

# Three-dimensional topology optimization of thermal-fluid-structural problems for cooling system design

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# Contents

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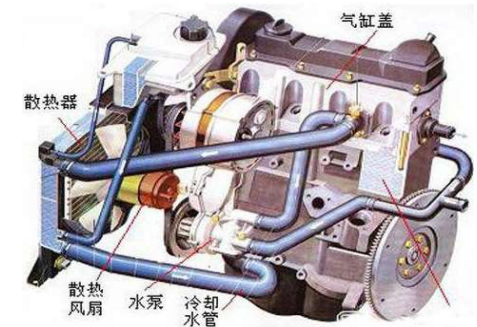
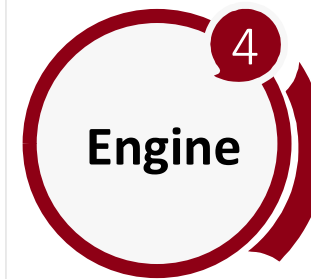
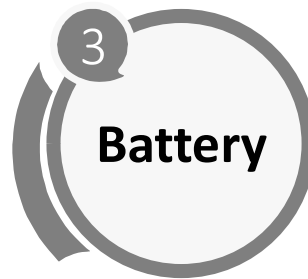
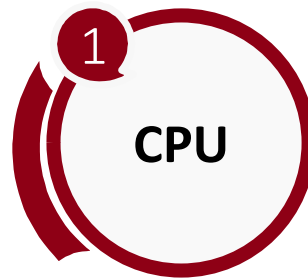
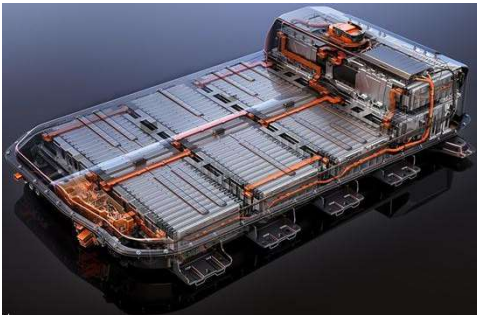
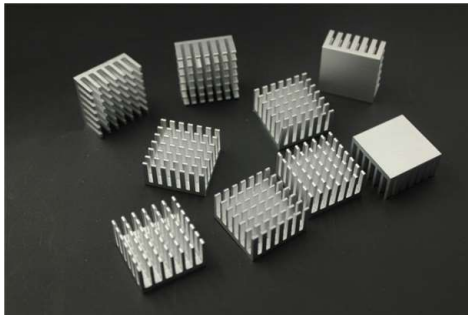


- I. Back Ground**
- II. Basic methods**
- III. Fluid-thermal design**
- IV. Fluid-thermal-structural design**
- V. Conclusion**

# Back Ground



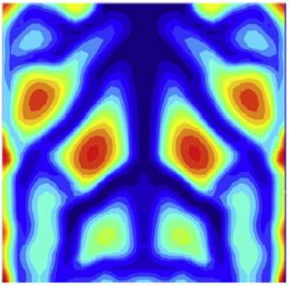
## Liquid-cooled heat sink



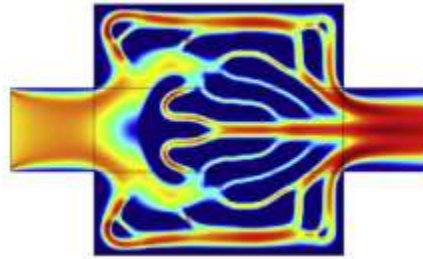
# Back Ground



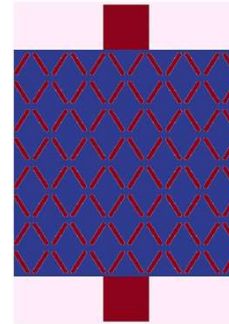
## Topology optimization for liquid-cooled heat sink



