Body-fitted topology optimization of 2D and 3D fluid-to-fluid heat exchangers

Florian Feppon

PhD advisors : Grégoire Allaire, Charles Dapogny

Safran Tech advisors : Julien Cortial, Felipe Bordeu.

10th TOP Webinar
2021, February 23rd

Problem at hand

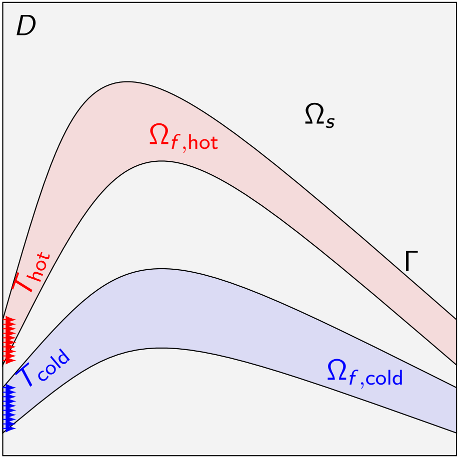

Figure: Settings of the heat exchanger topology optimization problem.

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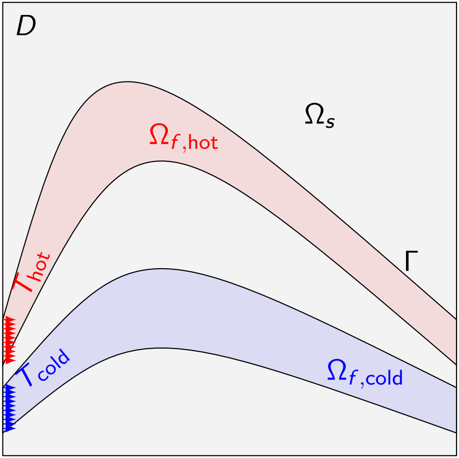

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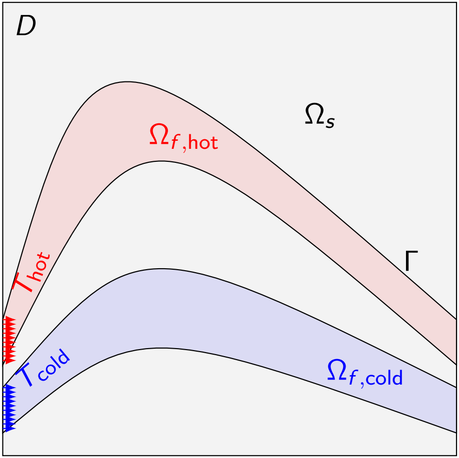

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- Thermal diffusion in Ω_s and Ω_f with conductivities $k_s >> k_f$.

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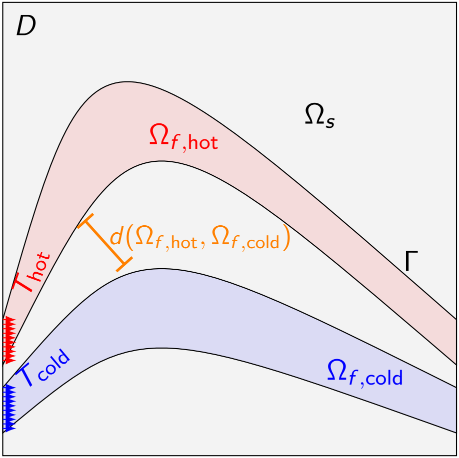

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$$d(\Omega_{f,\text{hot}}, \Omega_{f,\text{cold}}) \geqslant d_{\min}.$$

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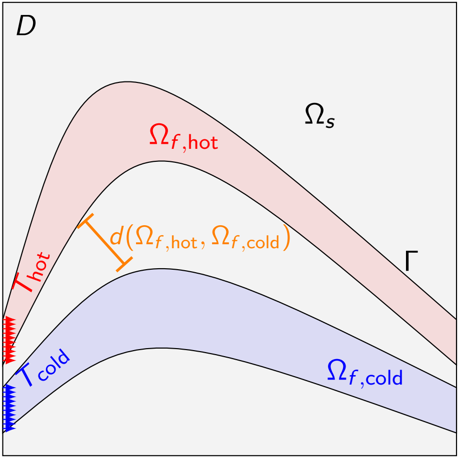

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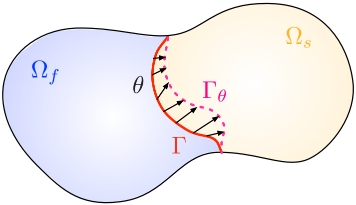
- In 3D!

The boundary variation method of Hadamard

$$\min_{\Gamma} \; J(\Gamma)$$



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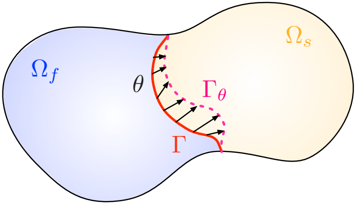

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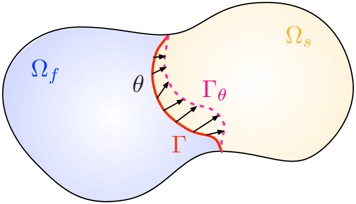

