Stress-based topology optimization of compliant mechanisms design using geometrical and material nonlinearities

https://doi.org/10.1007/s00158-019-02484-4

De Leon, Daniel M., Gonçalves, Juliano F.*, de Souza, Carlos E.**

Federal University of Rio Grande do Sul (UFRGS) - Brazil
*University of Sao Paulo (USP) - Brazil
**Federal University of Santa Maria (UFSM) - Brazil

Introduction

This work addresses the problem of computing stress in problems dealing with large displacements for compliant mechanisms:

- the goal is to investigate the effect of using nonlinear materials (a policonvex model) for stress-based optimization formulations, and not only for equilibrium stabilization;
- formulations using linear and nonlinear materials and linear and nonlinear equilibrium are compared;
- a benchmark problem is applied in this presentation (two examples are used in the original work);

Constitutive Equations

In this work a Neo-Hookean model is applied (unless clear stated):

$$\Psi = \frac{1}{2}\lambda\left(\ln J\right)^2 + \frac{\mu}{2}\left(\mathrm{tr}(\mathbf{C}) - 3\right) - \mu \ln J, \tag{1}$$

where $J = \det(\mathbf{F})$ is the Jacobian of the deformation gradient, $\mathrm{tr}(\mathbf{C})$ is the trace of the right Cauchy-Green tensor.

$$\mathbf{G} = \frac{1}{2}\left(\mathbf{C} - \boldsymbol{\delta}\right) \quad \rightarrow \quad \mathbf{S} = \frac{\partial \Psi}{\partial \mathbf{G}} \quad \mathbf{D} = \frac{\partial^2 \Psi}{\partial \mathbf{G} \partial \mathbf{G}} \tag{2}$$

The material interpolation scheme is written as:

$$E_i = E_{min} + \bar{\tilde{\rho}}_i^{\,k}\left(E_0 - E_{min}\right) \tag{3}$$

Optimization problem for compliant mechanism design

The optimization problem is defined as:

$$
\begin{aligned}
\min_{\boldsymbol{\rho}} &: f(\boldsymbol{\rho}) = \mathbf{l}^T \mathbf{u} \\
\text{s.t.} &: \mathbf{r}(\mathbf{u}(\boldsymbol{\rho})) = \mathbf{0} \\
&: f_v(\boldsymbol{\rho}) = \frac{\sum_{i \in \mathbb{N}_e} \bar{\tilde{\rho}}_i(\rho) v_i}{V} \leqslant V^* \\
&: f_s(\boldsymbol{\rho}) = \max(\boldsymbol{\sigma}(\mathbf{u}(\boldsymbol{\rho}))) \leqslant \sigma^* \\
&: 0 \leqslant \rho_i \leqslant 1, \quad i \in \mathbb{N}_e
\end{aligned} \tag{4}
$$

$$
\tilde{\rho}_i = \frac{\sum_{j \in \mathbb{N}_{e,i}} w(x_j) v_j \rho_j}{\sum_{j \in \mathbb{N}_{e,i}} w(x_j) v_j} \qquad w(x_j) = \frac{R - |x_j - x_i|}{R} \tag{5}
$$

$$
\bar{\tilde{\rho}}_i = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{\rho}_i - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))} \tag{6}
$$

Optimization problem for compliant mechanism design

The equivalent stress measure (von Mises) is computed from Cauchy stress:

$$\tau = J^{-1}\mathbf{FSF}^T \rightarrow \sigma_{VM} = \left(\tau^T V \tau\right)^{1/2} \tag{7}$$

Stress relaxation is computed as:

$$\sigma_i = \bar{\tilde{\rho}}_i{}^q \sigma_{VM,i} \tag{8}$$

For the stress constraint, a global p-norm (Le et al., 2010) is applied:

$$\sigma_{pn} = c\left(\sum_{i \in \mathbb{N}_\sigma} v_i {\sigma_i}^p\right)^{\frac{1}{p}} \qquad \rightarrow \qquad p = 8 \tag{9}$$

$$\max(\boldsymbol{\sigma}) \approx \sigma_{pn} \rightarrow f_s = \sigma_{pn} \tag{10}$$

Numerical Examples

In this presentation, the displacement inverter is used as test case:

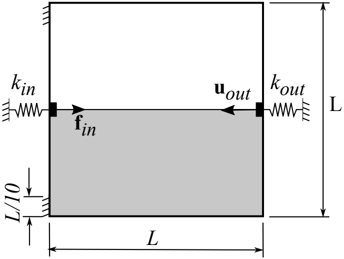


Figure: Design domain definition and boundary conditions. Dimension and spring values are depicted in the paper.