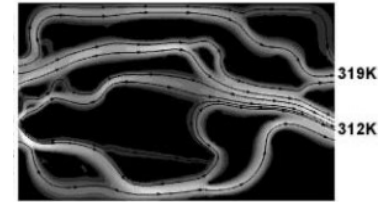
Adriano et al SIMP



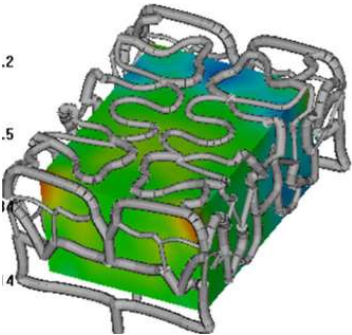
Yaji et al Level set



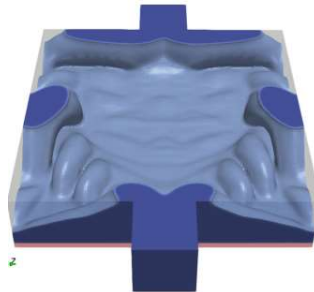
Yu et al MMC



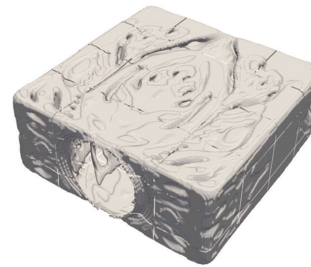
Kontoleonos et al Turbulent



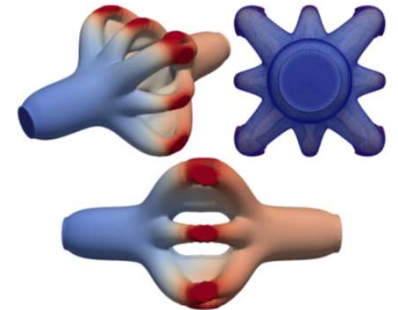
Li et al BEM



Dilgen et al Turbulent



Yaji et al Unsteady flow



Feppon et al Level set

# Back Ground

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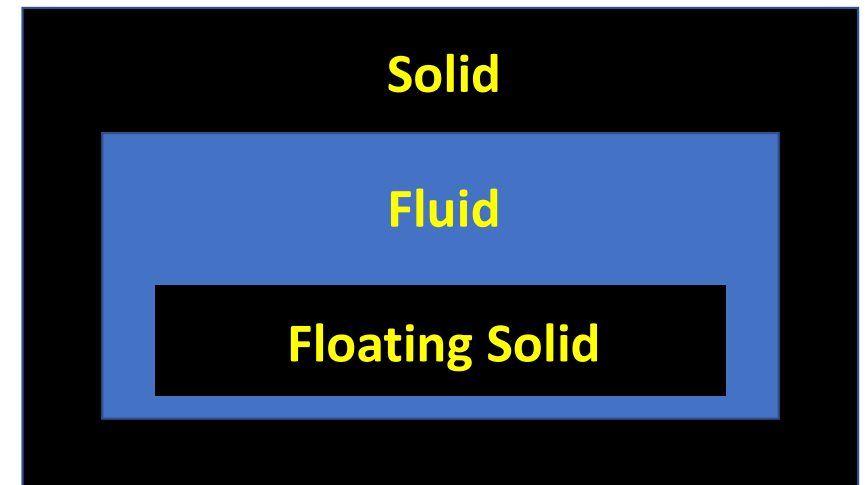
## Motivations & highlights

- **Fluid-thermal-structural design**

Not only for the load bearing ability, but also to avoid impractical structure (floating solid).

- **Choice of Darcy number for 3D case**

The Darcy number which can be well used in fluid topology optimization may cause a problem in 3D fluid-thermal design.



# Contents

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**I. Back Ground**

**II. Basic methods**

**III. Fluid-thermal design**

**IV. Fluid-thermal-structural design**

**V. Conclusion**

# Basic methods

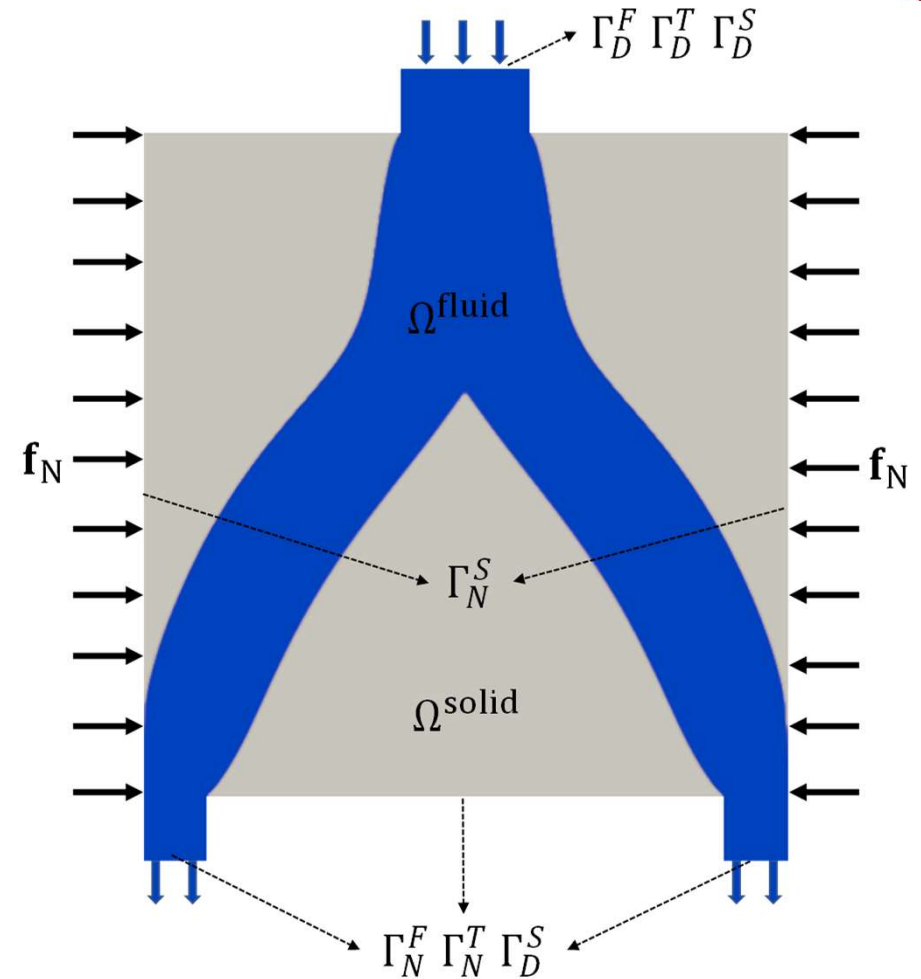


## Assumptions:

- Laminar flow
- Newtonian fluid
- Ignoring viscous dissipation

## Boundary conditions:

- Constant velocity and temperature of fluid in inlets
- Zero fluid pressure in outlets
- Uniform pressure on the surface



# Basic methods



## Governing equations:

### N-S equations

$$\begin{cases} -\nabla \cdot \mathbf{u} = 0 & \text{in } \Omega \\ \rho(\mathbf{u} \cdot \nabla) \mathbf{u} - \eta \nabla \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \nabla p - \alpha_f \mathbf{u} = 0 & \text{in } \Omega \\ \mathbf{u} = \mathbf{u}_D & \text{on } \Gamma_D^F \\ \left[ \eta (\nabla \mathbf{u} + \nabla \mathbf{u}^T) - p \mathbf{I} \right] \cdot \mathbf{n} = \mathbf{g}_N & \text{on } \Gamma_N^F \end{cases}$$

$\mathbf{u}$ : velocity

$p$ : pressure

$\eta$ : dynamic viscosity

$\rho$ : fluid density

### Heat transfer equation

$$\begin{cases} \rho c (\mathbf{u} \cdot \nabla T) = \nabla \cdot (k \nabla T) + Q & \text{in } \Omega \\ T = T_D & \text{on } \Gamma_D^T \\ k \nabla T \cdot \mathbf{n} = q_N & \text{on } \Gamma_N^T \end{cases}$$

$T$ : temperature

$k$ : thermal conductivity

$c$ : specific heat capacity

$Q$ : heat source

### Linear elasticity equations

$$\begin{cases} \nabla \cdot \left[ \mu (\nabla \mathbf{D} + \nabla \mathbf{D}^T) + \lambda (\nabla \cdot \mathbf{D}) \mathbf{I} - \alpha_T (3\lambda + 2\mu) (T - T_{ref}) \mathbf{I} \right] = 0 & \text{in } \Omega \\ \mathbf{D} = \mathbf{D}_D & \text{on } \Gamma_D^S \\ \left[ \mu (\nabla \mathbf{D} + \nabla \mathbf{D}^T) + \lambda (\nabla \cdot \mathbf{D}) \mathbf{I} - \alpha_T (3\lambda + 2\mu) (T - T_{ref}) \mathbf{I} \right] \cdot \mathbf{n} = \mathbf{f}_N & \text{on } \Gamma_N^S \end{cases}$$

