DTU

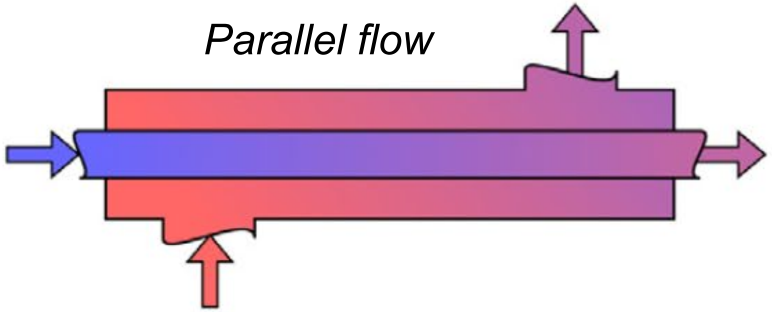
Topology Optimization of Two Fluid Heat Exchangers

L. C. Høghøj, D. R. Nørhave, J. Alexandersen, O. Sigmund, and C. S. Andreasen, "Topology optimization of two fluid heat exchangers," *Int. J. Heat Mass Transf.*, vol. 163, p. 120543, Dec. 2020. 10.1016/j.ijheatmasstransfer.2020.120543

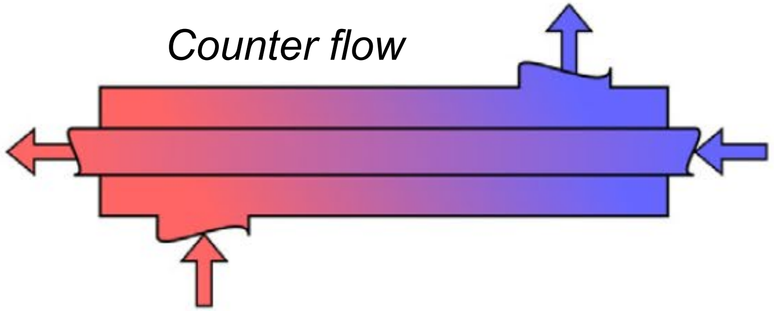
Topology Optimization of Two Fluid Heat Exchangers

Shell and tube design

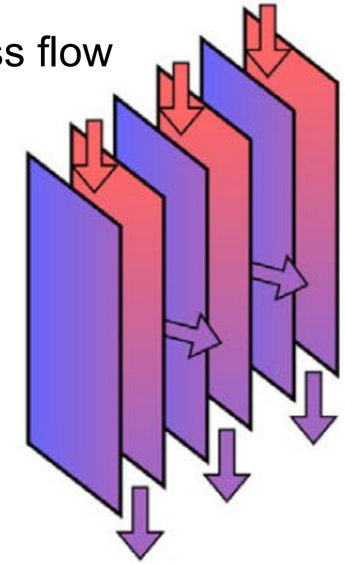
Parallel flow

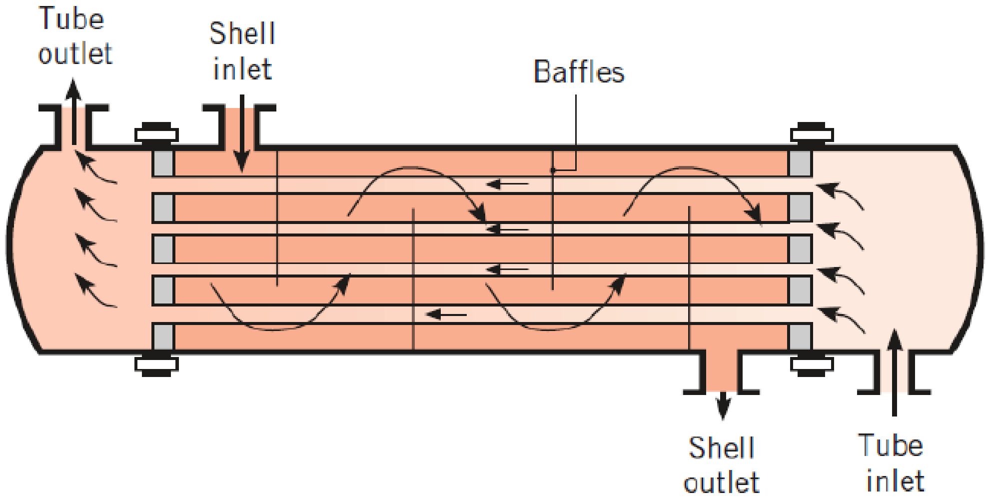


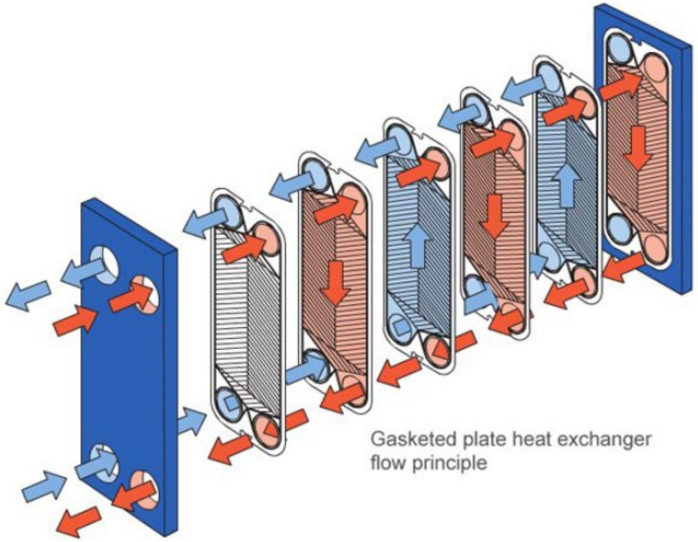
Counter flow



Cross flow



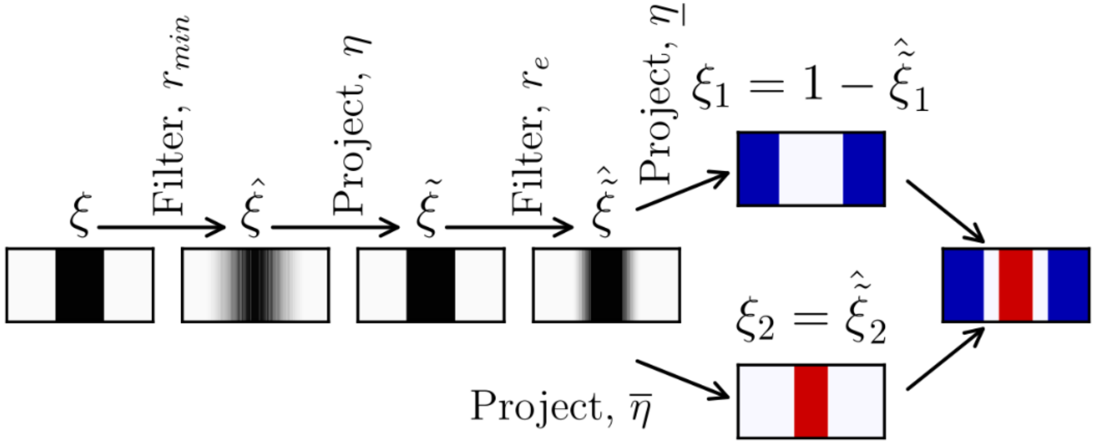


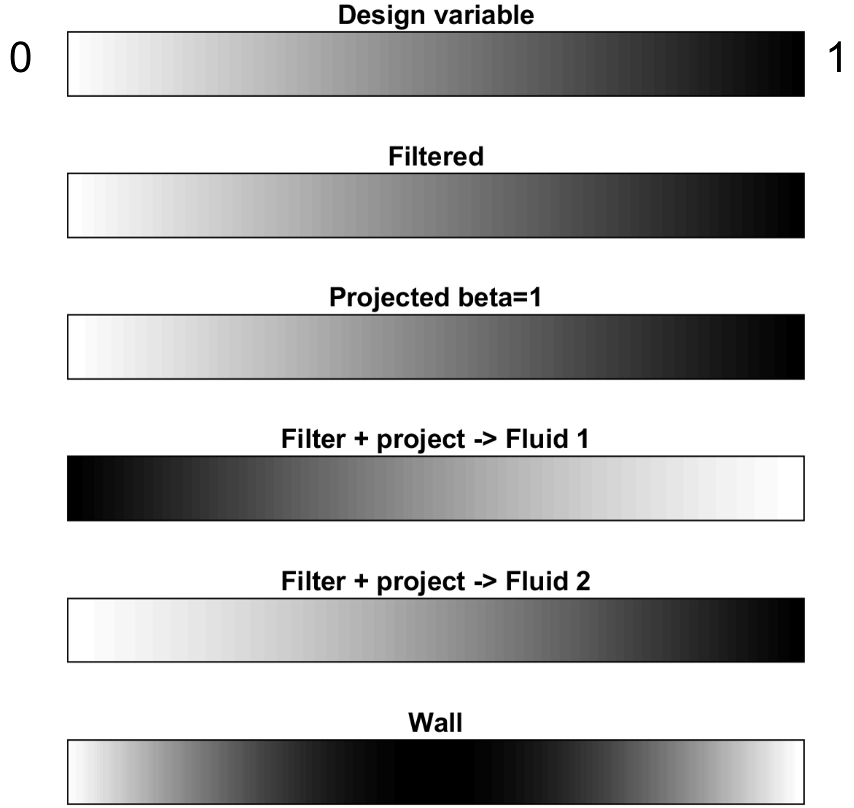


Bergman, T.L. Fundamentals of Heat and Mass Transfer, Wiley, 2011

Alfa Laval, Types of gasketed plate heat exchangers, http://www.alfalaval.com/microsites/gphe/types