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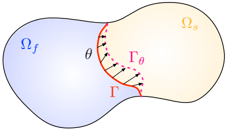
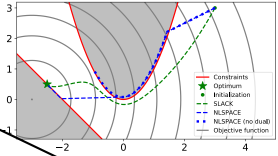
$$J(\Gamma_{\boldsymbol{\theta}}) = J(\Gamma) + \frac{\mathrm{d}J}{\mathrm{d}\boldsymbol{\theta}}(\boldsymbol{\theta}) + o(\boldsymbol{\theta}), \quad \text{with } \frac{|o(\boldsymbol{\theta})|}{||\boldsymbol{\theta}||_{W^{1,\infty}(D,\mathbb{R}^d)}} \xrightarrow{\boldsymbol{\theta}\to 0} 0.$$

Ingredients

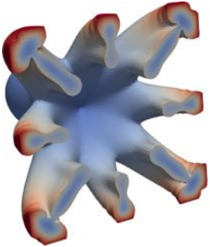
I. Shape derivatives and mesh evolution method



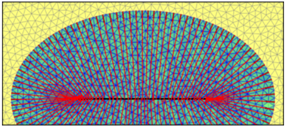
II. Null space optimization algorithm



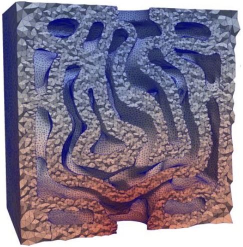
III. Parallel computing



IV. Treatment of geometric constraints



V. Heat exchangers



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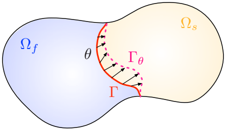

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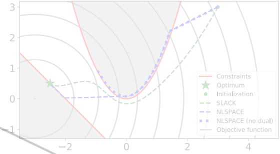

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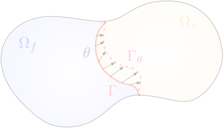

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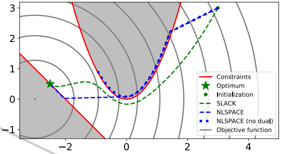

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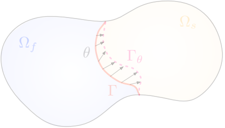

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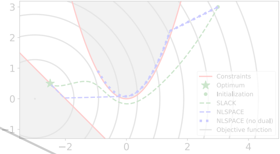

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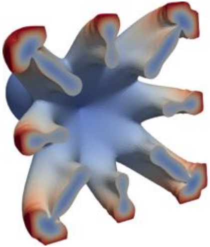

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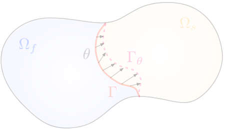

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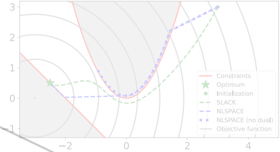

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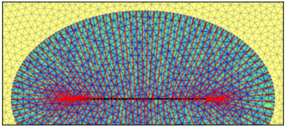

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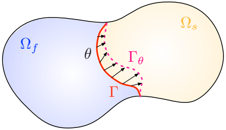

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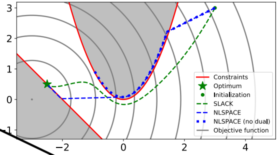

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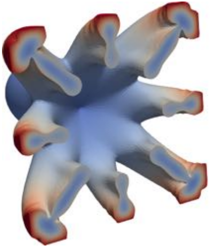

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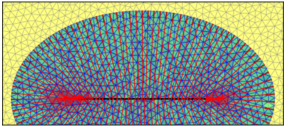

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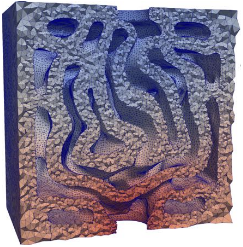

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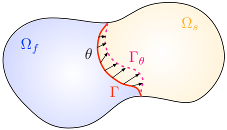

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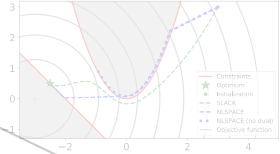

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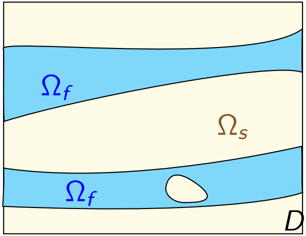
Coupled physics system



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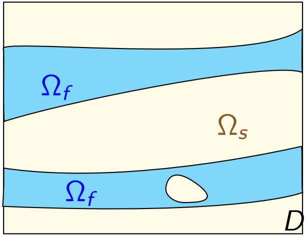


- Incompressible Navier-Stokes system for the velocity and pressure $(\boldsymbol{v}, p)$ in Ω_f

$$-\mathrm{div}(\sigma_f(\boldsymbol{v}, p)) + \rho \nabla \boldsymbol{v}\, \boldsymbol{v} = \boldsymbol{f}_f \text{ in } \Omega_f$$

- Convection-diffusion for the temperature T in Ω_f and Ω_s:

$$\begin{aligned} -\mathrm{div}(k_f \nabla T_f) + \rho \boldsymbol{v} \cdot \nabla T_f = Q_f \quad &\text{in } \Omega_f \\ -\mathrm{div}(k_s \nabla T_s) = Q_s \quad &\text{in } \Omega_s \end{aligned}$$

- Boundary conditions on $\Gamma = \partial\Omega_f$:

$$\begin{cases} T_f = T_s \text{ on } \Gamma \\ k_f \nabla T_f \cdot \boldsymbol{n} = k_s \nabla T_s \cdot \boldsymbol{n} \text{ on } \Gamma \\ \boldsymbol{v} = 0 \text{ on } \Gamma. \end{cases}$$

