TOP Webinar

Topology Optimization Webinar

Data-driven Approaches in Topology Optimization (DATO), January 26, 2021

A new data-driven topology optimization framework for structural optimization

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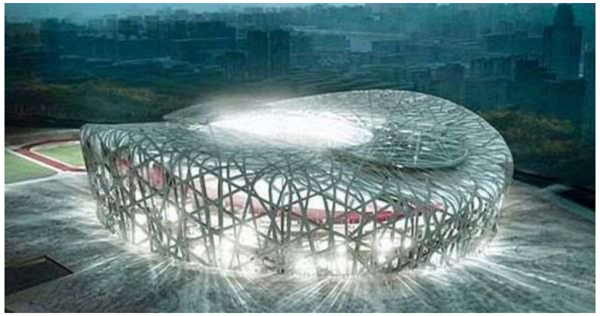
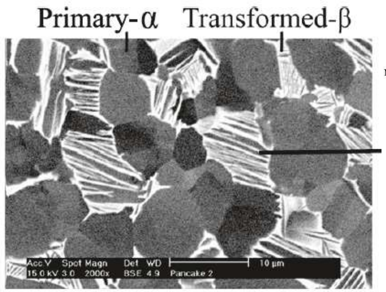
Content

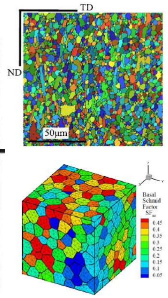
1. Research background

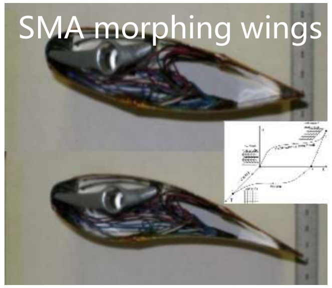
Topology optimization: lightweight and high-performance structures

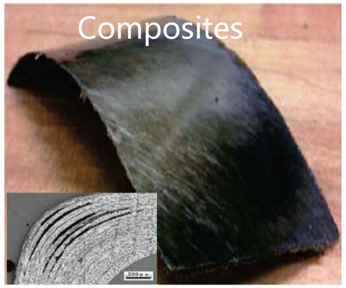


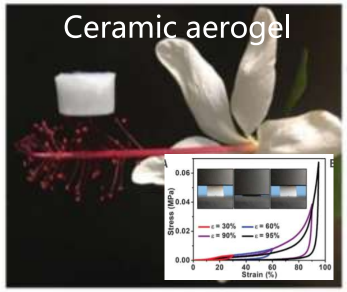
Material mechanical behavior description: a long-term open question due to their intrinsic complexities and multiphysics effects





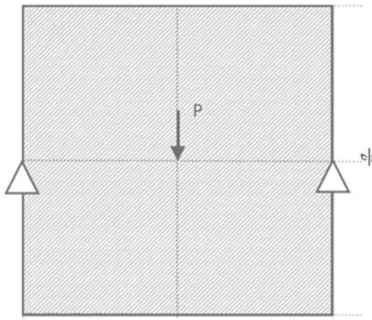


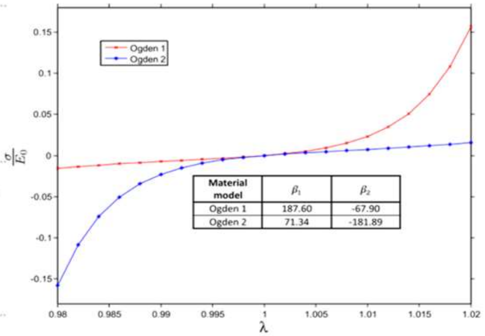


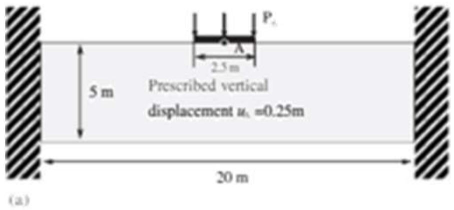


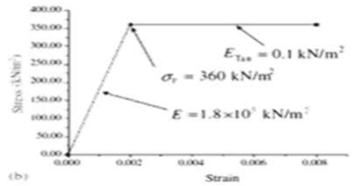
1. Research background

Material nonlinearity: One of the main concerns in nonlinear topology optimization