Interface identification





- Filtering using a Helmoltz equation type PDE filter:

$$-R^2\nabla^2\hat{\xi} + \hat{\xi} = \xi \quad R = \frac{r}{2\sqrt{3}}$$

- Smooth Heaviside projection:

$$\tilde{\xi}(\hat{\xi}, \beta, \eta) = \frac{\tanh(\beta\eta) + \tanh(\beta(\hat{\xi} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$$

- Wall thickness w_e is obtained by setting:

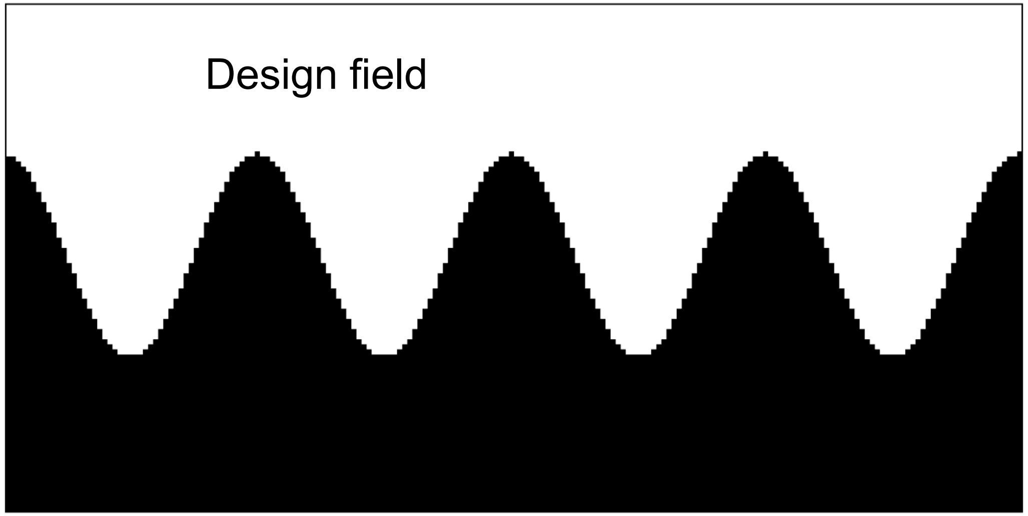
$$\eta = 0.5 \pm \Delta\eta, \quad \Delta\eta = 0.45, \quad r_e \approx 0.75 w_e, \quad r_{min} > w_e$$

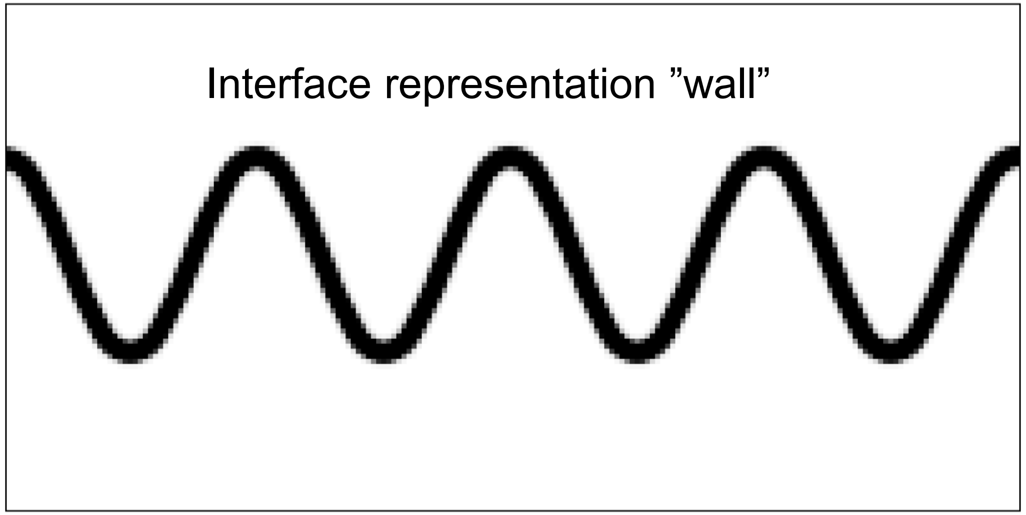
Inspired by the works:

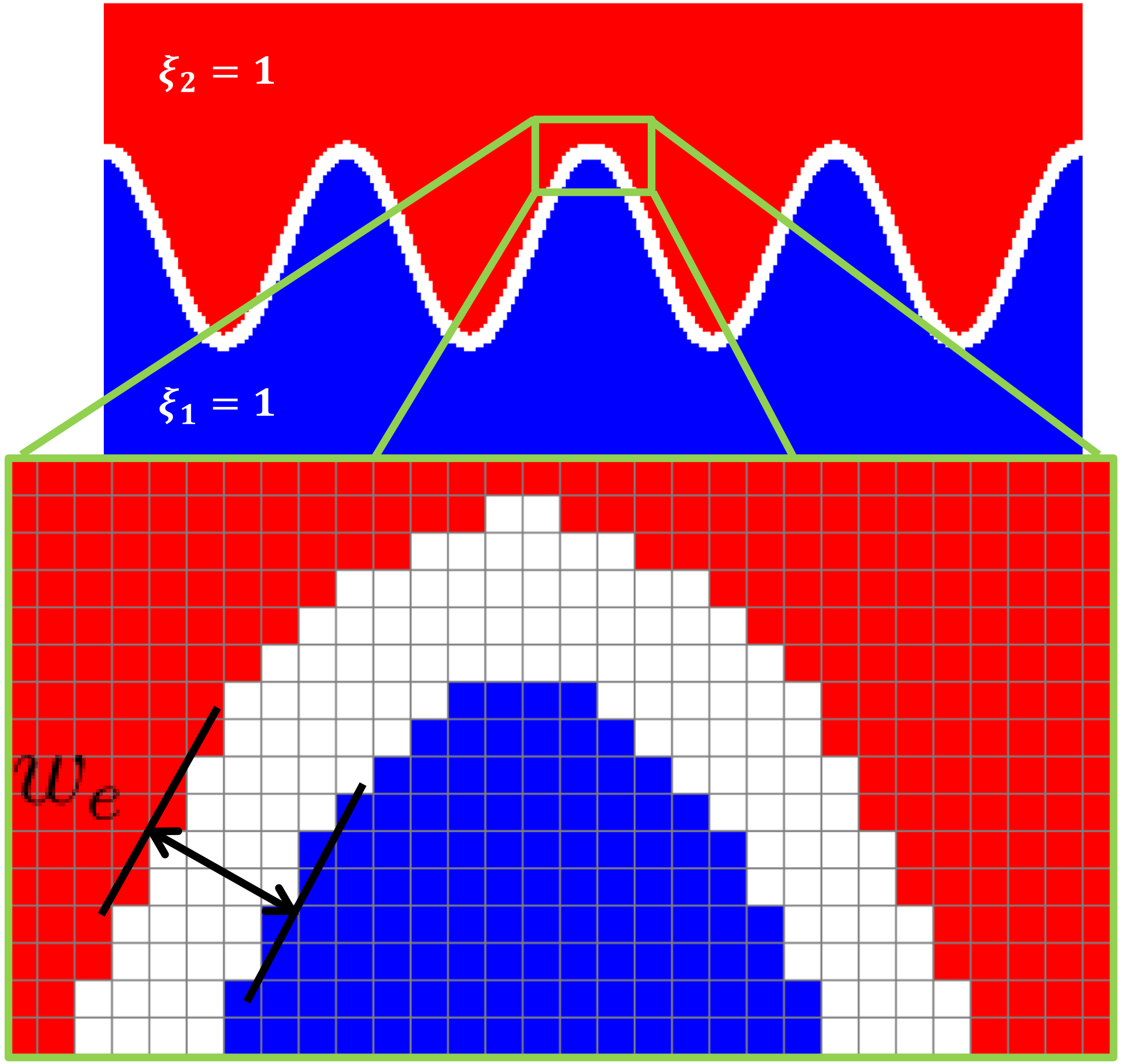
Clausen, A., Aage, N. & Sigmund, O. Topology optimization of coated structures and material interface problems. *Comput. Methods Appl. Mech. Eng.* **290**, 524–541 (2015).

Luo, Y., Li, Q. & Liu, S. Topology optimization of shell–infill structures using an erosion-based interface identification method. *Comput. Methods Appl. Mech. Eng.* **355**, 94–112 (2019).