Shape derivatives

Proposition

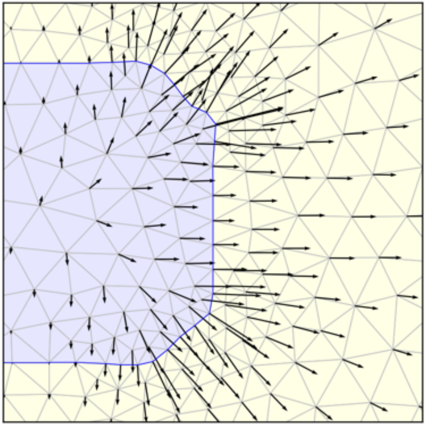
Let $J(\Gamma, T, \boldsymbol{v}, p)$ an arbitrary cost function. If J has continuous partial derivatives, then $\Gamma \mapsto J(\Gamma, \boldsymbol{u}(\Gamma), T(\Gamma), \boldsymbol{v}(\Gamma), p(\Gamma))$ is shape differentiable and the shape derivative reads[1]*:*

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\theta}}\Big[J(\Gamma_{\boldsymbol{\theta}}, \boldsymbol{v}(\Gamma_{\boldsymbol{\theta}}), p(\Gamma_{\boldsymbol{\theta}}), T(\Gamma_{\boldsymbol{\theta}}), \boldsymbol{u}(\Gamma_{\boldsymbol{\theta}}))\Big](\boldsymbol{\theta}) \\
&\quad = \overline{\frac{\partial \mathfrak{J}}{\partial \boldsymbol{\theta}}}(\boldsymbol{\theta}) + \int_\Gamma (\boldsymbol{f}_f \cdot \boldsymbol{w} - \sigma_f(\boldsymbol{v}, p) : \nabla \boldsymbol{w} + \boldsymbol{n} \cdot \sigma_f(\boldsymbol{w}, q) \nabla \boldsymbol{v} \cdot \boldsymbol{n} + \boldsymbol{n} \cdot \sigma_f(\boldsymbol{v}, p) \nabla \boldsymbol{w} \cdot \boldsymbol{n})(\boldsymbol{\theta} \cdot \boldsymbol{n}) \mathrm{d}s \\
&\quad + \int_\Gamma \left(k_s \nabla T_s \cdot \nabla S_s - k_f \nabla T_f \cdot \nabla S_f + Q_f S_f - Q_s S_s - 2k_s \frac{\partial T_s}{\partial \boldsymbol{n}} \frac{\partial S_s}{\partial \boldsymbol{n}} + 2k_f \frac{\partial T_f}{\partial \boldsymbol{n}} \frac{\partial S_f}{\partial \boldsymbol{n}} \right) (\boldsymbol{\theta} \cdot \boldsymbol{n}) \mathrm{d}s
\end{aligned}
$$

[1] Feppon et al., *Shape optimization of a coupled thermal fluid–structure problem in a level set mesh evolution framework* (2019)

Body-fitted meshes

We rely on body fitted meshes[2,3].



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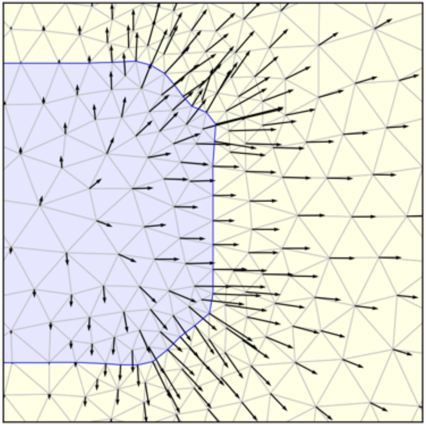
- Fluid-Solid interface Γ exactly captured, no need of physics interpolation because no porous regions.



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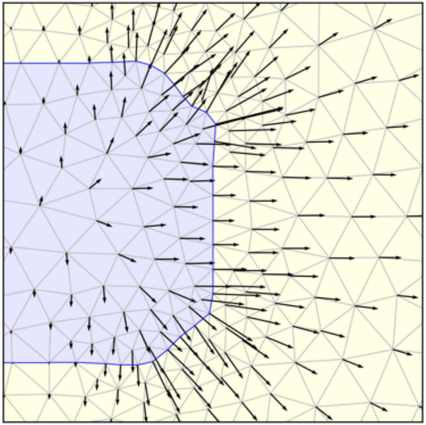

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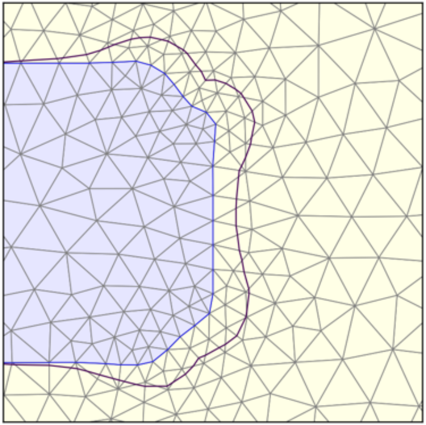

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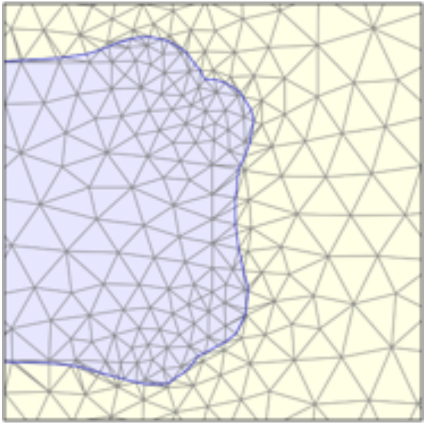