$\mathbf{D}$ : solid displacement

$\mu, \lambda$ : Lamé parameters

$\alpha_T$ : thermal expansion coefficient

$\mathbf{I}$ : second order unit tensor



# Basic methods



## SIMP interpolation of material properties

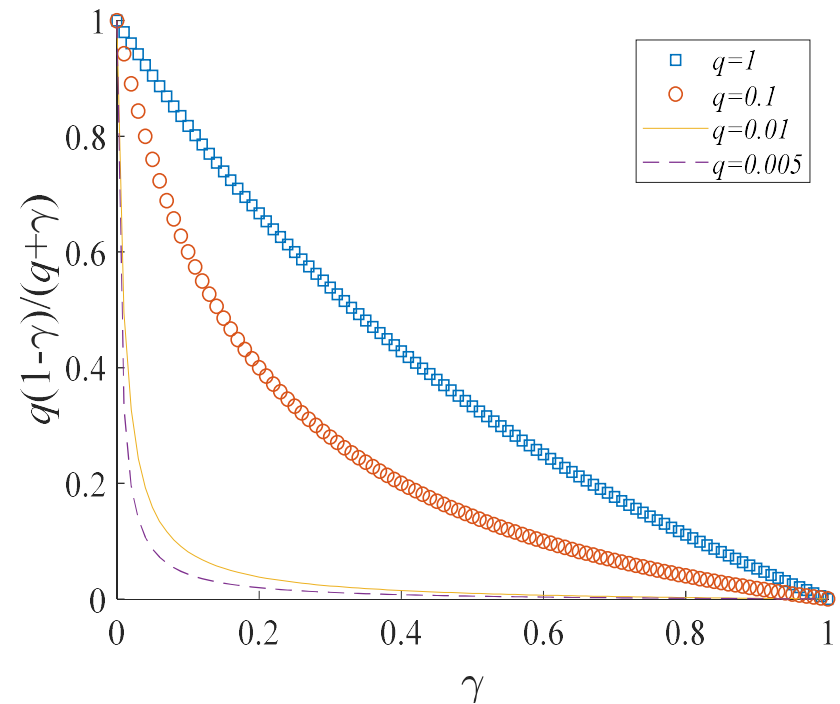
**Pseudo density:** solid  $0 \leq \gamma \leq 1$  fluid

$$\rho(\mathbf{u} \cdot \nabla) \mathbf{u} - \eta \nabla \cdot (\nabla \mathbf{u} + \nabla \mathbf{u}^T) + \nabla p - \alpha_f \mathbf{u} = 0$$

**Inverse permeability:**  $\alpha_f = \alpha_{\max} \frac{q_\alpha (1 - \gamma)}{q_\alpha + \gamma}$

**Heat conductivity:**  $k = k_s + (k_f - k_s) \gamma \frac{1 + q_k}{q_k + \gamma}$

**Elastic modulus:**  $E = E_{\max} \frac{q_E (1 - \gamma)}{q_E + \gamma}$



# Basic methods



## Optimization model

Find  $\gamma$

To minimize  $\Psi = \int_{\Omega} T d\Omega$  (mean temperature)

Subjected to  $J = -\int_{\Gamma} (p + \mathbf{u} \cdot \mathbf{u}/2) \mathbf{u} \cdot \mathbf{n} d\Gamma \leq \bar{J}$  (power dissipation)

$V = \int_{\Omega} \gamma d\Omega \leq \bar{V}$  (volume of fluid region)

$S = -\int_{\Gamma_{force}} (\mathbf{D} \cdot \mathbf{n})^2 d\Gamma \leq \bar{S}$  (deformation)

Governing equations

- Sensitivities are calculated by solving continuous adjoint equations.
- Performed with OpenFOAM
- Code has been uploaded to [github.com/MTopOpt/MTO](https://github.com/MTopOpt/MTO)

# Contents

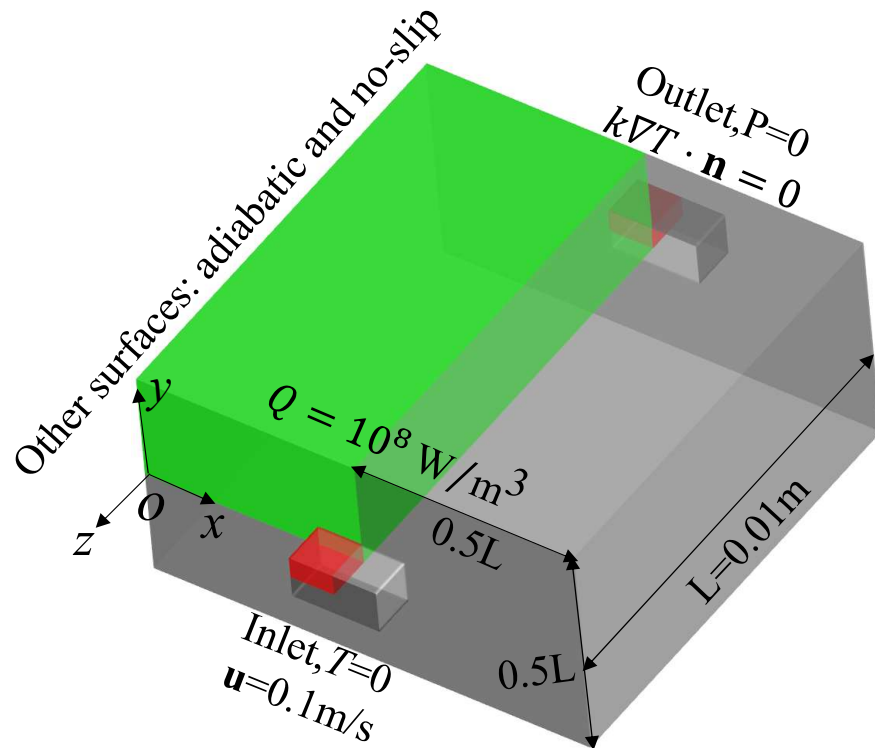
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- I. Back Ground
- II. Basic methods
- III. **Fluid-thermal design**
- IV. Fluid-thermal-structural design
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# Fluid-thermal design