Interface identification





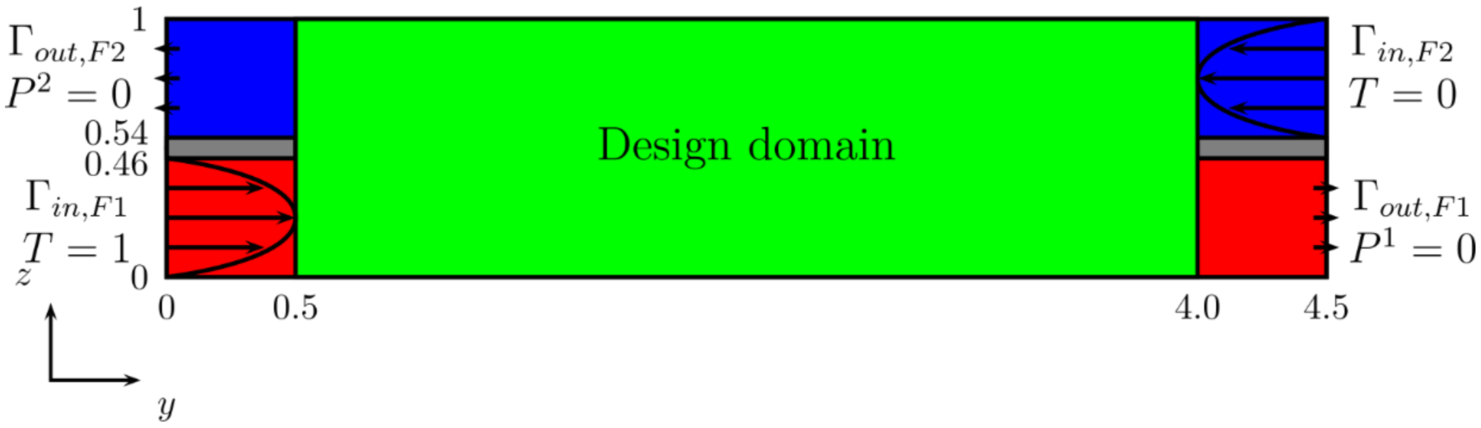


Parametrization

- Two fluid domains, required to be separated by a solid wall of a minimum thickness
  – Using an erosion – dilation based approach

- Solve Navier-Stokes equations for each fluid, in the entire domain
  – Velocities outside of the domain are penalized with Brinkman penalization

- Convection diffusion equation solved with summation of convective terms and varying conductivity



Physical modeling

- Non dimensional parameters describing the problem:

$$Re = \frac{UL\rho}{\mu} \qquad Pe_s^\gamma = \frac{\rho^\gamma c_p^\gamma U^\gamma L}{k_{solid}} \qquad C_k^\gamma = \frac{k^\gamma}{k_{solid}} \qquad Pe^\gamma = \frac{Pe_s^\gamma}{C_k^\gamma}$$

- Incompressibe Navier–Stokes with Brinkman penalization (one for each fluid):

$$u_i \frac{\partial u_j}{\partial x_i} - \frac{1}{Re}\frac{\partial}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \frac{\partial P}{\partial x_i} + \alpha u_i = 0 \qquad \frac{\partial u_i}{\partial x_i} = 0$$

- Convection diffusion equation (single equation combining both fluids and solid) :

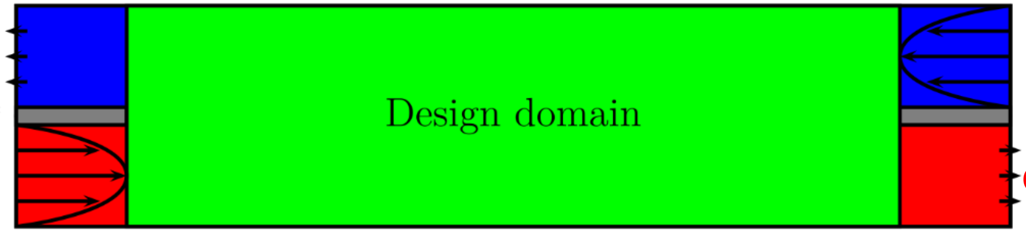
$$\sum_{\gamma=1}^{NF} (Pe_s^\gamma u_i^\gamma) \frac{\partial T}{\partial x_i} - \frac{\partial}{\partial x_i}\left(C_k(\mathbf{x}) \frac{\partial T}{\partial x_i}\right) = 0$$

$$C_k(\mathbf{x}) = \begin{cases} C_k^\gamma & \text{if } (\mathbf{x} \in \Omega^\gamma \\ 1 & \text{if } \mathbf{x} \in \Omega^s \end{cases}$$

Fluid:
low conductivity

Solid material :
high conductivity

Optimization



Objective function: maximize heat transferred from cooled- to coolant fluid

$$\Phi = \frac{1}{\int_{\Gamma_{F2}} dA}\left(Pe_s^1 \int_{\Gamma_{F1}} n_i u_i^1 T dA - Pe_s^2 \int_{\Gamma_{F2}} n_i u_i^2 T dA \right)$$

Enthalpy out F1   Enthalpy out F2

Expressed on both fluids outlets

Constraints:

$$g_\gamma = \frac{1}{\Delta P_{max}^\gamma \int_{\Gamma_{in,\ F\gamma}} dA} \int_{\Gamma_{in,\ F\gamma}} P^\gamma dA - 1$$

Constraints on inlet pressure of both fluids

Interpolations:

$$\alpha^\gamma(\xi_\gamma) = \overline{\alpha}^\gamma \frac{1-\xi_\gamma}{1+\xi_\gamma q_\alpha}$$

RAMP

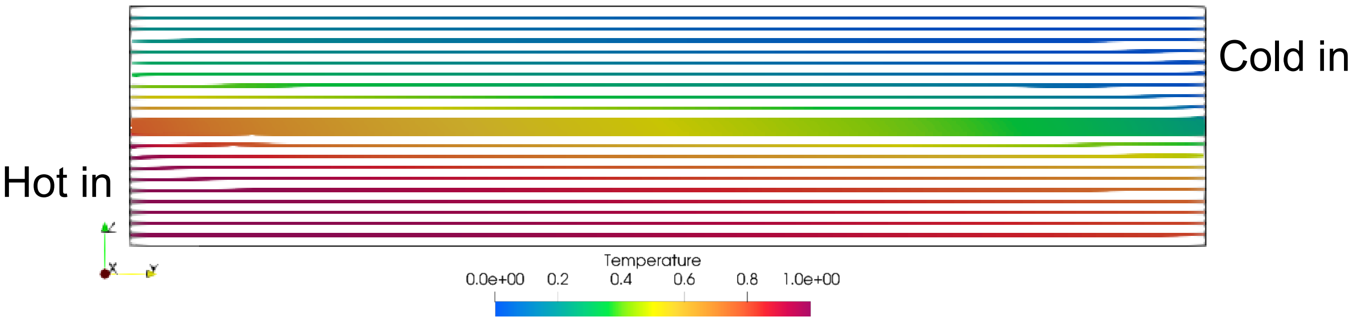
$$C_k(\xi_1,\ \xi_2) = (1-\xi_1-\xi_2)^{p_k} + C_k^1 \xi_1 + C_k^2 \xi_2$$