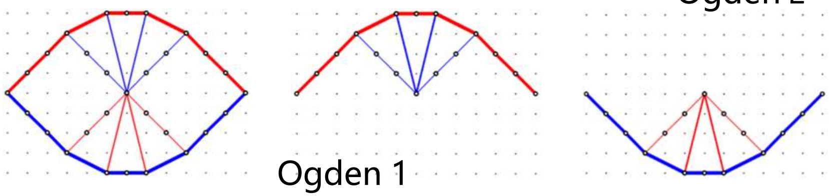








Linear elasticity **Material nonlinearity** **Linear elasticity** **Material nonlinearity**



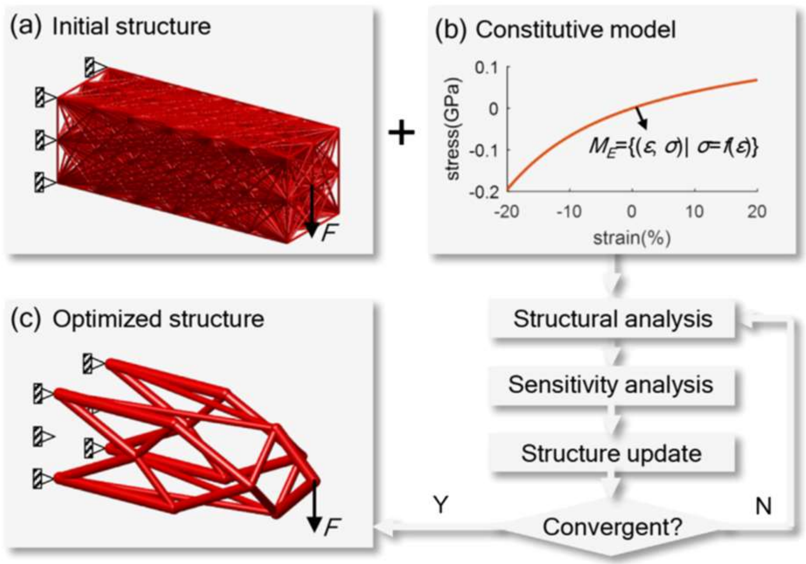
A. Ramos, G. Paulino. SMO (2015) 51, 287-304

G.H. Yoon, Y.Y. Kim, IJNME (2007) 69, 2196-2218

1. Research background

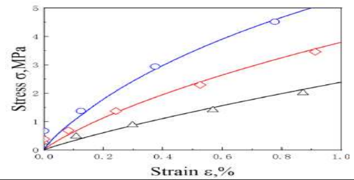
Traditional topology optimization framework

- Material nonlinearity: Described with phenomenological or physical-based constitutive models calibrated from experimental and computational data