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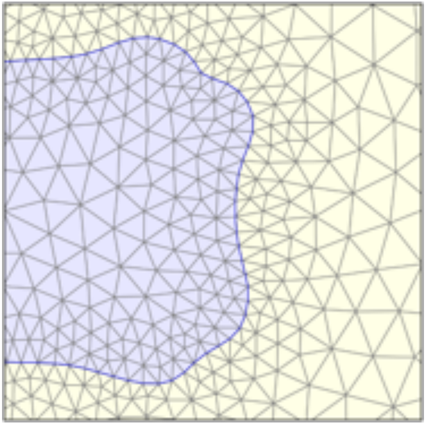

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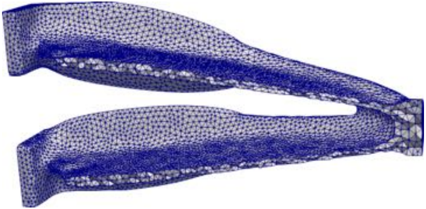

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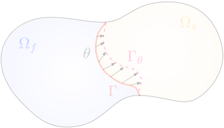

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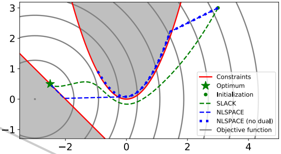

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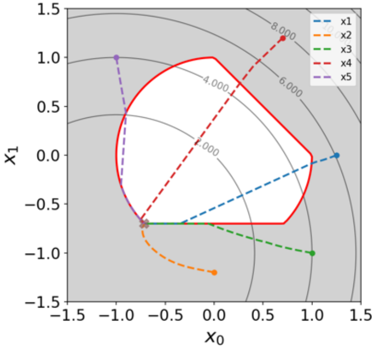
Null space optimization algorithm

- Nonlinear constrained optimization on manifolds with a moderate number of constraints



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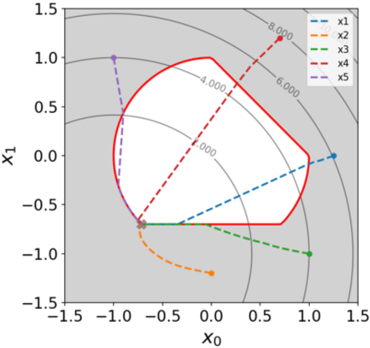

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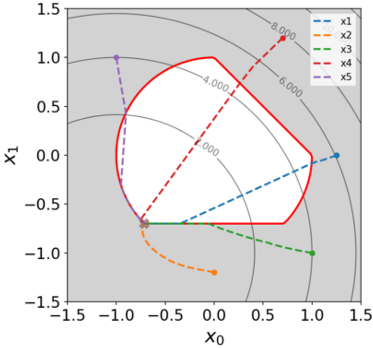
- Generalization of the unconstrained gradient flow: no hard tuning of parameters
- Adapted to the infinite dimensional setting of the method of Hadamard



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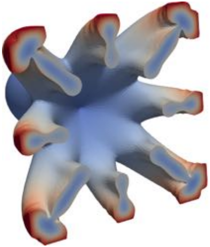

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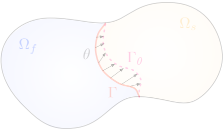

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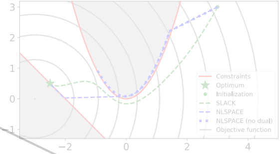

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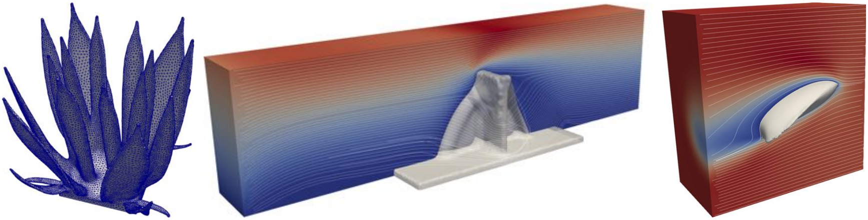
Parallel computing

- Use of Domain Decomposition and adapted preconditioners for solving finite element problems : all FEM related operations are achieved in parallel[5].



[5] Feppon et al., *Topology optimization of thermal fluid–structure systems using body-fitted meshes and parallel computing* (2020)

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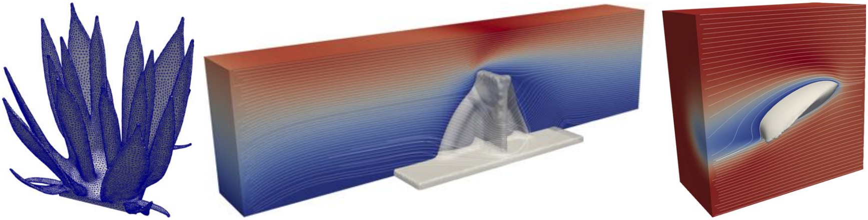

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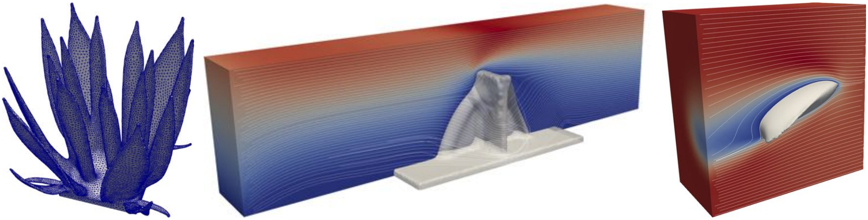
Parallel computing

- Use of Domain Decomposition and adapted preconditioners for solving finite element problems : all FEM related operations are achieved in parallel[5].
- We solve fluid FEM problems on meshes up to 4.8 millions of Tetrahedra with 30 CPUs.
- Mesh adaptation and Isosurface discretization is still sequential. A future release of (Par)Mmg will allow to do it in parallel.