## Model



Power dissipation

$$J \leq \bar{J} \leftarrow 4 \text{ times of straight channel}$$

Volume

$$V \leq \bar{V} \leftarrow 20\% \text{ of total volume}$$

- Quarter-symmetric model
- Materials: water and aluminum
- Uniform heat source in the domain

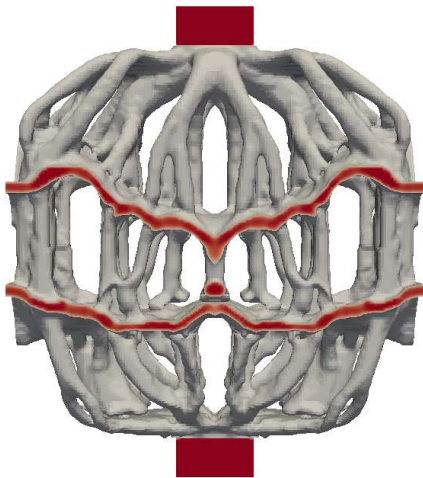
# Fluid-thermal design

## Inverse permeability in solid

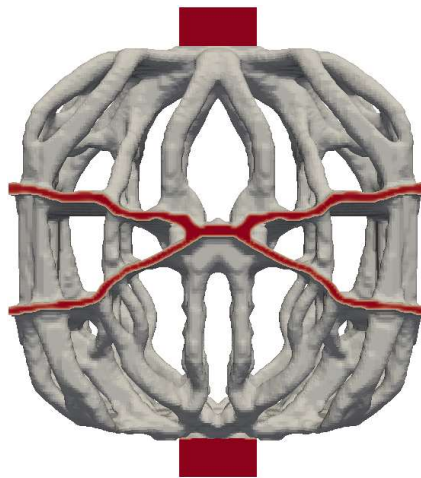
$$\alpha_{\max} = \frac{\eta}{l^2 Da}$$

Viscosity  $\eta$   
 Characteristic length  $l$   
 Darcy number  $Da$

$Da = 10^{-5}$



$Da = 10^{-6}$



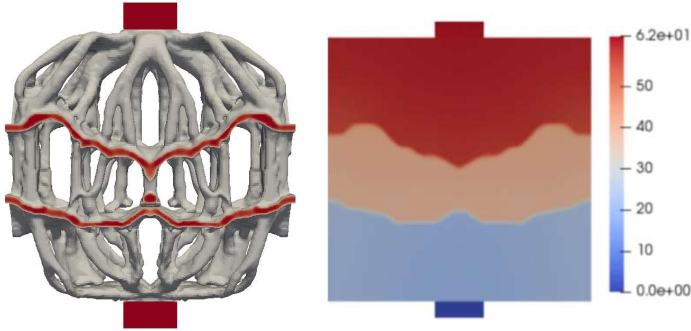
$Da = 10^{-7}$



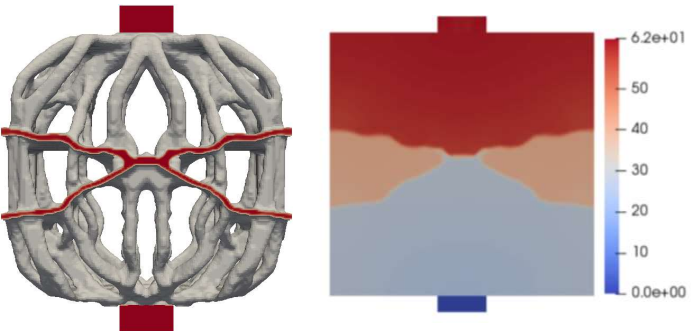
With higher values of  $Da$ , wall-like fluid regions are observed.

# Fluid-thermal design

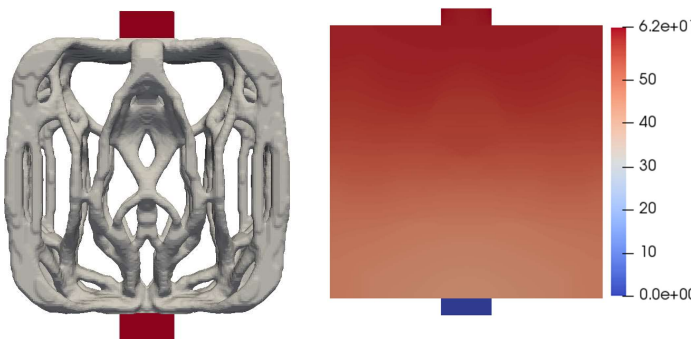
$$Da = 10^{-5}$$



$$Da = 10^{-6}$$



$$Da = 10^{-7}$$



- Wall-like fluid regions divide the domain into subdomains
  - Each subdomain has uniform temperature with low gradient
- ↓
- Low heat conduction