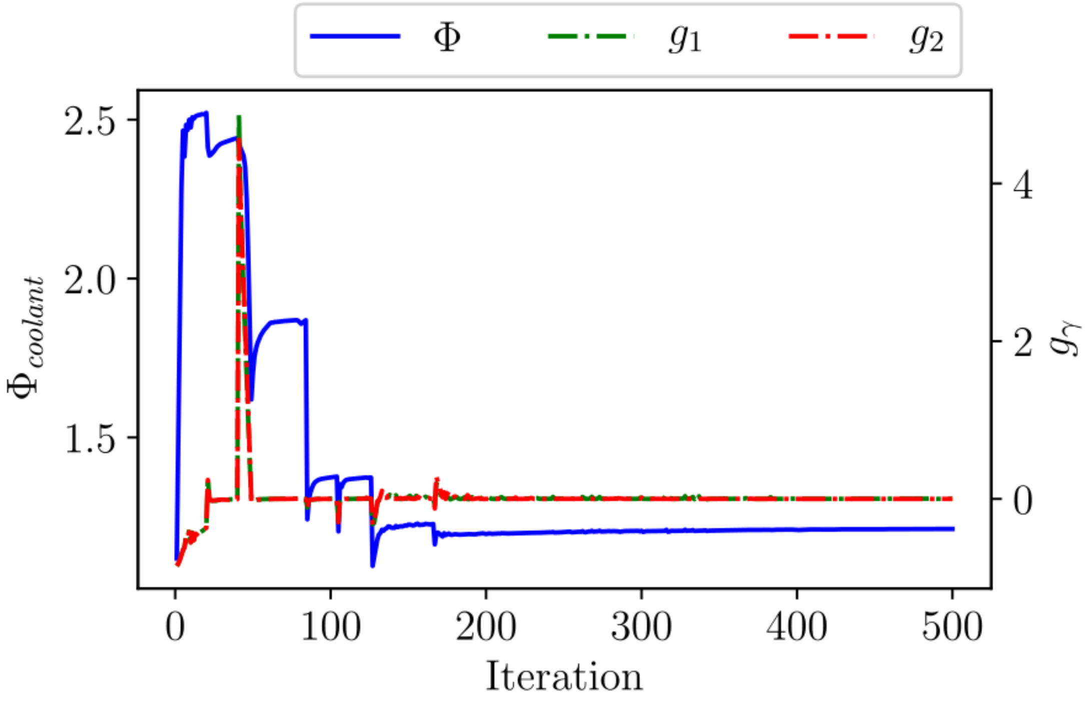
Multi material SIMP

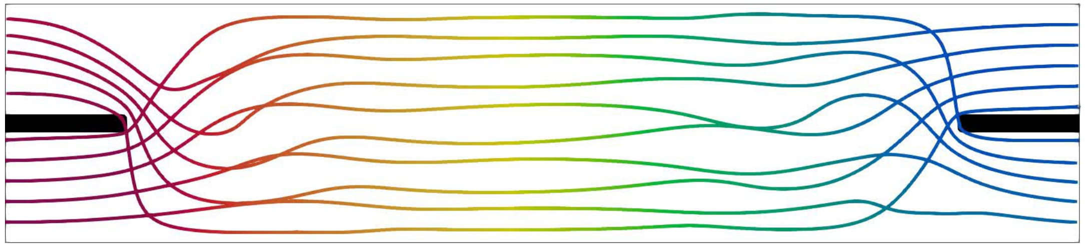
Results in 2D

- Optimization of a counterflow heat exchanger
- Two identical fluids with

$$Re = 100 \quad Pe = 100 \quad k_{solid} = 25k_{fluid}$$



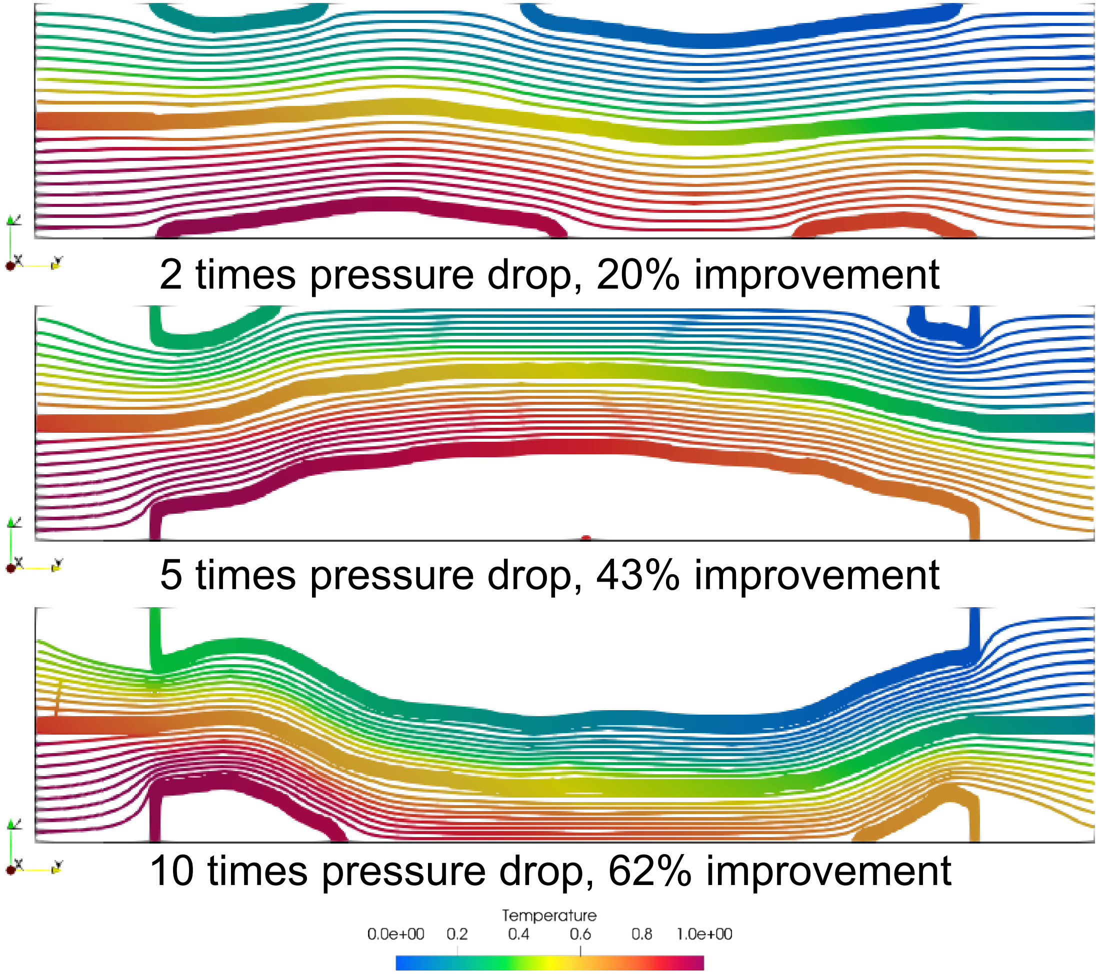




Five times higher pressure drop, 43% more transferred heat

Step	1	2	3	4	5	6	7
β	1	2	2	4	8	16	32
q_α	10^4	10^4	10^3	10^3	10^3	10^3	10^3
p_k	1	1	1	2	2	3	3

Features in optimized 2D designs



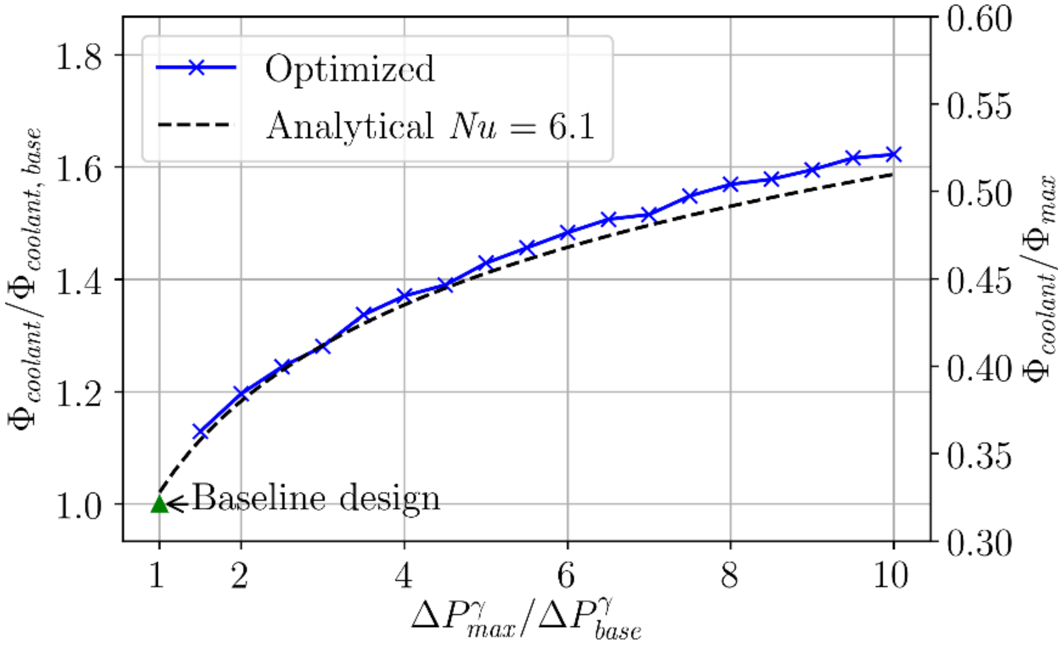
2 times pressure drop, 20% improvement

5 times pressure drop, 43% improvement

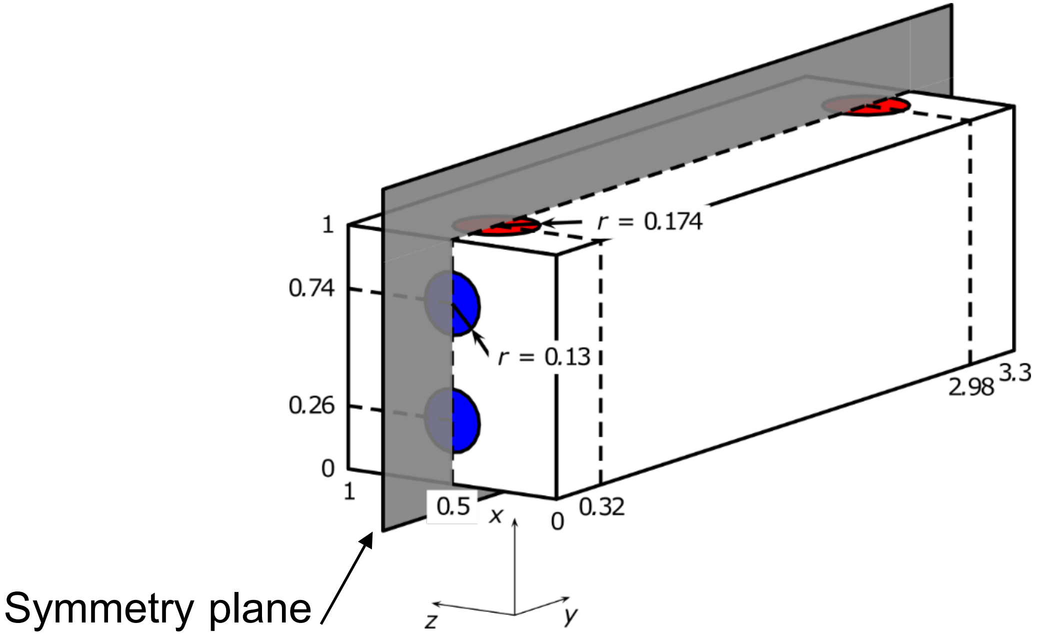
10 times pressure drop, 62% improvement

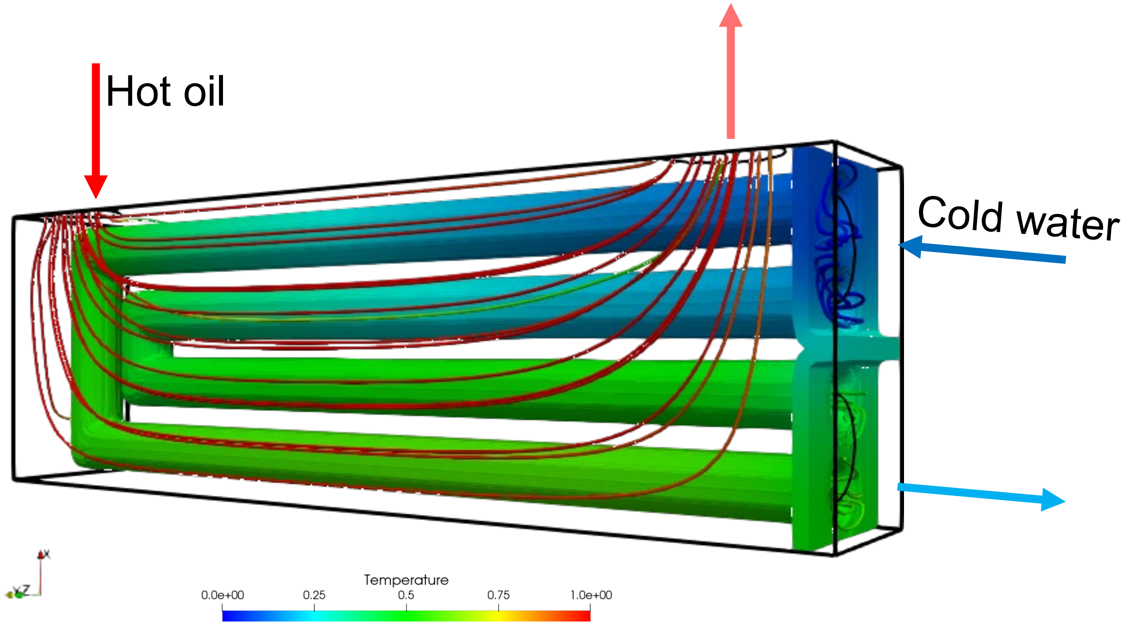
- Channels get narrowed by the optimization
- The bend of the channel results in a longer path, greater surface area