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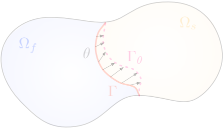
Ingredients

III. Parallel computing

I. Shape derivatives and mesh evolution method



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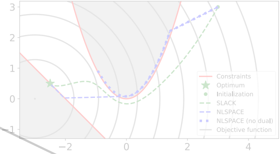
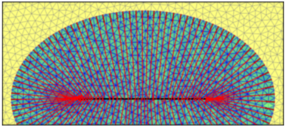


V. Heat exchangers

IV. Treatment of geometric constraints



Non mixing constraint

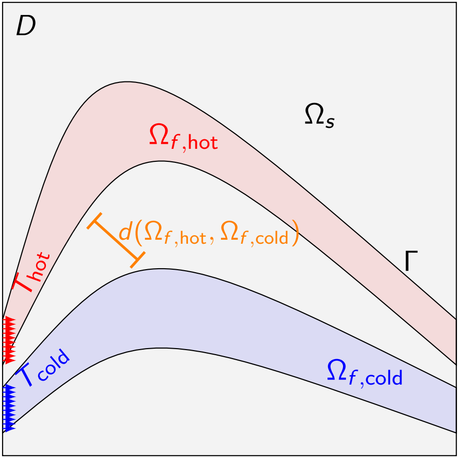


Figure: Settings of the heat exchanger topology optimization problem .

Non-penetration constraint:

$$d(\Omega_{f,\text{hot}}, \Omega_{f,\text{cold}}) \geqslant d_{\text{min}}.$$

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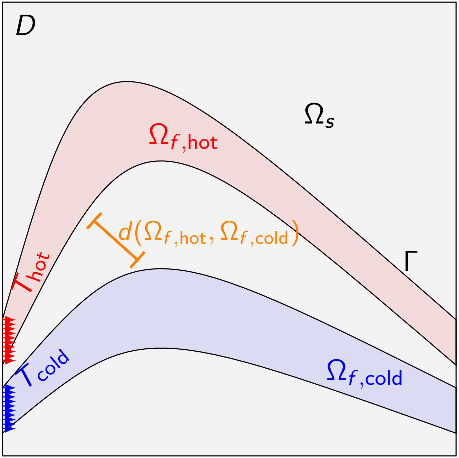


Figure: Settings of the heat exchanger topology optimization problem .

Non-penetration constraint:

$$d(\Omega_{f,\text{hot}}, \Omega_{f,\text{cold}}) \geqslant d_{\min}.$$

We enforce it by imposing

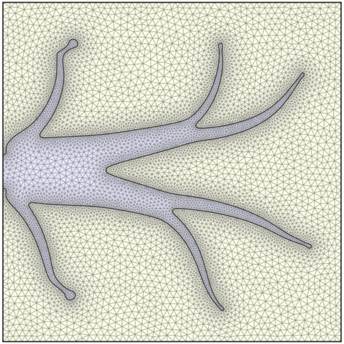
$$\forall x \in \Omega_{f,\text{cold}},\ d_{\Omega_{f,\text{hot}}}(x) \geqslant d_{\min},$$

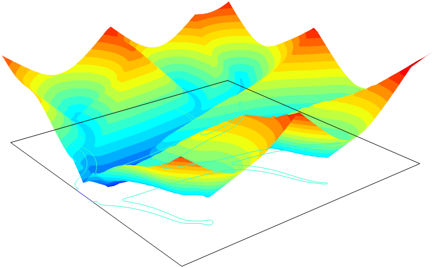
where $d_{\Omega_{f,\text{hot}}}$ is the signed distance function to the domain $\Omega_{f,\text{hot}}$.

The signed distance function

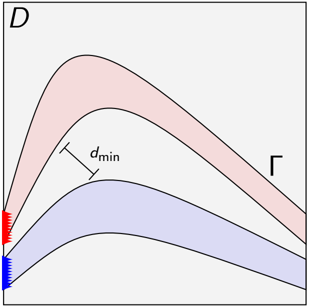
The signed distance function d_Ω to the domain $\Omega \subset D$ is defined by:

$$\forall x \in D, \;\; d_\Omega(x) = \begin{cases} -\min\limits_{y \in \partial\Omega} ||y - x|| & \text{if } x \in \Omega, \\ \min\limits_{y \in \partial\Omega} ||y - x|| & \text{if } x \in D \backslash \Omega. \end{cases}$$





2D heat exchangers



Heat exchanger problem with limited pressure loss and non-mixing constraint:

$$\min_{\Gamma} \quad J(\Omega_f) = -\left(\int_{\Omega_{f,cold}} \rho c_p \boldsymbol{v} \cdot \nabla T \mathrm{d}x - \int_{\Omega_{f,hot}} \rho c_p \boldsymbol{v} \cdot \nabla T \mathrm{d}x \right)$$

$$s.c. \begin{cases} \mathrm{DP}(\Omega_f) = \int_{\partial\Omega_f^D} p \mathrm{d}s - \int_{\partial\Omega_f^N} p \mathrm{d}s \leq \mathrm{DP}_0 \\ \qquad\qquad Q_{hot\leftrightarrow cold}(\Omega_f) \geqslant d_{\min}. \end{cases}$$