# Fluid-thermal design

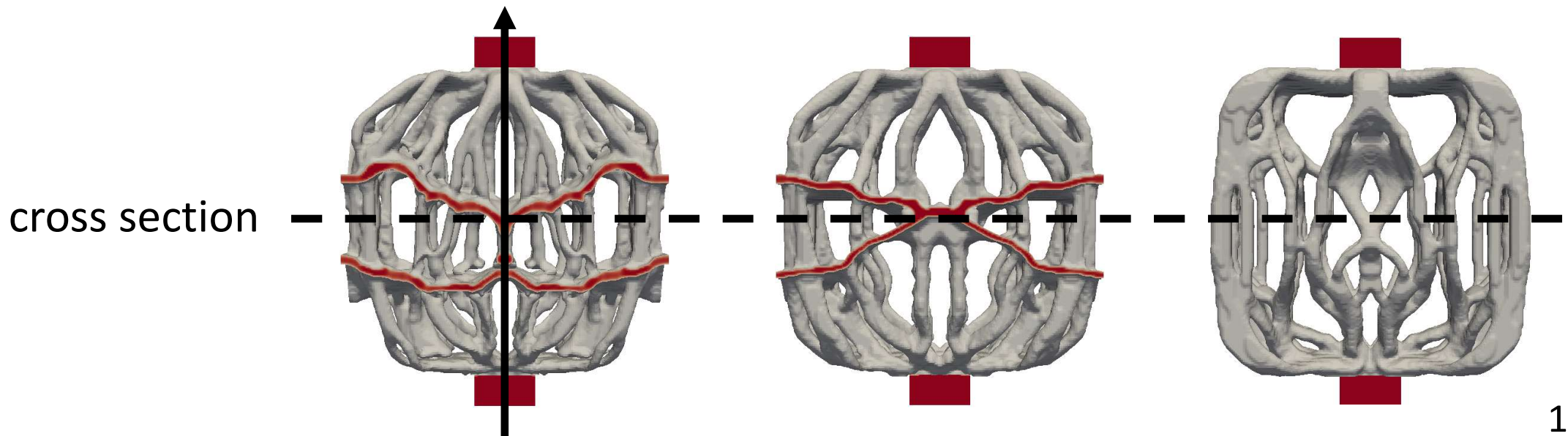
Define  $f = \frac{|k \nabla T \cdot \mathbf{n}|}{|k \nabla T \cdot \mathbf{n}| + |\rho c T \mathbf{u} \cdot \mathbf{n}|}$

Heat conduction  $\rightarrow$   $|k \nabla T \cdot \mathbf{n}|$

Heat convection  $\rightarrow$   $|\rho c T \mathbf{u} \cdot \mathbf{n}|$

$\begin{cases} f \rightarrow 0, & \text{in fluid region} \\ f \rightarrow 1, & \text{in solid region} \end{cases}$

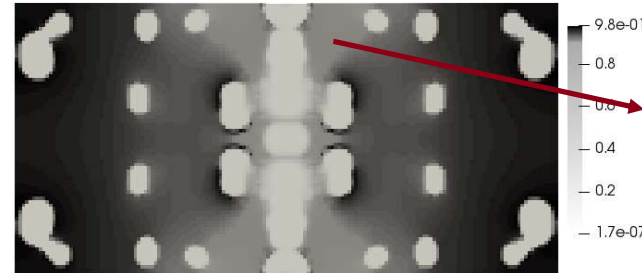
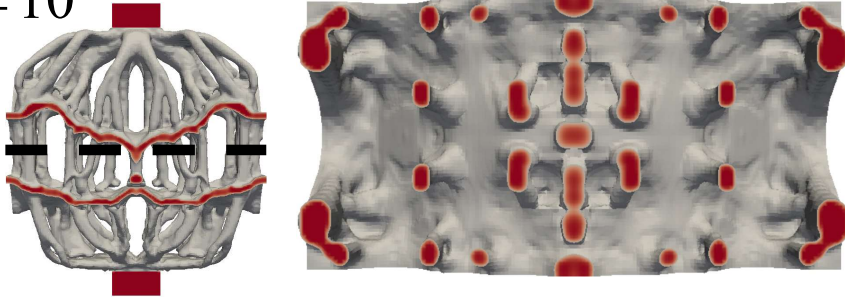
$\mathbf{n}$ : direction from inlet to outlet.





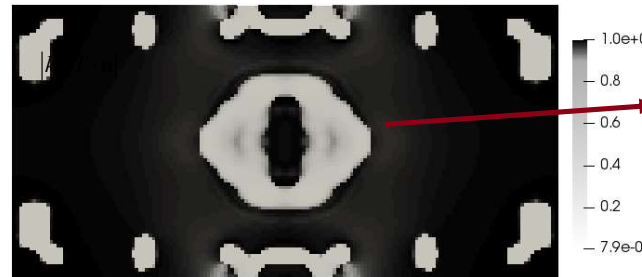
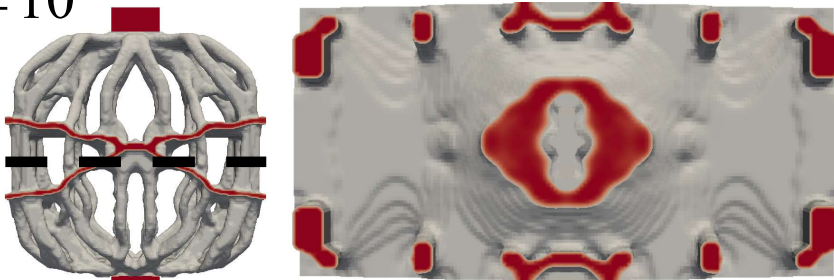
# Fluid-thermal design

$Da = 10^{-5}$



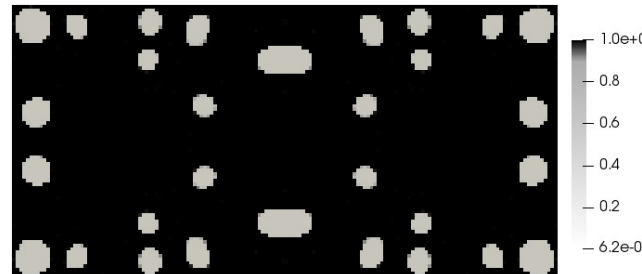
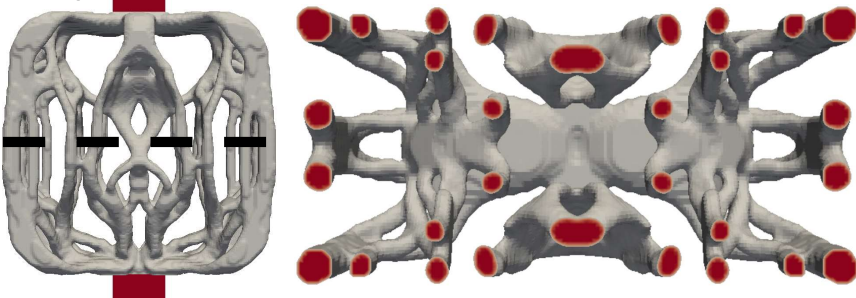
$$|\rho c T \mathbf{u} \cdot \mathbf{n}| \approx 0.2 |k \nabla T \cdot \mathbf{n}|$$

$Da = 10^{-6}$



$$|\rho c T \mathbf{u} \cdot \mathbf{n}| \approx 0.1 |k \nabla T \cdot \mathbf{n}|$$