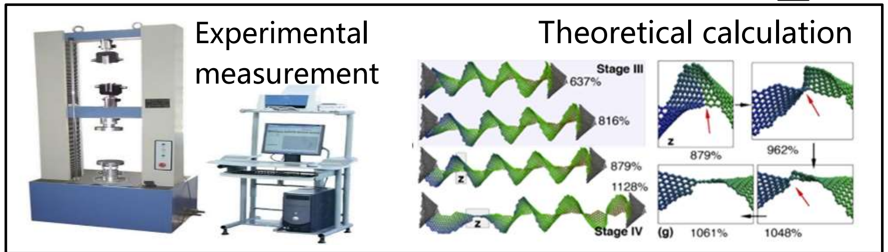


Phenomenological or physical based constitutive modeling

Type	Strain energy density W	Constitutive model
Ogden	$W=\sum_{i=1}^{I}\frac{\gamma_i}{\beta_i}(\lambda^{\beta_i}-1)$	$\sigma=\sum_{i=1}^{I}\gamma_i\lambda^{\beta_i-1}$
St Venant	$W=\frac{E}{8}(\lambda^2-1)^2$	$\sigma=\frac{E}{2}(\lambda^3-\lambda)$
Neo-Hookean	$W=\frac{E}{4}(\lambda^2-1-2\ln\lambda)$	$\sigma=\frac{E}{2}(\lambda-\lambda^{-1})$
Henky	$W=\frac{E}{2}(\ln\lambda)^2$	$\sigma=\frac{E}{\lambda}\ln\lambda$



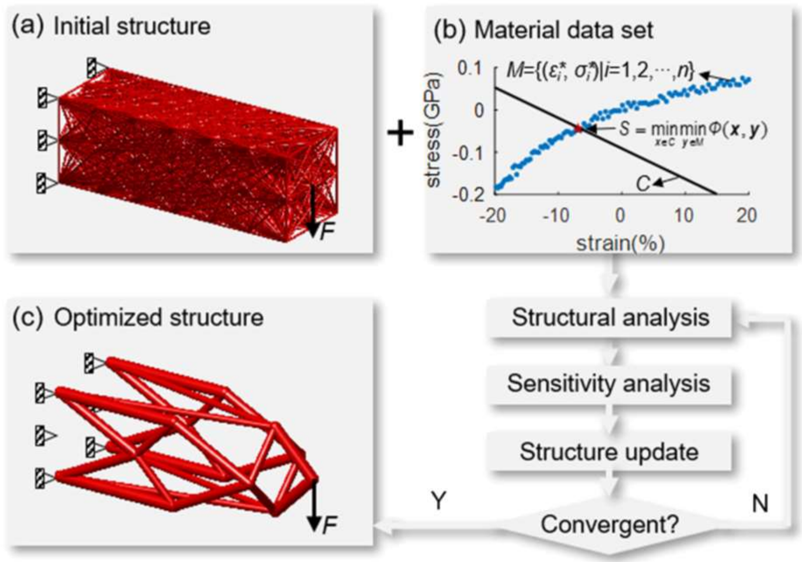
Data



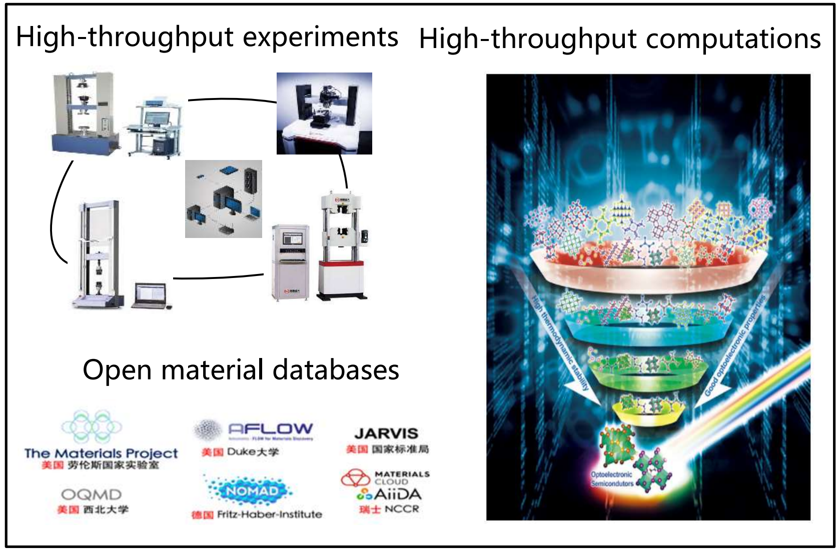
1. Research background

Data-driven topology optimization framework

- ❑ Instead of using empirical constitutive models, material data sets are directly used to describe the nonlinear constitutive relationship