Comparson with analytical expression for narrow straight channels



Optimization of shell-and-tube heat exchanger





Baseline design

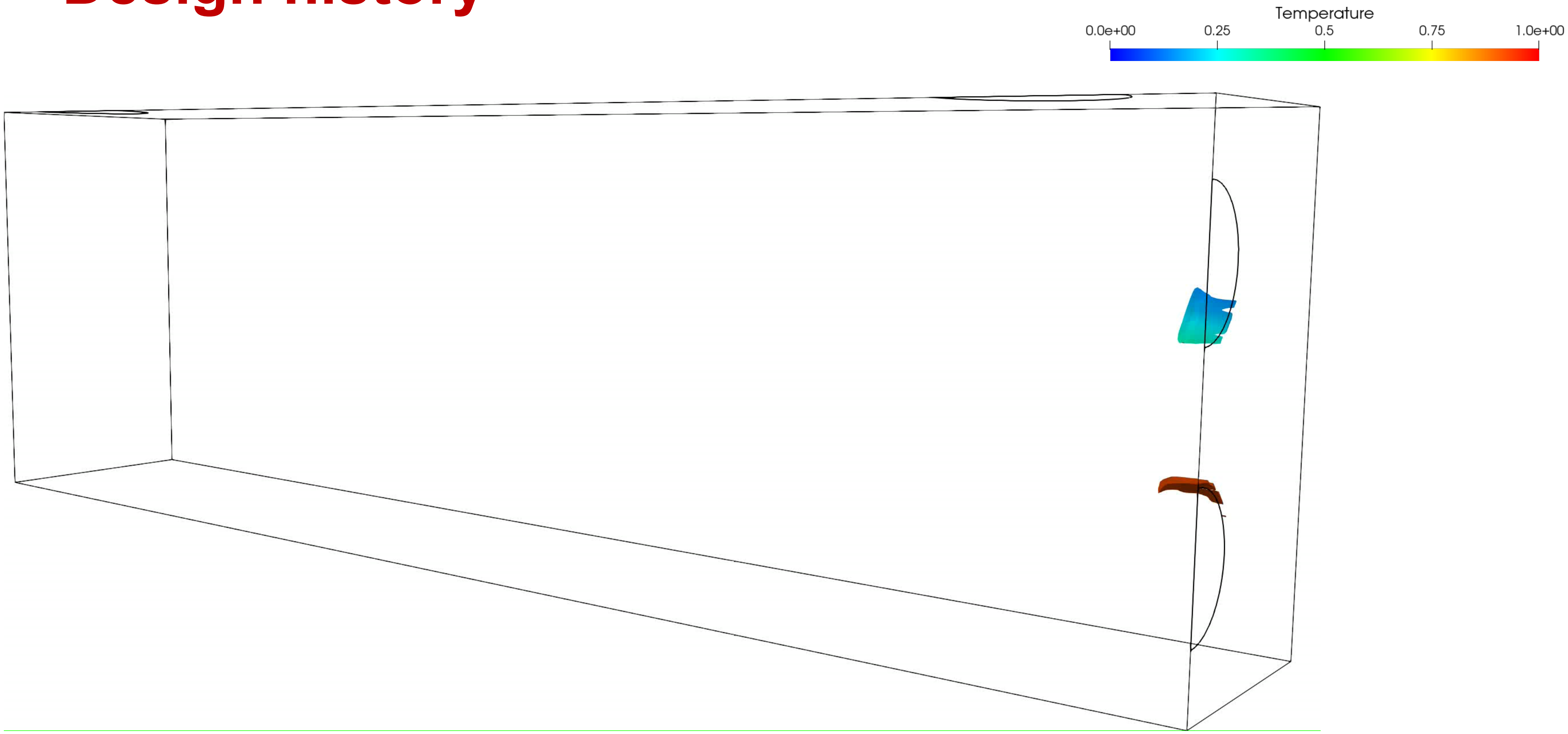
- Baseline design on the half domain
- Optimize using the same pressure drops as in the baseline design
- Physical parameters:

$$Re_{cooled} = 10.9,\ Pe_{cooled} = 3973,\ k_{cooled} = 0.1233\,[\mathrm{W}\cdot\mathrm{m}^{-1}\cdot\mathrm{K}^{-1}]$$ Oil

$$Re_{coolant} = 150,\ Pe_{coolant} = 453,\ k_{coolant} = 0.6\,[\mathrm{W}\cdot\mathrm{m}^{-1}\cdot\mathrm{K}^{-1}]$$ Water

$$k_{solid} = 30\,[\mathrm{W}\cdot\mathrm{m}^{-1}\cdot\mathrm{K}^{-1}]$$ Stainless Steel

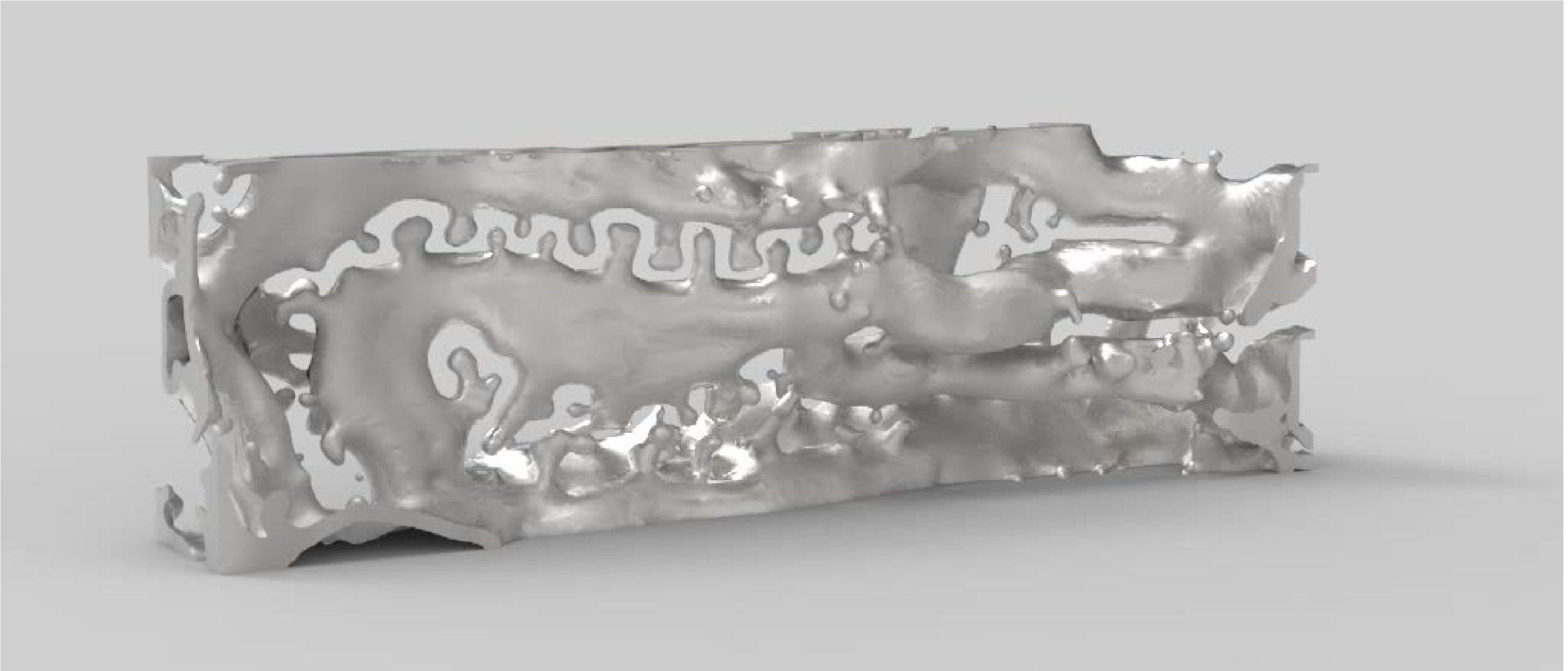
Design history



Discretization: 120x400x60 = 2,880,000 elements. NS systems 11.8Mdof each, CD: 2.9Mdof

7-15 hours on 320 cores, using max 350 design iter. Wall thickness approx. 5.6 elements.