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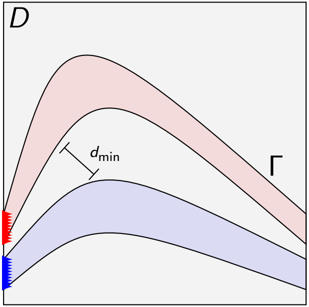

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Shape derivatives of geometric constraints

For instance

$$Q_{hot\leftrightarrow cold}(\Omega_f) := \left(\int_{\Omega_{f,cold}} \frac{1}{|d_{\Omega_{f,hot}}|^p} \mathrm{d}x \right)^{-\frac{1}{p}} \simeq \left\| \frac{1}{d_{\Omega_{f,hot}}} \right\|^{-1}_{L^\infty(\Omega_{f,cold})}.$$

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This reduces to the setting of computing the shape derivative of some penalty functional $Q_{hot\leftrightarrow cold}(\Omega_f)$ with:

$$Q_{hot\leftrightarrow cold}(\Omega_f) := \int_D j(d_{\Omega_{f,hot}}) \mathrm{d}x.$$

Shape derivatives of geometric constraints

The shape derivative of $Q_{hot\leftrightarrow cold}(\Omega_f)$ is given by[6]:

$$Q'_{hot\leftrightarrow cold}(\Omega)(\boldsymbol{\theta}) = \int_{\partial\Omega_{f,hot}} u(y)\ \boldsymbol{\theta}\cdot\boldsymbol{n}\,\mathrm{d}y$$

$$\text{with } u(y) = -\int_{z\in\mathrm{ray}(y)} j'(d_{\Omega_{f,hot}}(z)) \prod_{1\leq i\leq n-1}(1+\kappa_i(y)d_{\Omega_{f,hot}}(z))\mathrm{d}z, \qquad \forall y\in\partial\Omega.$$

[6] Allaire, Jouve, and Michailidis, *Thickness control in structural optimization via a level set method* (2016)

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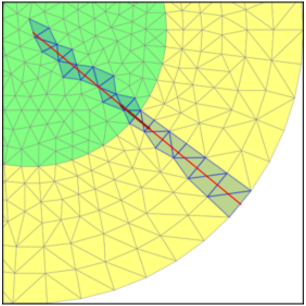

The computation of $u(y)$ requires a priori integration along the normal rays and the computation of curvatures $\kappa_i(y)$.

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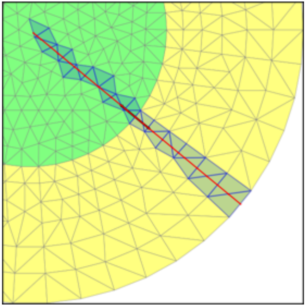

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Shape derivatives of geometric constraints

It turns out that it is possible to compute u without integrating along the rays[7]:

[7] Feppon, Allaire, and Dapogny, *A variational formulation for computing shape derivatives of geometric constraints along rays* (2019)

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Proposition

Let $\hat{u} \in V_\omega$ be the solution to the variational problem

$$\forall v \in V_\omega,\ \int_{\partial\Omega_{f,hot}} \hat{u} v \mathrm{d}s + \int_D \omega(\nabla d_{\Omega_{f,hot}} \cdot \nabla \hat{u})(\nabla d_{\Omega_{f,hot}} \cdot \nabla v)\mathrm{d}x = -\int_D j'(d_{\Omega_{f,hot}}) v \mathrm{d}x.$$

Then $u(y) = \hat{u}(y)$ for any $y \in \partial\Omega_{f,hot}$.

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- This variational problem can easily be solved with FEM in 2D and 3D
- This allows to handle conveniently geometric constraints (e.g. maximum thickness, minum distance, etc...) in 2D and 3D level set based topology optimization.

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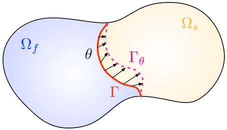

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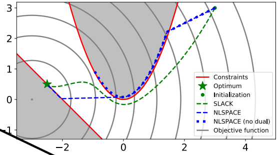

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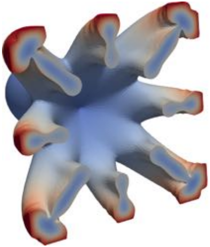

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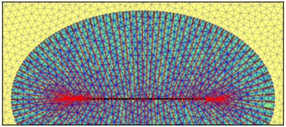

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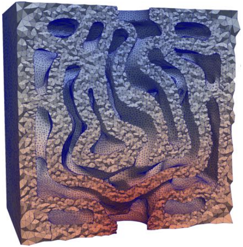