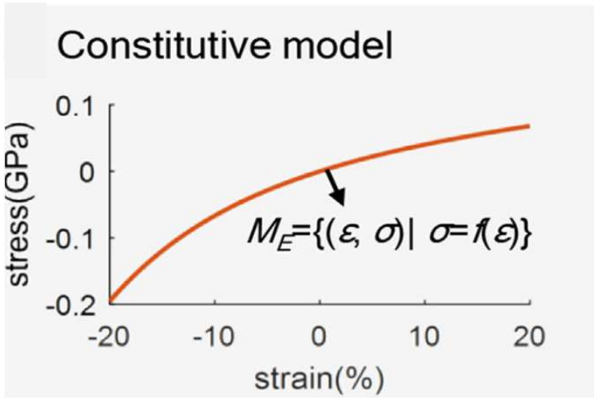


Material big data



2. Design methodology

Structural analysis with empirical constitutive model



❖ **Conservation laws**

❑ **Equilibrium + Neumann**

$$\mathrm{div}\boldsymbol{\sigma}+\boldsymbol{f}=\mathbf{0}\quad \text{in}\ \Omega$$

$$\boldsymbol{\sigma}\cdot\boldsymbol{n}=\boldsymbol{t}\quad \text{on}\ \Gamma_N$$

❑ **Kinematics + Dirichlet**

$$\varepsilon(\boldsymbol{u})=1/2(\nabla\boldsymbol{u}+\nabla^T\boldsymbol{u})$$

$$\boldsymbol{u}=\bar{\boldsymbol{u}}\quad \text{on}\ \Gamma_D$$

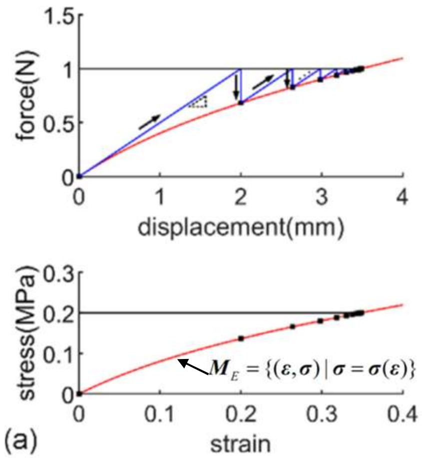
- **Newton-Raphson solver**

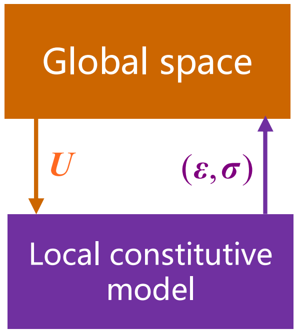
Principal of minimum potential energy $\quad \boldsymbol{\varepsilon}=\boldsymbol{BU}\quad \boldsymbol{u}=\boldsymbol{NU}$

$$\min\quad \Pi=\Pi^{\text{int}}-\Pi^{\text{ext}}=\frac{1}{2}\int_\Omega \boldsymbol{\sigma}^T\boldsymbol{\varepsilon}d\Omega-\int_\Omega \boldsymbol{f}^T\boldsymbol{u}d\Omega-\int_{\Gamma_N}\boldsymbol{t}^T\boldsymbol{u}d\Gamma$$

Variational theory $\quad \delta\Pi(\boldsymbol{U})=0\ \Rightarrow\ \int_\Omega \boldsymbol{B}^T\boldsymbol{\sigma}d\Omega=\int_\Omega \boldsymbol{N}^T\boldsymbol{f}d\Omega-\int_{\Gamma_N}\boldsymbol{N}^T\boldsymbol{t}d\Gamma\ \Rightarrow\ \boldsymbol{F}_{\text{int}}(\boldsymbol{U})=\boldsymbol{F}_{\text{ext}}$

