gamma, x \in D, \\ \lambda\partial_n y = 0, \qquad x \in \partial D. \end{cases}$$



Derivatives with respect to the shape and path

$$D_\Omega Cply(\Omega,\Gamma)(\theta_\Omega) = \int_{\partial\Omega_F} (-Ae(u){:}\,e(u))\theta_\Omega \cdot n_\Omega \, ds \qquad D_\Omega L_F(\Omega,\Gamma)(\theta_\Omega) = 0$$

$$D_\Omega V(\Omega,\Gamma)(\theta_\Omega) = \int_{\partial\Omega_F} \theta_\Omega \cdot n_\Omega \, ds$$

$$D_\Omega C(\Omega,\Gamma)(\theta_\Omega) = \int_{\partial\Omega_F} \left((y_\phi - y)^+ + (y - y_{M,D_S})^+ - (y - y_{M,D\setminus D_S})^+ \right) \theta_\Omega \cdot n_\Omega \, ds$$

$$D_\Gamma Cply(\Omega,\Gamma)(\theta_\Gamma) = 0 \qquad D_\Gamma L_F(\Omega,\Gamma)(\theta_\Gamma) = \int_\Gamma \kappa\theta_\Gamma \cdot n_\Gamma \, ds + \theta_\Gamma(B)\cdot\tau_\Gamma(B) - \theta_\Gamma(A)\cdot\tau_\Gamma(A)$$

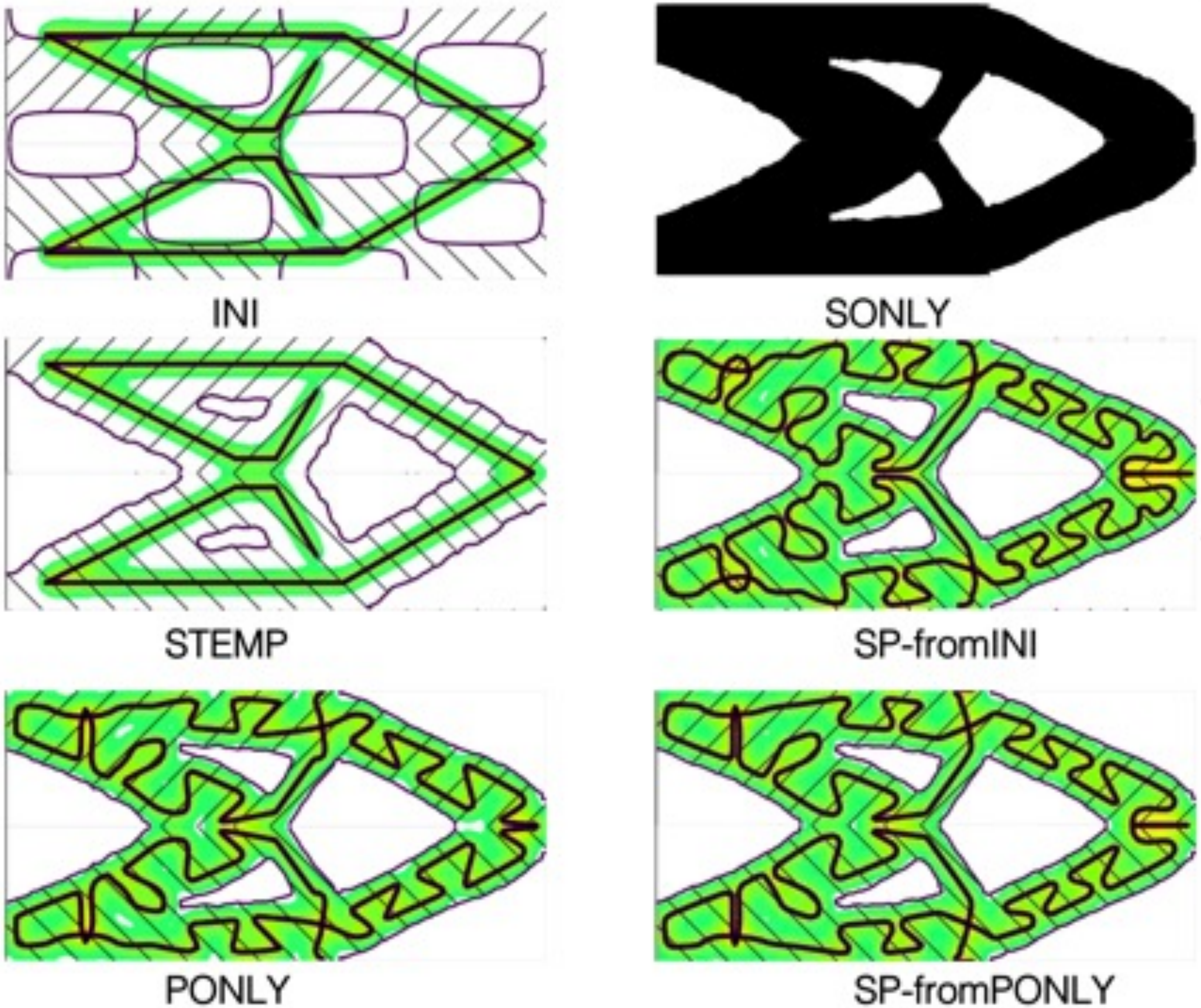
$$D_\Gamma V(\Omega,\Gamma)(\Omega,\Gamma)(\theta_\Gamma) = 0$$