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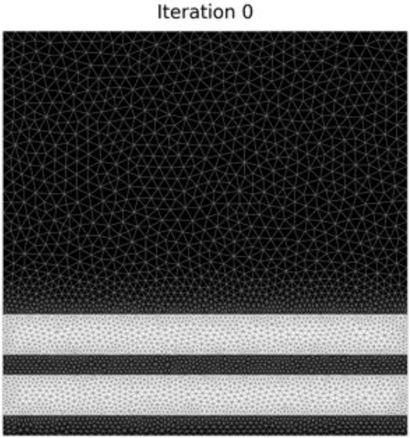
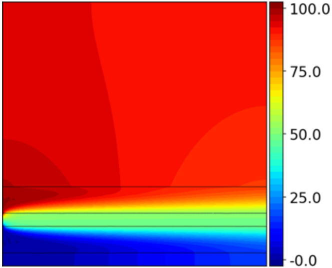
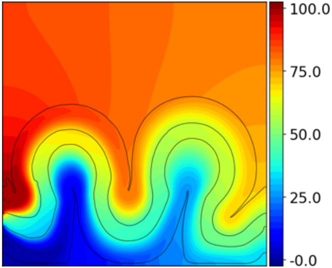


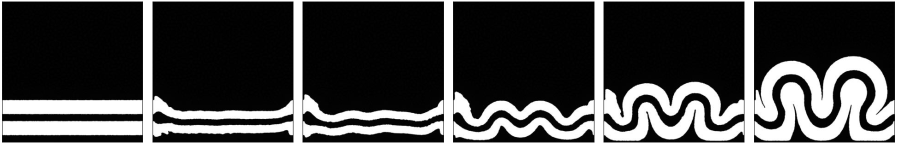
2D Heat Exchangers with non-mixing constraint



(a) Initial temperature



(b) Final temperature.



(c) Intermediate iterations 0, 8, 20, 50, 88 et 200.

3D fluid-to-fluid heat exchanger

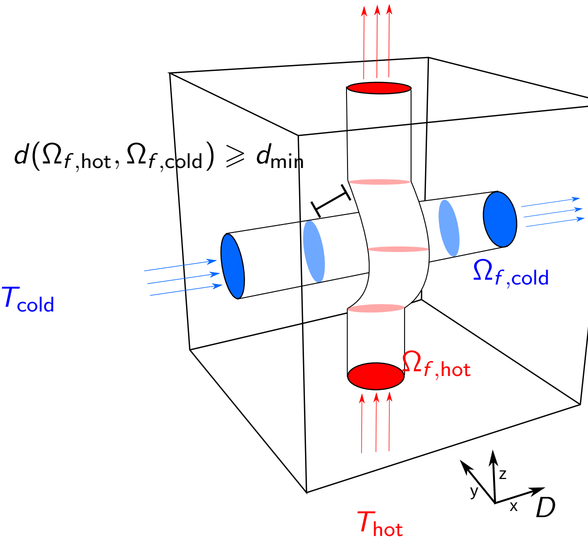


Figure: Schematic of the 3D setting.

3D fluid-to-fluid heat exchanger

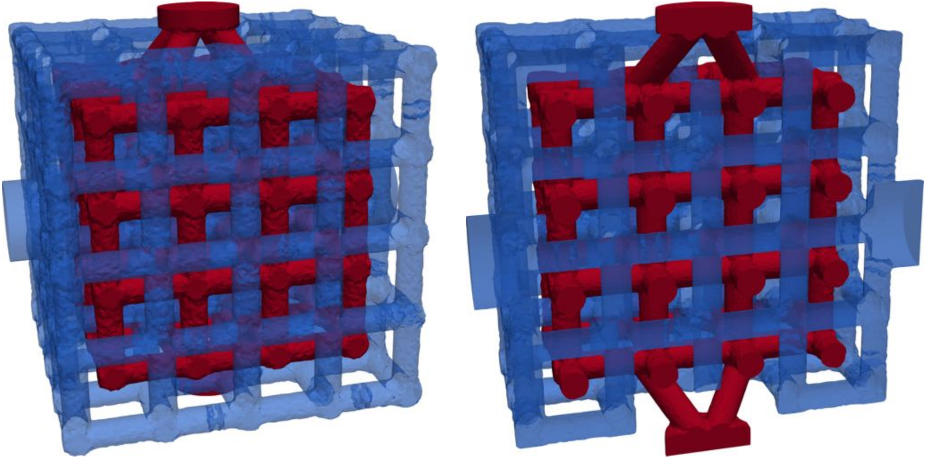
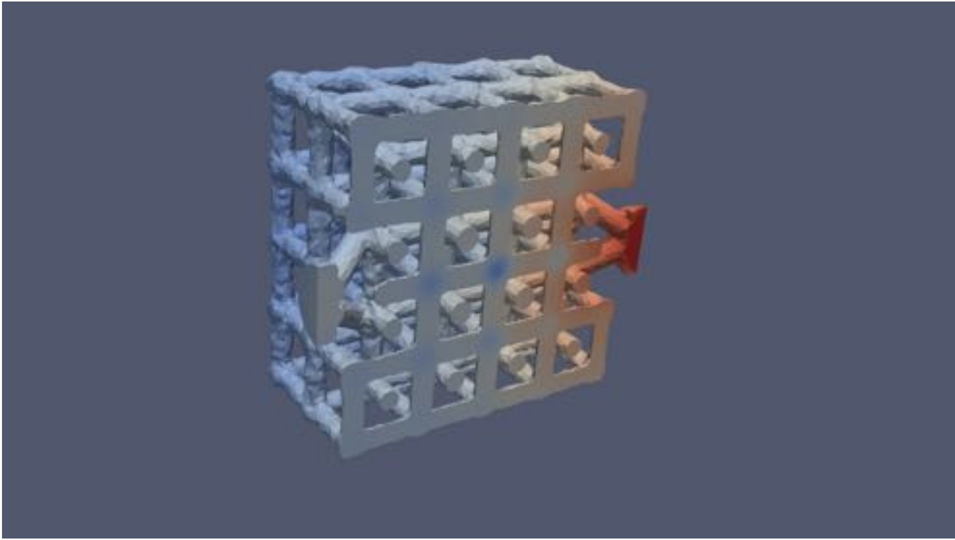


Figure: Initial distribution of fluid considered for the 3D heat exchanger test case.