Numerical Examples

Here the stress constraint is not active ($\sigma^* = \infty$):

(a) $f = -2.89\ \mu m$
$\sigma_{max} = 2.94$ GPa

(b) $f = -3.46\ \mu m$
$\sigma_{max} = 4.67$ GPa

Figure: Optimized design topologies using (a) linear and (b) nonlinear framework. The cost functions values (f) are in μm. The input force is $f_{in} = 5 \times 10^{-3}$ N.

Linear x nonlinear geometric problem

The topologies shown previously were run replacing FE framework (first topology) and material (second topology):

Table: Equilibrium analyses of optimized mechanisms.

Inverter mechanism			
FE solution	material	f [μm]	max. stress [GPa]
(a) linear	linear	-2.89	2.94
NR	linear	-3.03	3.62
(b) NR	Neo-Hookean	-3.46	4.67
NR	linear	-3.46	4.68

Effect of Stress Constraint

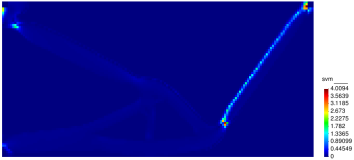
Now the stress constraint is active:

(a) $f = -3.42\ \mu m$
$\sigma^* = 4.00$ GPa

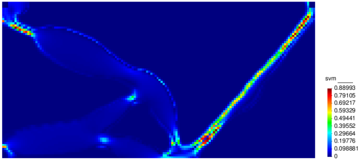
(b) $f = -3.09\ \mu m$
$\sigma^* = 900$ MPa

Figure: Optimized design topologies for different levels of stress (σ^*). The volume constraint of 20% is active in all designs.

Effect of Stress Constraint



(a) $f = -3.42\ \mu m$
$\sigma^* = 4.00$ GPa



(b) $f = -3.09\ \mu m$
$\sigma^* = 900$ MPa

Figure: Equivalent von Mises plots for different levels stress thresholds (σ^*). The volume constraint of 20% is active in all designs.

Nonlinear material in stress constraint problems

The next investigation is on the effect of using a nonlinear material in stress-constrained formulation

- here the topology obtained with the nonlinear model (first line in the table) is analyzed replacing the nonlinear material by a linear model (second line) and replaced with linear model for geometry and material (third line):

Table: Comparison for the stress constrained inverter.

Inverter mechanism			
FE solution	material	f [μm]	max. stress [GPa]
NR	Neo-Hookean	-3.09	0.89
NR	linear	-2.44	1.70
linear	linear	-2.46	1.82

It is interesting to notice that, despite the extra stiffness given by the nonlinear model, the optimized design using the Neo-Hookean model is kinematically superior.

Concluding Remarks

- by examples shown in this research, it is clear that a material nonlinear assumption is not effective for problems that do not take stress into consideration. However, in stress-based problems a policonvex model plays an important role not only to convergence stability;
- a more detailed explanation on this research can be found in **Stress-based topology optimization of compliant mechanisms design using geometrical and material nonlinearities**. Structural and Multidisciplinary Optimization. https://doi.org/10.1007/s00158-019-02484-4

Financial Support

- Daniel M. De Leon is grateful to Research Support Foundation of the State of Rio Grande do Sul (FAPERGS), process number 19/2551-0001255-1;
- Juliano F. Gonçalves received support from the RCGI (Research Centre for Gas Innovation) hosted by the University of Sao Paulo (USP) and sponsored by FAPESP (Sao Paulo Research Fundation (2014/50279-4)) and Shell Brasil.

Bibliography cited in this presentation

- Le C, Norato J, Bruns TE, Ha C, Tortorelli DA (2010) Stress-based topology optimization for continua. Struct Multidiscip Optim.
- Bruns TE, Tortorelli DA (2001) Topology optimization of non-linear elastic structures and compliant mechanisms. Comput Methods Appl Mech Eng. https://doi.org/10.1016/S0045-7825(00)00278-4
- De Leon DM, Alexandersen J, Fonseca JSO, Sigmund O (2015). Stress-constrained topology optimization for compliant mechanism design. Structural and Multidisciplinary Optimization. https://doi.org/10.1007/s00158-015-1279-z.

Thanks

Thanks for Watching!