Linearization $\quad \boldsymbol{F}_{\text{int}}(\boldsymbol{U}^{k+1})\approx\boldsymbol{F}_{\text{int}}(\boldsymbol{U}^k)+\boldsymbol{K}_{\tan}^k(\boldsymbol{U}^k)\Delta\boldsymbol{U}^k=\boldsymbol{F}_{\text{ext}}$





Residual $\boldsymbol{R}$

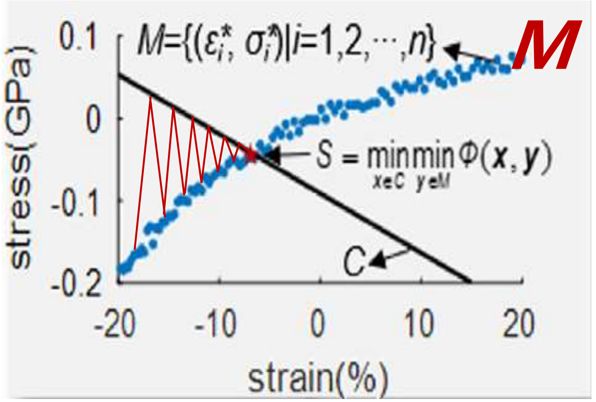
$$\Leftarrow\ \begin{cases}\boldsymbol{K}^{\tan}\Delta\boldsymbol{U}^k=\boxed{\boldsymbol{F}_{\text{ext}}-\boldsymbol{F}_{\text{int}}}\\ \boldsymbol{U}^{k+1}=\boldsymbol{U}^k+\Delta\boldsymbol{U}^k\end{cases}$$

$$\Leftarrow\ \begin{cases}\boldsymbol{\varepsilon}=\boldsymbol{BU}\\ \boldsymbol{\sigma}=\boldsymbol{\sigma}(\boldsymbol{\varepsilon})\end{cases}$$

Convergence criteria $\|\boldsymbol{R}\|\le 10^{-\gamma}$ or $\|\Delta\boldsymbol{U}\|\le 10^{-\gamma}$

2. Design methodology

Structural analysis with discrete material dataset



❖ **Conservation laws** ***C***

❑ **Equilibrium + Neumann**

$$\mathrm{div}\boldsymbol{\sigma} + \boldsymbol{f} = \mathbf{0} \quad \text{in } \Omega$$

$$\boldsymbol{\sigma}\cdot\boldsymbol{n} = \boldsymbol{t} \quad \text{on } \Gamma_N$$

❑ **Kinematics + Dirichlet**

$$\boldsymbol{\varepsilon}(\boldsymbol{u}) = 1/2(\nabla\boldsymbol{u} + \nabla^T\boldsymbol{u})$$

$$\boldsymbol{u} = \bar{\boldsymbol{u}} \quad \text{on } \Gamma_D$$

- **Data-driven solver**

Local strain and stress pair $z = (\boldsymbol{\varepsilon}, \boldsymbol{\sigma}) \in C$ and $z^* = (\boldsymbol{\varepsilon}^*, \boldsymbol{\sigma}^*) \in M$

Distance metric $d(z,z^*) = \sqrt{\Psi(\varepsilon-\varepsilon^*) + \Psi'(\sigma-\sigma^*)}$

where $\Psi(\boldsymbol{\varepsilon}) = \frac{1}{2}\boldsymbol{\varepsilon}^T\boldsymbol{D}\boldsymbol{\varepsilon}$ strain energy density

$\Psi'(\boldsymbol{\sigma}) = \frac{1}{2}\boldsymbol{\sigma}^T\boldsymbol{D}^{-1}\boldsymbol{\sigma}$ complementary energy density

Minimize an energy-based distance between *C* and *M*

Nested optimization problem

$$\min_{z^*\in M}\min_{z\in C} \Phi = \int_\Omega d(z,z^*)^2 d\Omega = \int_\Omega \Psi(\varepsilon-\varepsilon^*) + \Psi'(\sigma-\sigma^*) d\Omega$$