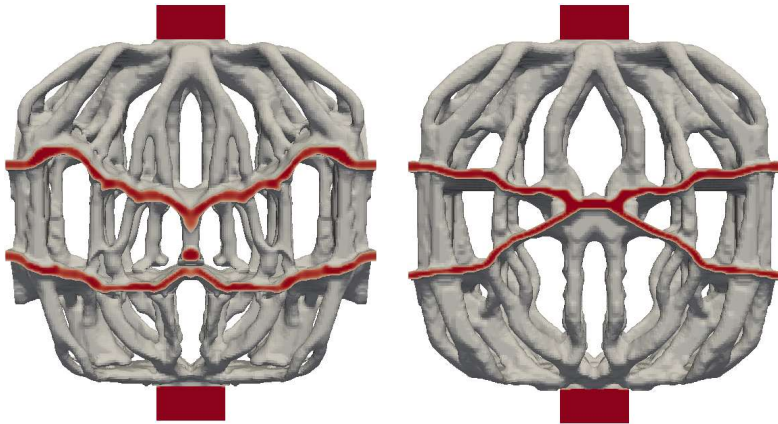
$Da = 10^{-7}$



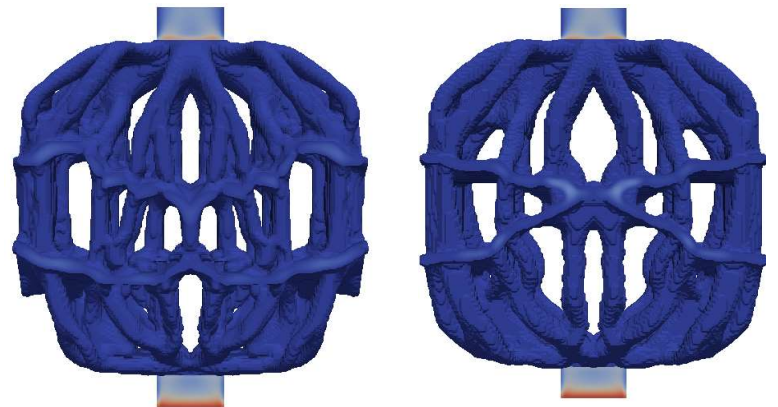


# Fluid-thermal design

Pseudo density field



Region of  $|\mathbf{u}| > |\mathbf{u}_{\text{inlet}}|/100$



- Low velocity  $\Rightarrow$  convection  $\ll$  conduction
- In the thermal-fluid topology optimization, we expect a better estimation for the  $\alpha_{max}$  than the old one which is used in pure fluid problem. Thermal properties must be considered.
- In the rest tests, we always set  $Da=10^{-7}$
- The wall-like fluid regions are meant to absorb the fluid with heat from solid region.



# Contents

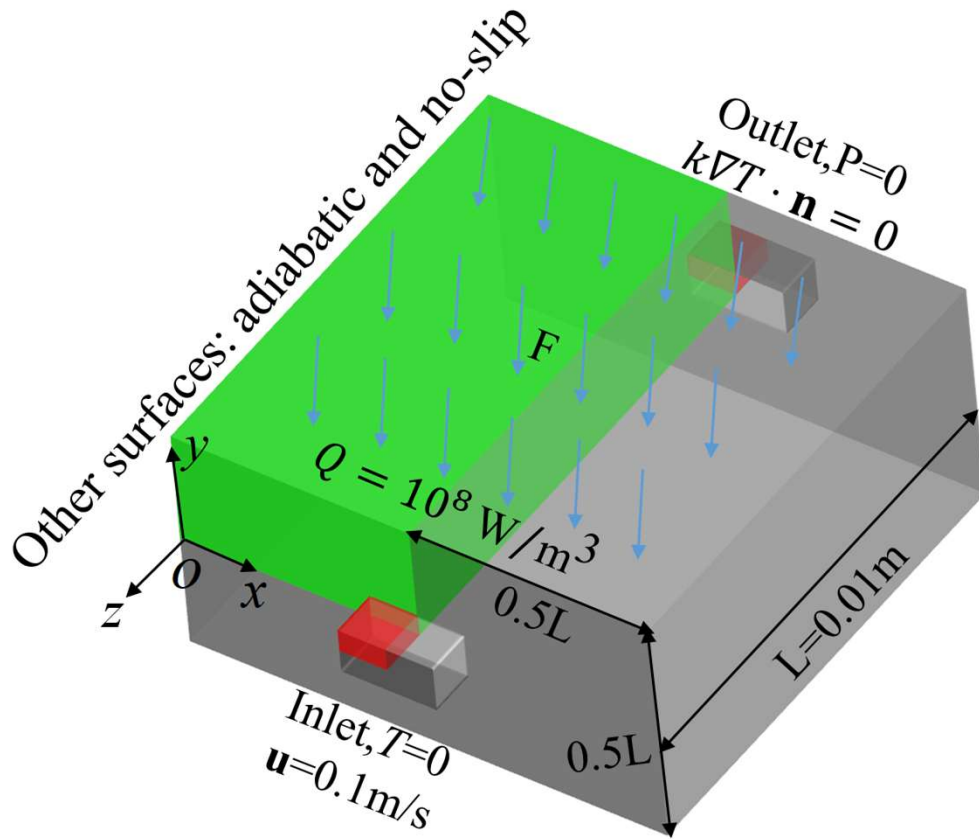
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- I. Back Ground
- II. Basic methods
- III. Fluid-thermal design
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# Fluid-thermal-structural design

## Model



■ Total force  $F=30\text{N}$

■ Deformation constraint

$$S = -\int_{\Gamma_{force}} (\mathbf{D} \cdot \mathbf{n})^2 d\Gamma \leq \bar{S}$$