Rendered topology



Initial design 4 channel baseline

Initial beta = 4

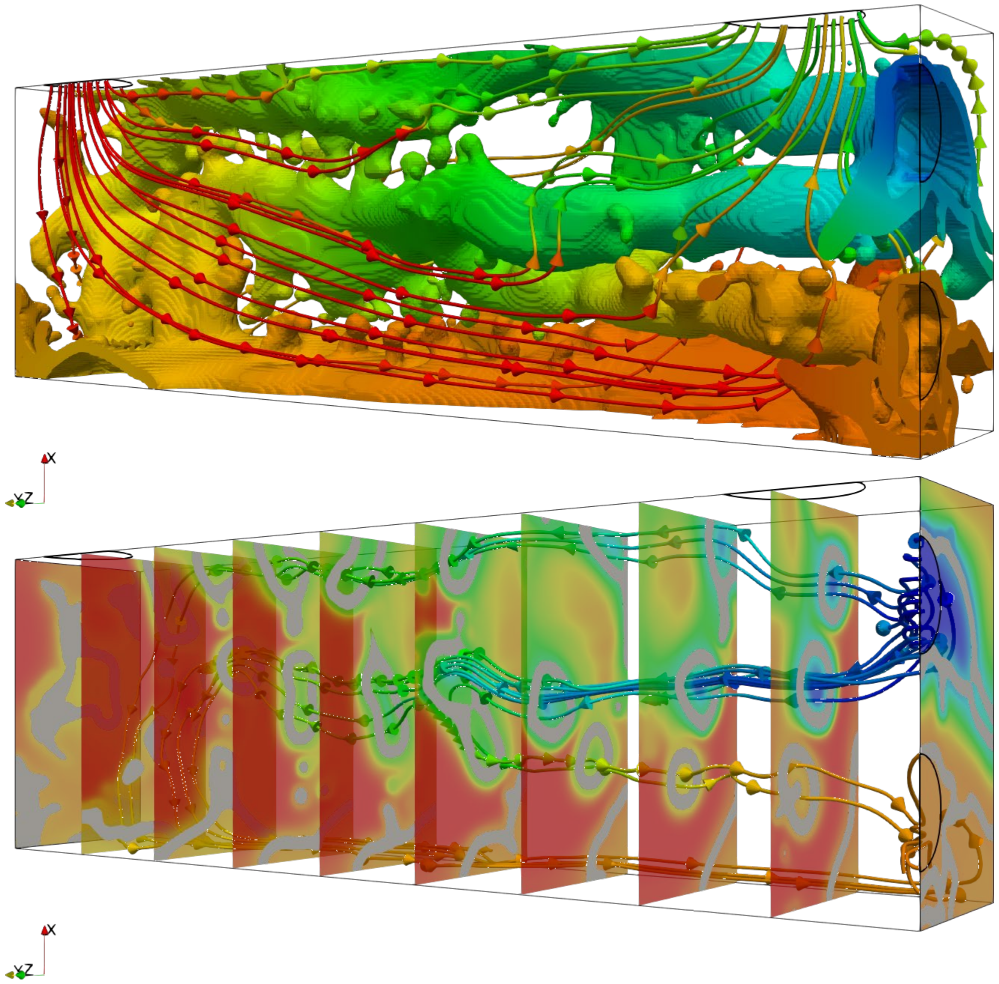
Phi = 15.49

Baseline design = 7.27

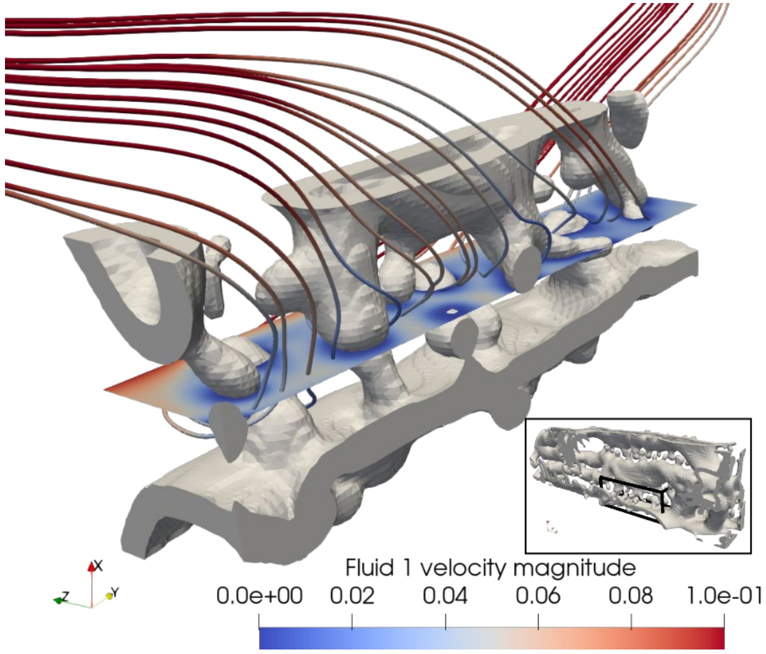
➔ Improvement of 113%

Visualization available as supplementary material: https://doi.org/10.1016/j.ijheatmasstransfer.2020.120543

Optimized design - details



- Heat transfer enhanced by 113% at same operational cost
- Microvilli-like features are obtained