$$D_\Gamma C(\Omega,\Gamma)(\theta_\Gamma) = \int_\Gamma -P(\partial_n p + \kappa p)\theta_\Gamma \cdot n_\Gamma ds - Pp(B)\theta_\Gamma(B)\cdot\tau_\Gamma(B) + Pp(A)\theta_\Gamma(A)\cdot\tau_\Gamma(A)$$

With $p \in H^1(D, \mathbb{R}^2)$ solution of

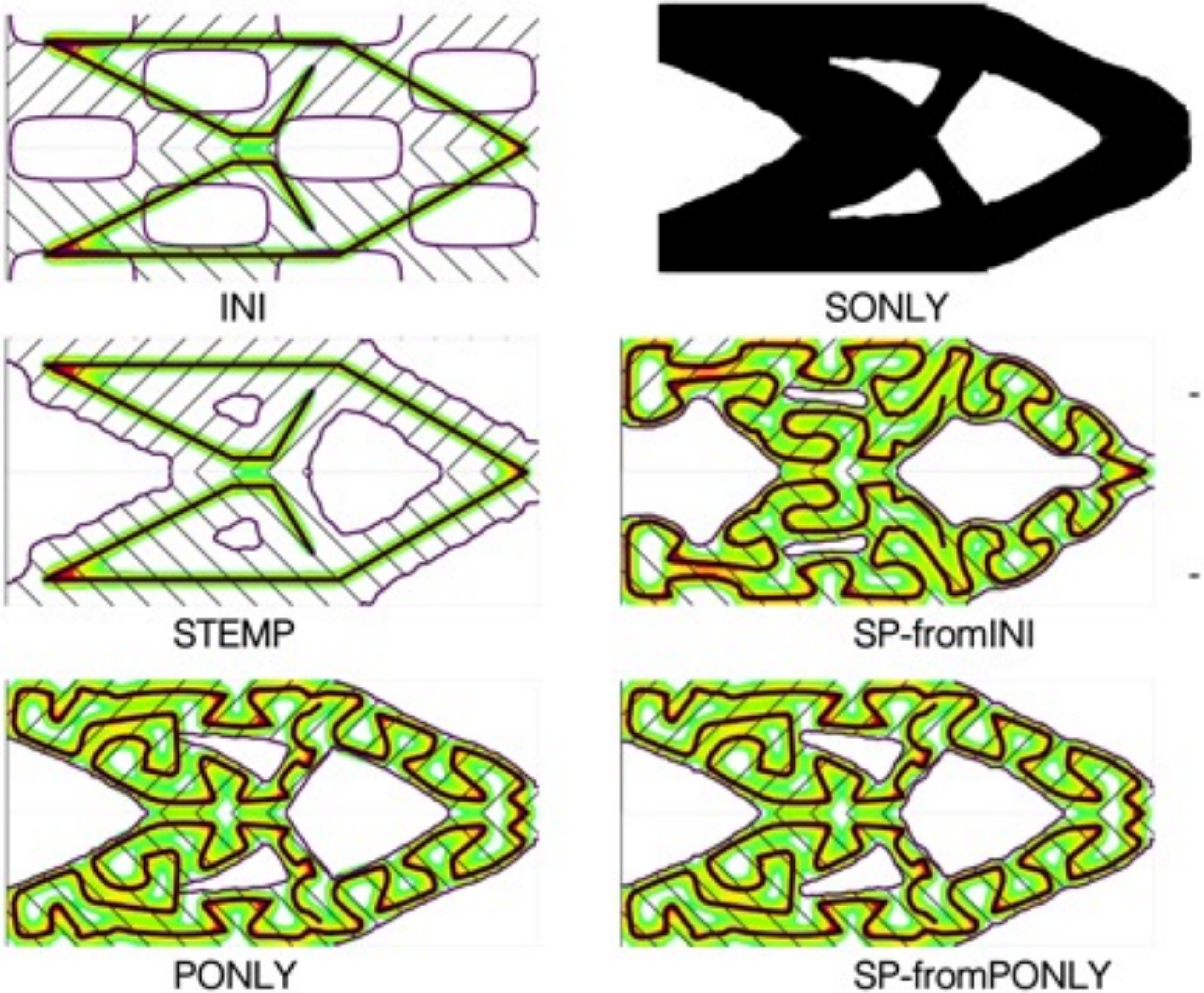
$$\begin{cases} -\nabla(\lambda\nabla p) + \beta p = 2\left[(y_\phi - y)^+ \mathbb{1}_{D_S} - (y - y_{M,in})^+ \mathbb{1}_{D_S} - (y - y_{M,out})^+ \mathbb{1}_{D\setminus D_S} \right], & x \in D \\ \lambda\partial_n p = 0, & x \in \partial D \end{cases}$$

Results – cantilever – aluminum – $V_{tar} = 1.1V^0$



- Intuition of a link between part's thickness and conductivity => Omega-shape path design
- Boundary adaptation to the path

Results – cantilever – titanium – $V_{tar} = 1.1V^0$



- Intuition of a link between part's thickness and conductivity => Omega-shape path design
  => Wave-shape path design
- Boundary adaptation to the path

Conclusions and perspectives

Concurrent path and shape optimization

- Confirms the importance of the link between thickness and conductivity
- Boundary adapted to the path

Perspectives

- Further test the optimization on other test cases, a complexified model
- Include path topology modifications into the concurrent optimization
- Adapt the constraint to reality: advantage the phase constraint, define “steady state” constraints modelling transient quantities (kinematics) to take benefit from the very easy resolution process and shape optimization theory
- Optimize in the transient (general) model
- Add the resolution of a mechanical problem (full resolution or inherent strain method)
- 3D considerations

References

References on which this talk is based on

- M.Boissier, G.Allaire, C.Tournier, *Additive Manufacturing Scanning Paths Optimization Using Shape Optimization Tools*, SMO, 61:6, pp. 2437-2466 (2020)

- M.Boissier, G.Allaire, C.Tournier, *Concurrent shape optimization of the part and scanning path for additive manufacturing*, submitted (2021). ⟨hal-03124075⟩.

- M.Boissier, Coupling structural optimization and trajectory optimization methods in additive manufacturing, PhD thesis (2020)

Further reference for path optimization considering unsteadiness :

- M.Boissier, G.Allaire, C.Tournier, *Time dependent scanning path optimization for the powder bed fusion additive manufacturing process*, submitted (2021). ⟨hal-03202102⟩.