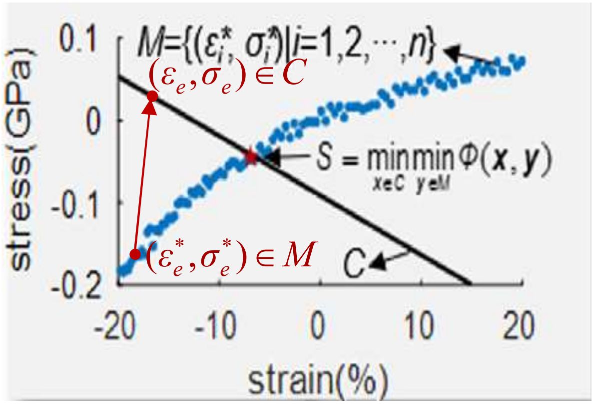
Inner subproblem

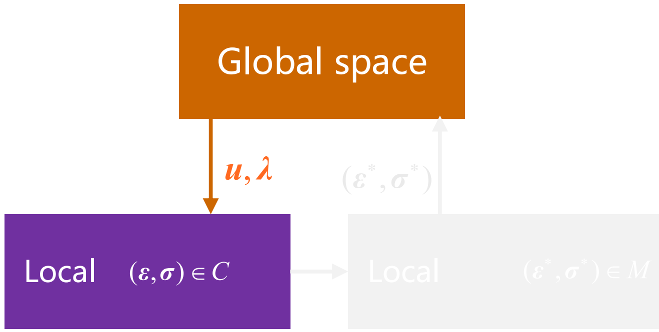
Outer subproblem

T. Kirchdoerfer, M. Ortiz. CMAME (2016) 304, 81-101

2. Design methodology

Structural analysis with discrete material dataset





- **Data-driven solver (Inner subproblem)**

given $(\varepsilon_e^*,\sigma_e^*)\in M \quad e=1,2,\cdots,m$ — Lagrange multiplier method

$$\min_{(\varepsilon_e,\sigma_e)} \Phi=\sum_{e=1}^{m}\int_{A_e}\int_{L_e}\Psi(\varepsilon_e-\varepsilon_e^*)+\Psi'(\sigma_e-\sigma_e^*)dAdL+\boldsymbol{\lambda}\left(\sum_{e=1}^{m}\int_{A_e}\int_{L_e}\boldsymbol{B}_e^T\sigma_e dAdL-\boldsymbol{F}_{\text{ext}}\right)=\tilde{\Phi}$$

s.t.

$$\sum_{e=1}^{m}\int_{A_e}\int_{L_e}\boldsymbol{B}_e^T\sigma_e dAdL=\boldsymbol{F}_{\text{ext}} \quad \text{(equilibrium)}$$

$$\varepsilon_e=\boldsymbol{B}_e\boldsymbol{u} \quad e=1,2,\cdots,m \quad \text{(compatibility)}$$

Taking all variations $\delta\tilde{\Phi}(\boldsymbol{u},\boldsymbol{\lambda},\sigma_e)=0$

$$\begin{cases}\delta\boldsymbol{u}=0 \Rightarrow \sum_{e=1}^{m}\int_{A_e}\int_{L_e}\boldsymbol{B}_e^T D_e(\boldsymbol{B}_e\boldsymbol{u}-\varepsilon_e^*)dAdL=0\\ \delta\boldsymbol{\lambda}=0 \Rightarrow \sum_{e=1}^{m}\int_{A_e}\int_{L_e}\boldsymbol{B}_e^T\sigma_e dAdL=\boldsymbol{F}_{\text{ext}}\\ \delta\sigma_e=0 \Rightarrow \sigma_e=D_e\boldsymbol{B}_e\boldsymbol{\lambda}+\sigma_e^* \quad e=1,2,\cdots,m\end{cases}$$