3D fluid-to-fluid heat exchanger



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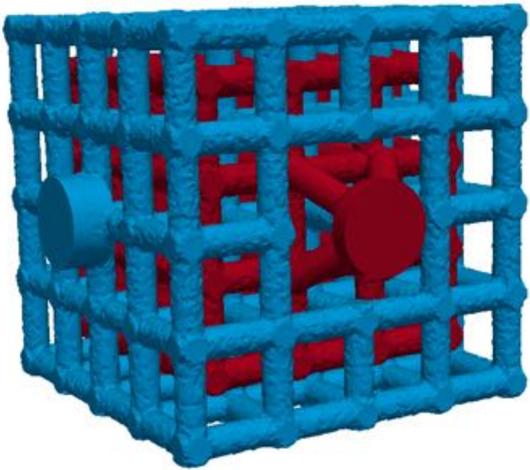

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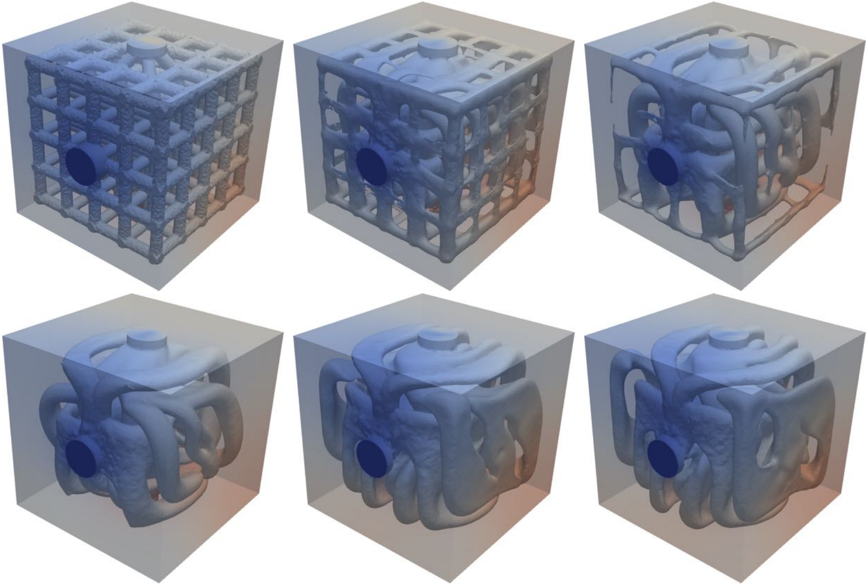

Figure: Intermediate iterations.

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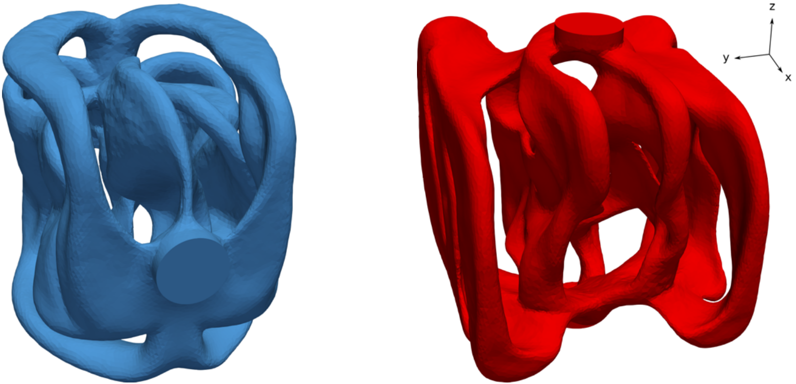

(a) Cold phase (b) Hot phase

Figure: Separate plots of the topologically optimized cold and hot fluid phases in the configuration $d_{\min} = 0.04$.

3D fluid-to-fluid heat exchanger

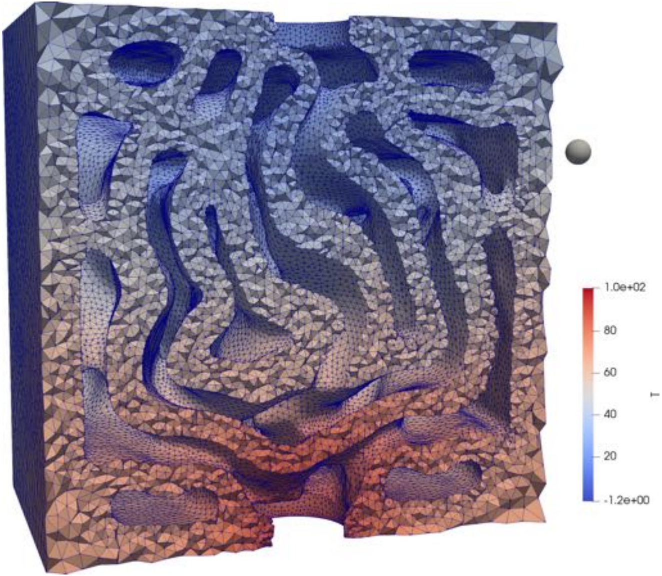

Figure: Cut of the resulting solid domain

Many thanks for your attention!