Initial design and continuation dependence

Uniform initial distribution

Initial beta=1

Phi = 13.69

Baseline design = 7.27

➔ Improvement of 88%

Initial design: 4 channel baseline

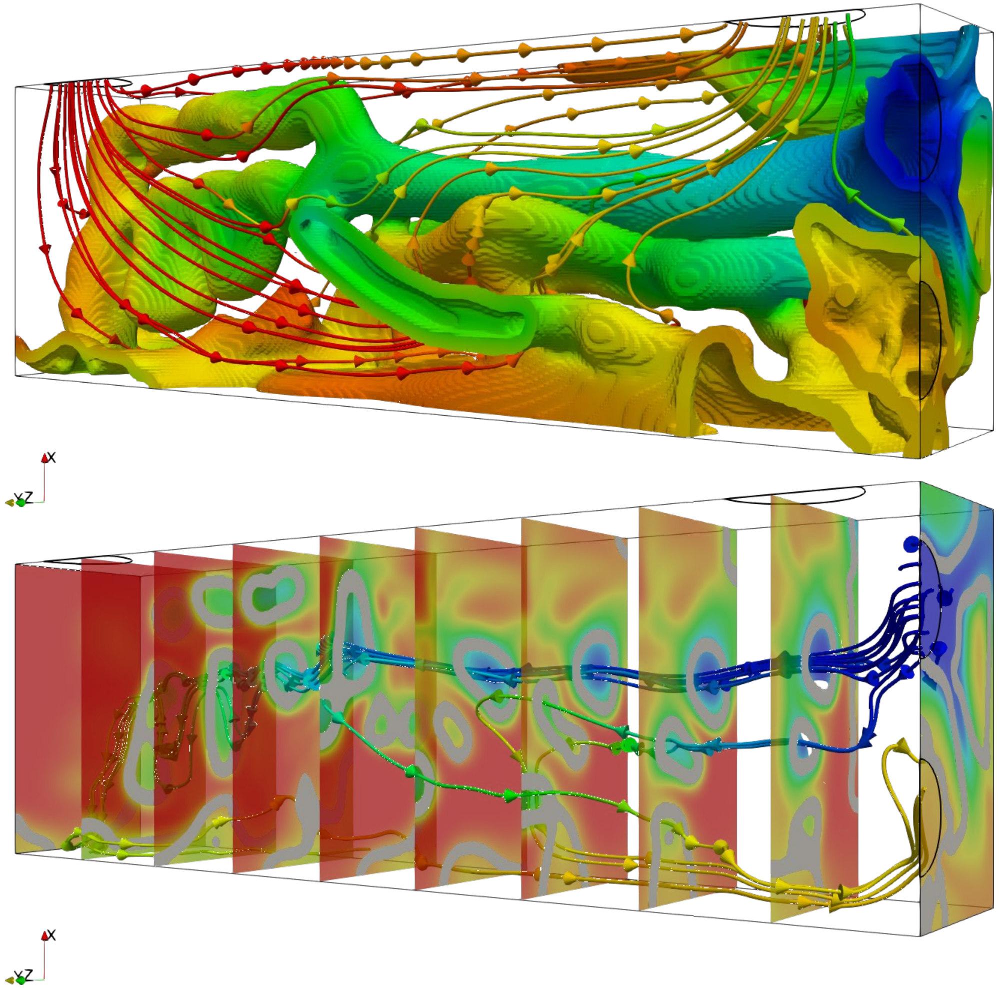
Initial beta = 8

Phi = 15.16

Baseline design = 7.27

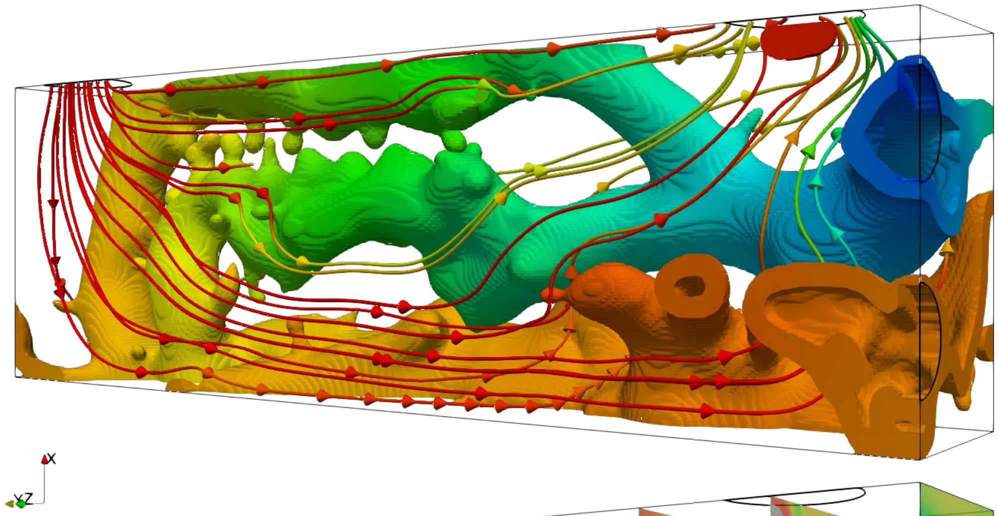
➔ Improvement of 109%

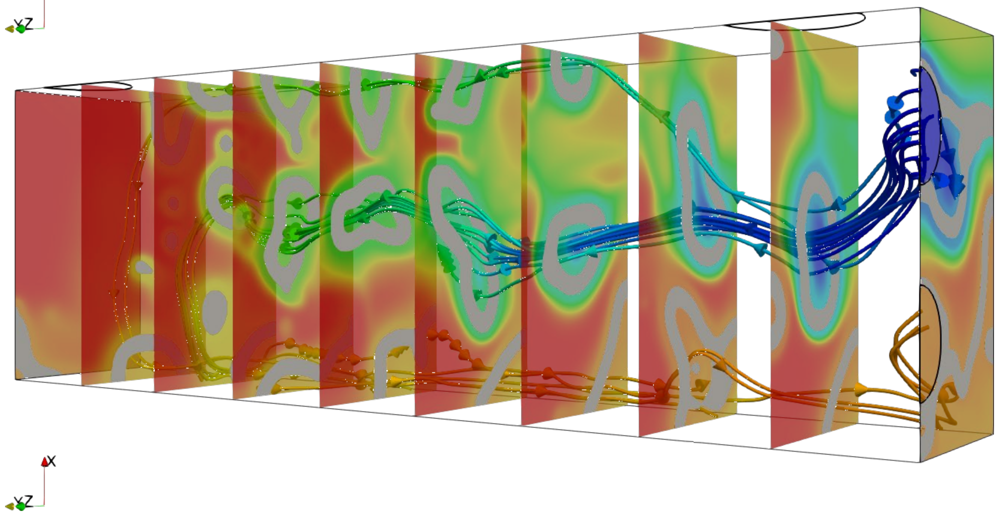
Lower solid conductivity



- Setting $k_{solid} = k_{coolant}$
- Extended surface are not advantageous under these conditions
- Serpentine channel is obtained in the design
- Improvement of 94.6% wrt. baseline

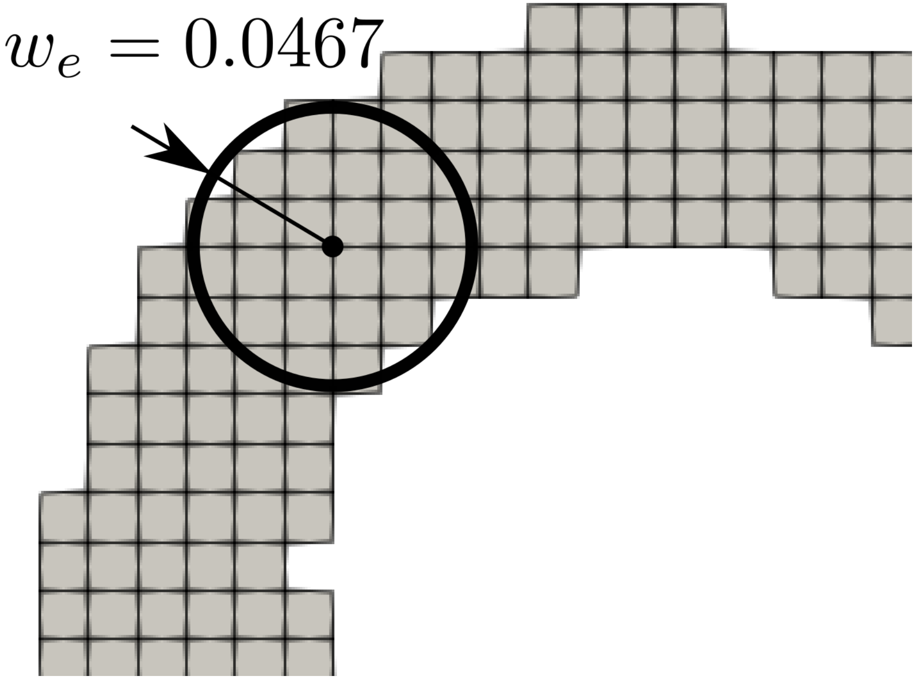
Higher wall thickness

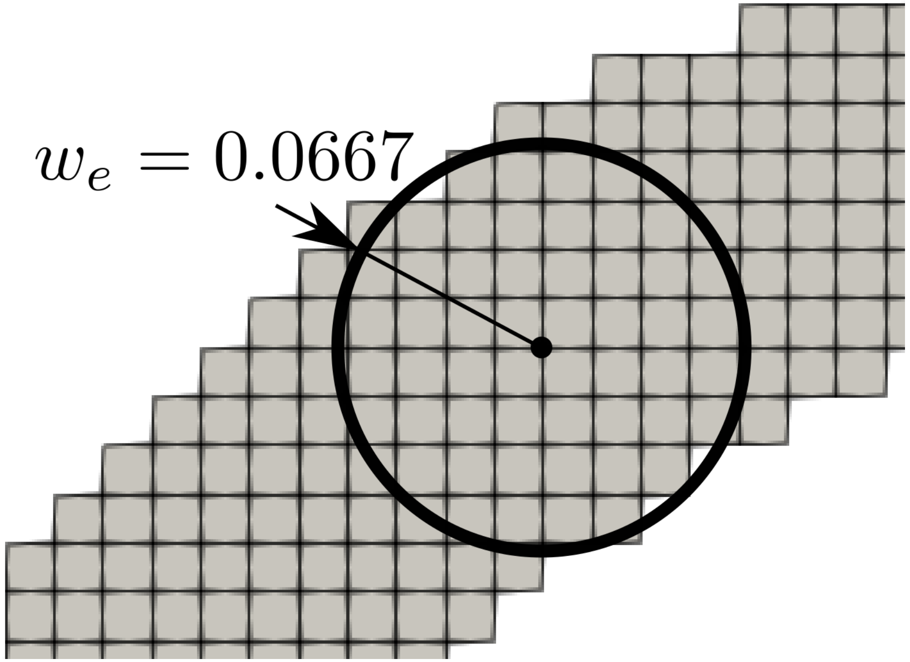




- Higher wall thickness can result in better manufaturability
- Features are generally larger
- Microvillies present in strongly reduced number
- Improvement of 105% wrt. baseline