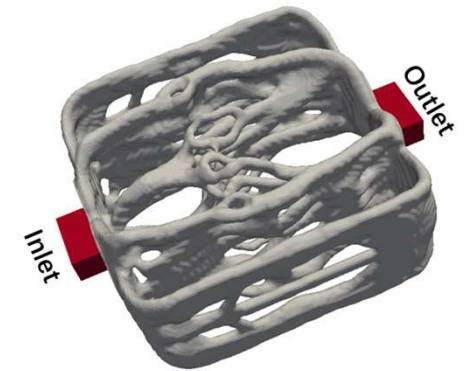
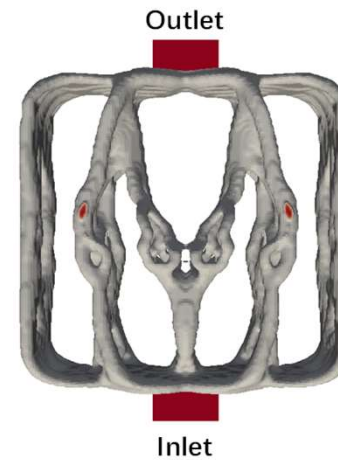
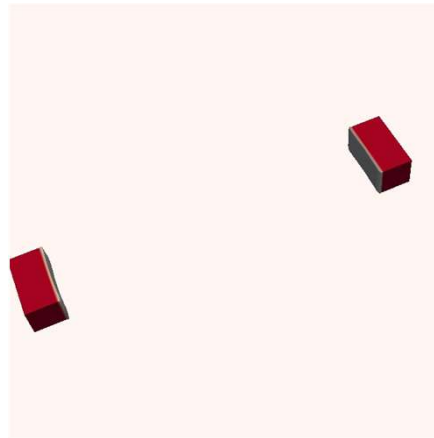
■ Elastic equation with **thermal expansion**

$$\nabla \cdot [\mu (\nabla \mathbf{D} + \nabla \mathbf{D}^T) + \lambda (\nabla \cdot \mathbf{D}) \mathbf{I}] = \nabla \cdot \alpha_T (3\lambda + 2\mu) (T - T_{ref}) \mathbf{I}$$

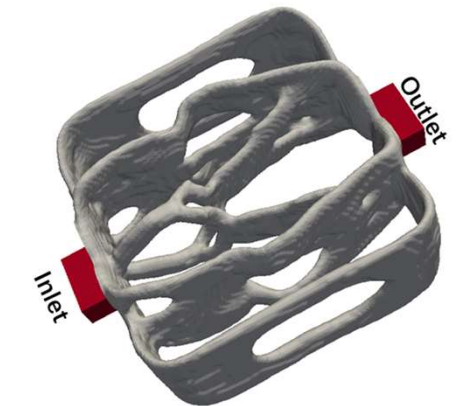
# Fluid-thermal-structural design

## Without considering thermal expansion

$$S \leq 1.5 * 10^{-16}$$



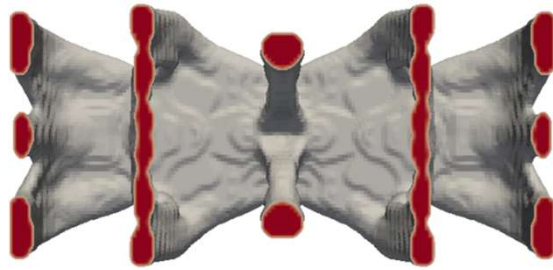
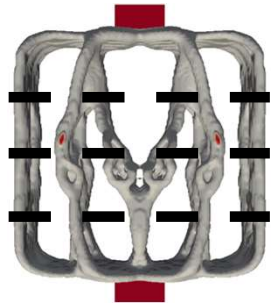
$$S \leq 10^{-16}$$



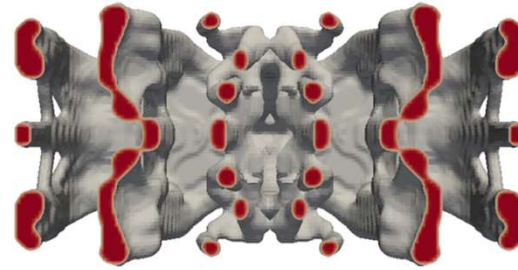


# Fluid-thermal-structural design

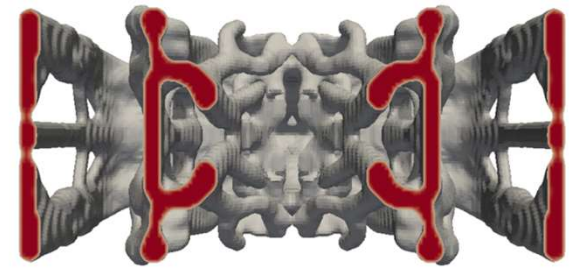
## Without considering thermal expansion



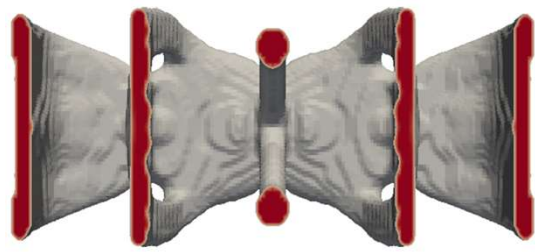
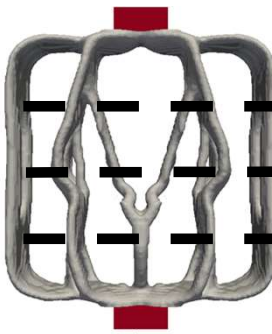
(a)  $z=-0.0025$



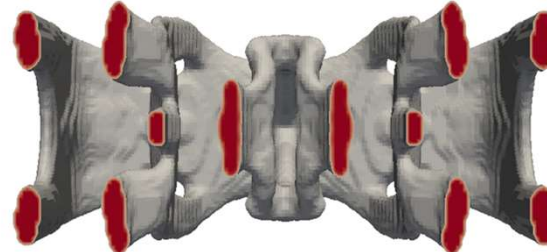
(b)  $z=-0.005$



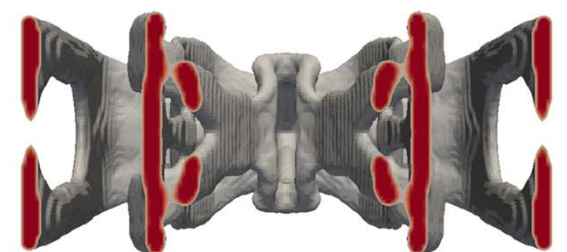
(c)  $z=-0.0075$



(a)  $z=-0.0025$



(b)  $z=-0.005$



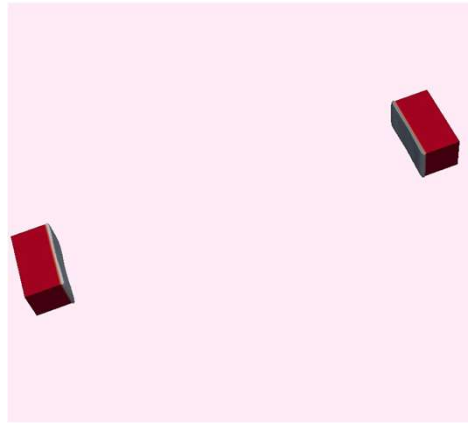
(c)  $z=-0.0075$

- Like 2D extruded design, the cross sections of channels are vertical (parallel to the direction of external force), which provide a high stiffness.