$$\boldsymbol{K}=\sum_{e=1}^{m}\int_{A_e}\int_{L_e}\boldsymbol{B}_e^T D_e\boldsymbol{B}dAdL$$

$$\begin{cases}\boldsymbol{K}\boldsymbol{u}=\sum_{e=1}^{m}\int_{A_e}\int_{L_e}\boldsymbol{B}_e^T D_e\varepsilon_e^* dAdL\\ \boldsymbol{K}\boldsymbol{\lambda}=\boldsymbol{F}_{\text{ext}}-\sum_{e=1}^{m}\int_{A_e}\int_{L_e}\boldsymbol{B}_e^T\sigma_e^* dAdL\end{cases}$$

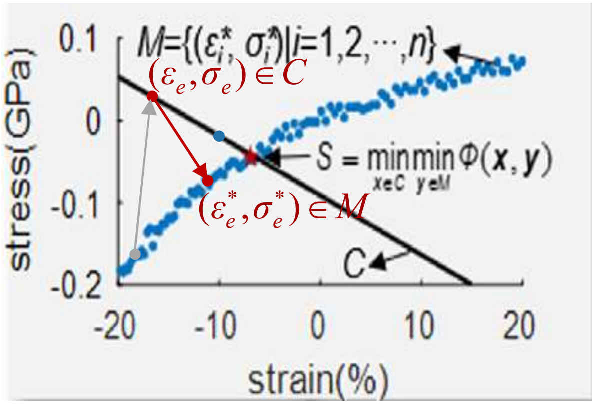
$\boldsymbol{F}_{\text{int}}$ correspond to guess σ_e^*

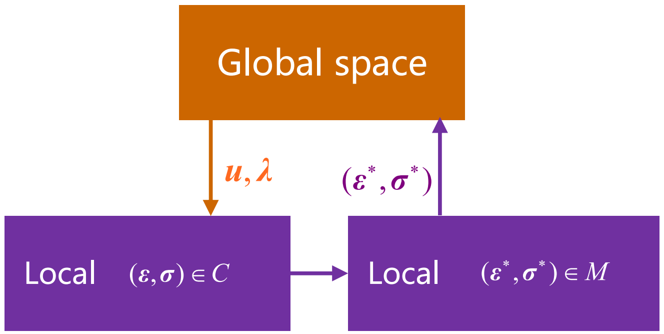
$$\begin{cases}\varepsilon_e=\boldsymbol{B}_e\boldsymbol{u}\\ \sigma_e=D_e\boldsymbol{B}_e\boldsymbol{\lambda}+\sigma_e^*\end{cases} \quad e=1,2,\cdots,m$$

T. Kirchdoerfer, M. Ortiz. CMAME (2016) 304, 81-101

2. Design methodology

Structural analysis with discrete material dataset





- **Data-driven solver (Outer subproblem)**

given $(\varepsilon_e,\sigma_e)\in C \quad e=1,2,\cdots,m$

$$\min_{(\varepsilon_e^*,\sigma_e^*)\in M} \quad \Phi=\sum_{e=1}^{m}\int_{A_e}\int_{L_e}\underbrace{\Psi(\varepsilon_e-\varepsilon_e^*)+\Psi'(\sigma_e-\sigma_e^*)}_{\text{Non-negativity}}dAdL$$

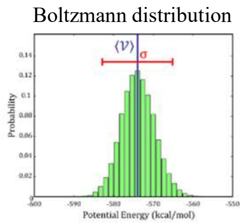
$$\min_{(\varepsilon_e^*,\sigma_e^*)\in M} \quad \varphi_e=d(z_e,z_e^*)^2=\Psi(\varepsilon_e-\varepsilon_e^*)+\Psi'(\sigma_e-\sigma_e^*) \quad e=1,2,\cdots,m$$

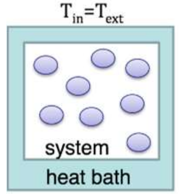
Nearest-neighbour-search (NNS)
- overwhelm the influence of the nearest data point
- sensitive to noises and outliers