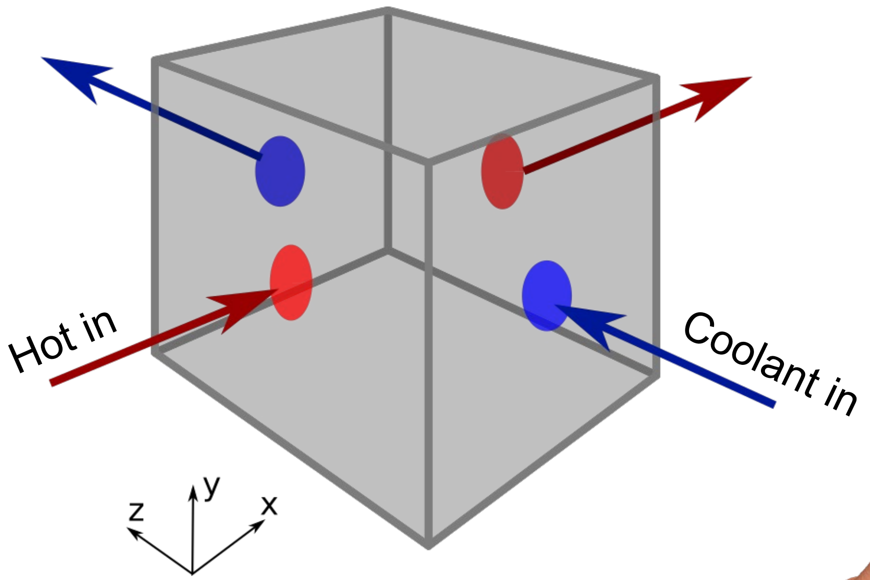
Thickness control



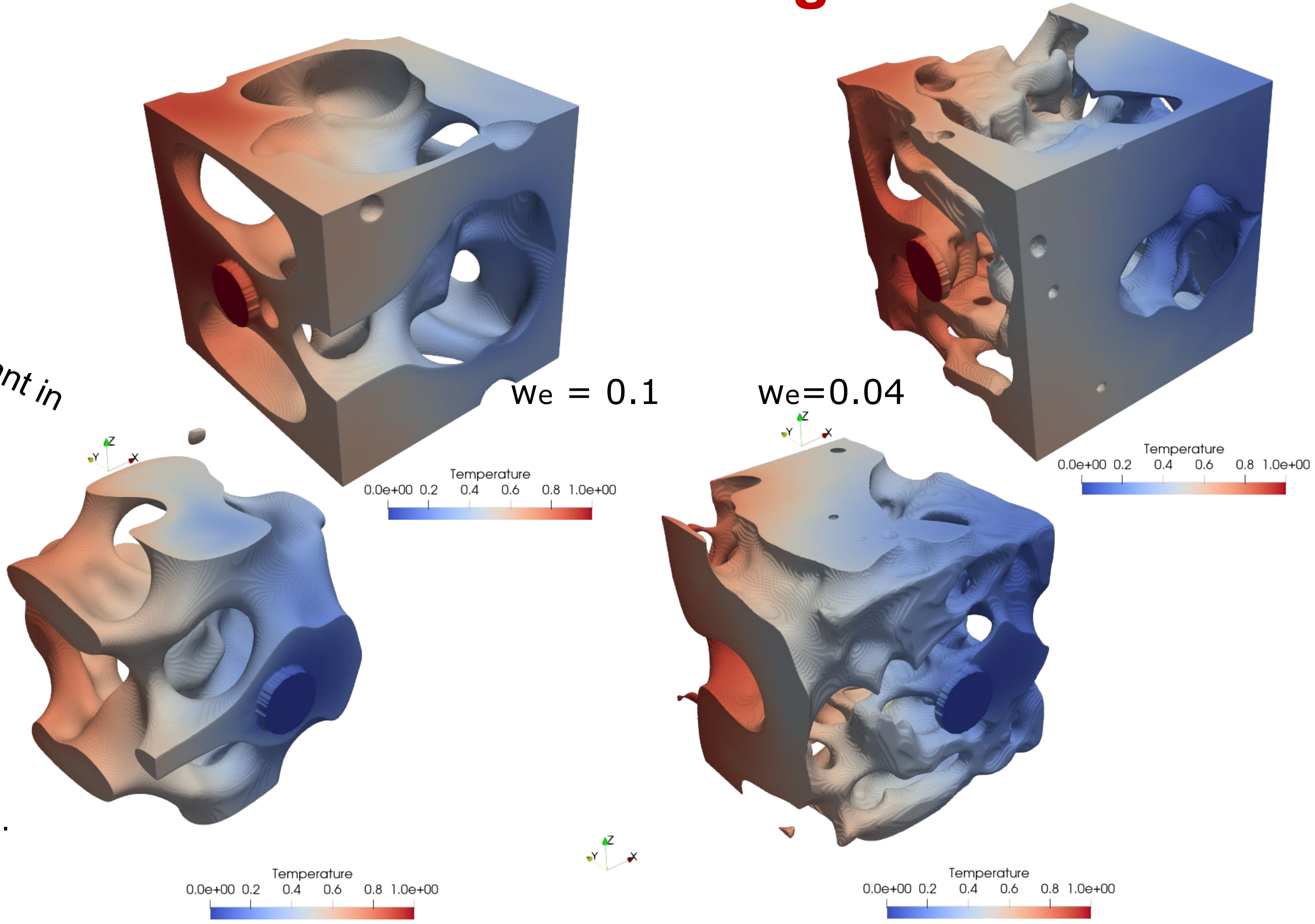


Slices of the wall in the shell-and-tube example

Application to cross flow heat exchanger



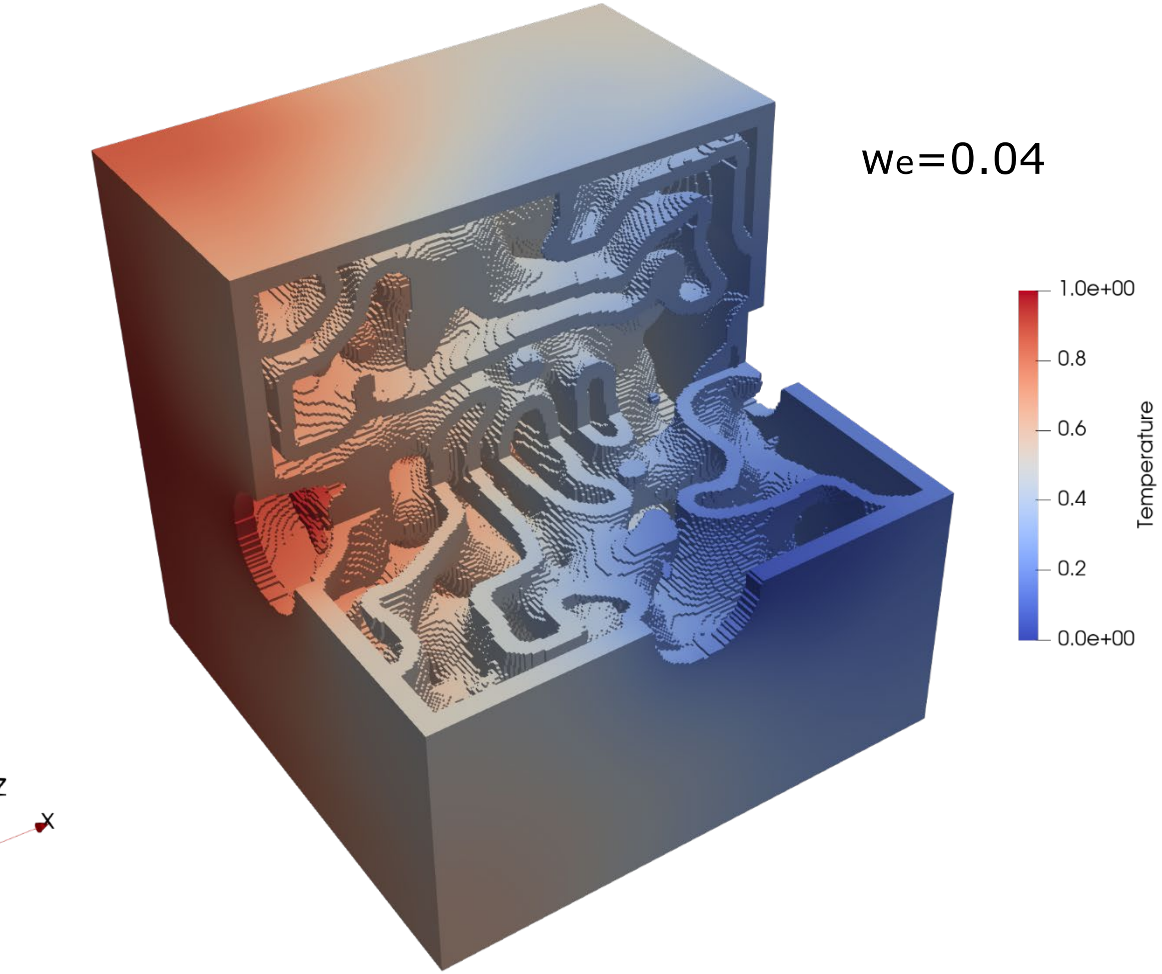
Pes = 200, Ck = 0.1, Re=100



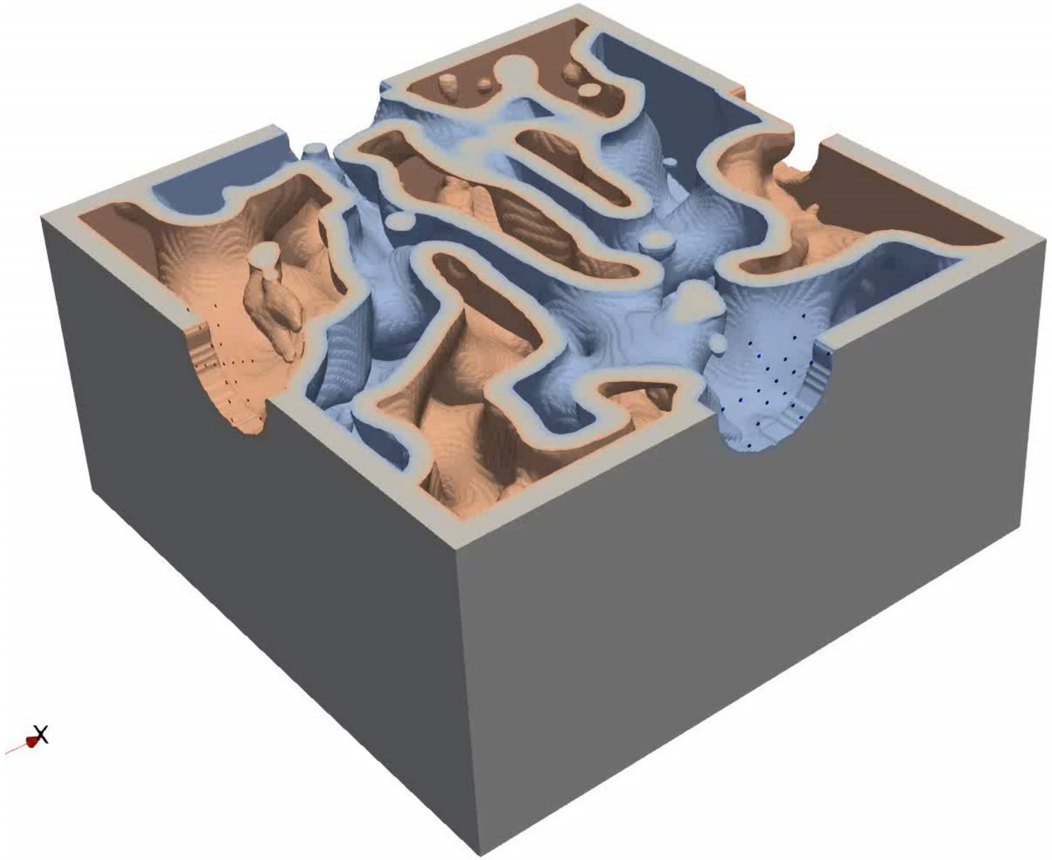
Design problem from:

Feppon, F., Allaire, G. ., Dapogny, C. & Jolivet, P. . Body-fitted topology optimization of 2D and 3D fluid-to-fluid heat exchangers. *Comput. Methods Appl. Mech. Eng.* **376**, 113638 (2020).

Solid structure and stream lines



Solid structure colored by temperature



Summary

- Two-fluids approach with clear interface identification – no constraints needed
- 2D optimization results strongly correlate with an analytical optimum
- Optimized 3D designs yield up to 113% improvement compared to baseline design
- Biological features such as microvillies appear in the optimized designs
  – Dependent on wall thickness and conductivity of solid material
- Applicable to other heat exchanger problems, e.g. Cross flow HEX
- Currently limited to laminar flow

Thank you for listening,

Questions?