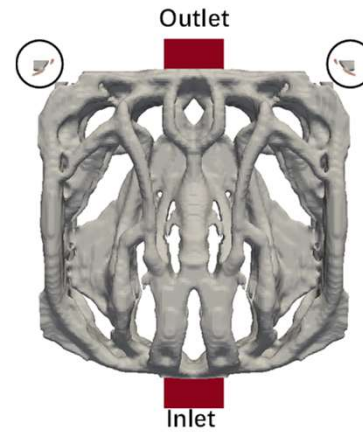
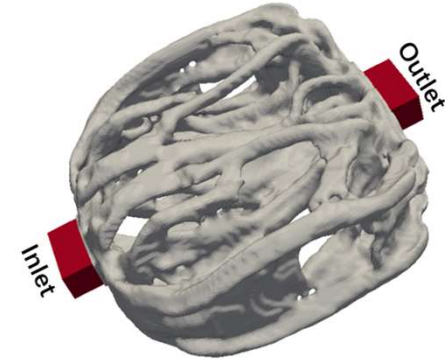
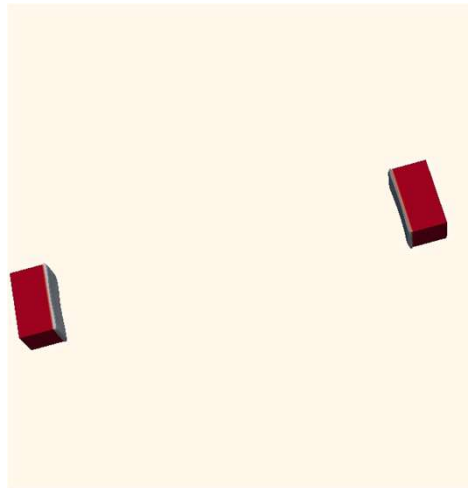
# Fluid-thermal-structural design

## Considering thermal expansion

$$S \leq 10^{-17}$$

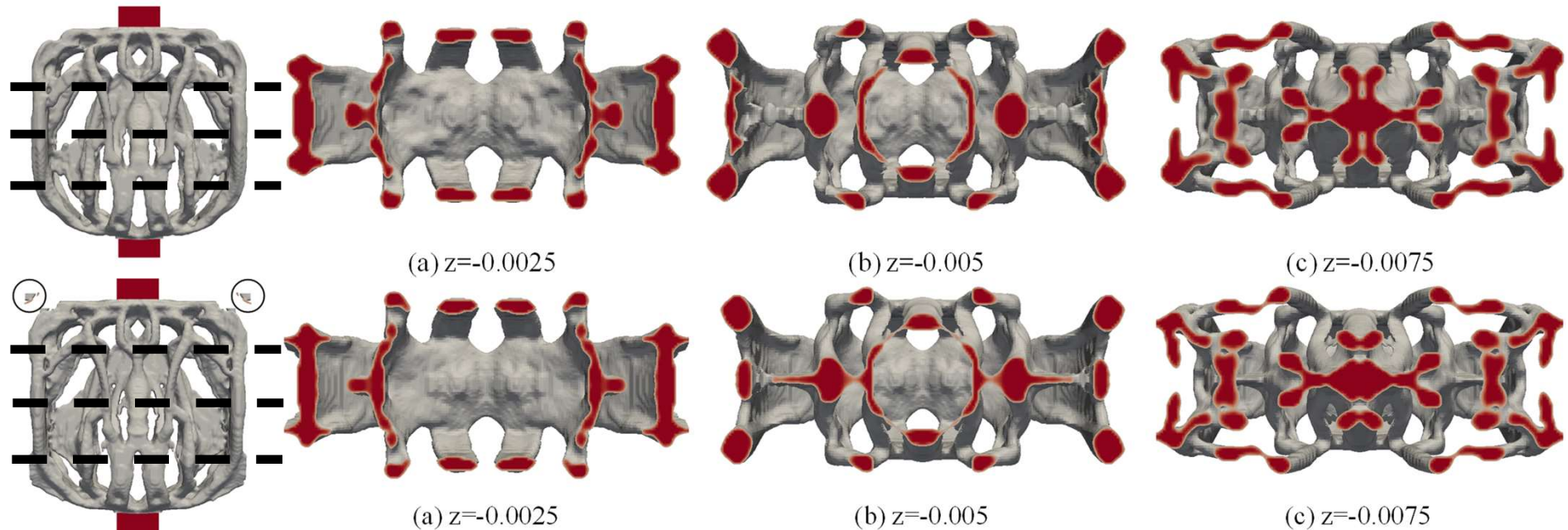


$$S \leq 5 * 10^{-18}$$



# Fluid-thermal-structural design

## Considering thermal expansion



- Displacements of expansion and load are opposite.
- Horizontal and isolated fluid regions reduce the stiffness so that the displacements can be neutralized.

## Conclusions

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- The fluid-thermal-structural topology optimizations are performed where the primal and analytical adjoint PDEs are solved by OpenFOAM. The structural features of the results in fluid-thermal design, fluid-thermal-structural without and with thermal expansions are discussed.
- The approach can be further extended to avoid floating solid result (by considering the weight) and solve the problem of thermal stress.



## Conclusions

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- In the fluid-thermal topology optimization, the velocity should be punished harder than pure fluid problem, so that the effect of the heat convection can be eliminated.
- A more appropriate formula of estimating the  $\alpha_{max}$  for 3D fluid-thermal problem is expected, which should be able to eliminate thermal convection in solid, therefore more properties (density, heat conductivity, heat capacity and characteristic velocity) should be considered.



***THANK YOU  
FOR YOUR ATTENTION!***