Probabilistic data cluster

$$p_i(z_e^k,\beta^k)=\frac{1}{Z(z_e^k,\beta^k)}e^{-\beta^k d^2(z_e^k,z_i^*)} \quad i=1,2,\cdots,n$$

where $Z(z_e^k,\beta^k)=\sum_{i=1}^{n}e^{-\beta^k d^2(z_e^k,z_i^*)}$





Minimize the free energy

$$F(z_e^k,\beta^k)=-\frac{1}{\beta^k}\log Z(z_e^k,\beta^k)$$

Expected local states
$$\begin{cases}\varepsilon_e^{*k+1}=\sum_{i=1}^{n}p_i(z_e^k,\beta^k)\varepsilon_i^* \\ \sigma_e^{*k+1}=\sum_{i=1}^{n}p_i(z_e^k,\beta^k)\sigma_i^*\end{cases}$$

Simulate annealing to adjust β

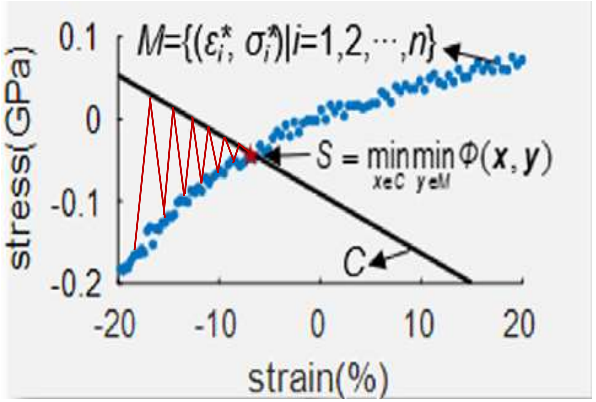
$$\frac{1}{\beta}\propto T$$

- Initially set β sufficient small that $F(z,\beta)$ is convex
  (sufficient high temperature T that a large data cluster accounts)
- Increase β to guide the solver towards minimum
  (decrease temperature T that a small data cluster accounts and the effect of outlier that is close to C than the data cluster is neglected)

T. Kirchdoerfer, M. Ortiz. CMAME (2017) 326, 622-641

2. Design methodology

Structural analysis with discrete material dataset



- **Data-driven solver (Convergence criteria)**

It is difficult to determine the threshold $10^{-\gamma}$

$\|\boldsymbol{R}^*\| \le 10^{-\gamma}$ or $\|\boldsymbol{\lambda}\| \le 10^{-\gamma}$

The data-driven solver terminates when

$(\varepsilon_e^{(K+1)}, \sigma_e^{(K+1)}) = (\varepsilon_e^{(K)}, \sigma_e^{(K)})$ for all $e = 1, 2, \cdots, m$

Solution

$$\begin{cases} \boldsymbol{K}\boldsymbol{u} = \sum_{e=1}^{m} \int_{A_e}\int_{L_e} \boldsymbol{B}_e^T D_e \varepsilon_e^* dAdL & ① \\ \boldsymbol{K}\boldsymbol{\lambda} = \boldsymbol{F}_{\text{ext}} - \sum_{e=1}^{m} \int_{A_e}\int_{L_e} \boldsymbol{B}_e^T \sigma_e^* dAdL & ② \end{cases}$$

$$\begin{cases} \varepsilon_e = \boldsymbol{B}_e \boldsymbol{u} & ③ \\ \sigma_e = D_e \boldsymbol{B}_e \boldsymbol{\lambda} + \sigma_e^* & ④ \end{cases} \quad e = 1, 2, \cdots, m$$

Structural equilibrium

$$\boldsymbol{F}_{\text{int}} = \sum_{e=1}^{m} \int_{A_e}\int_{L_e} \boldsymbol{B}_e^T \underbrace{\sigma_e^k}_{④} dAdL$$

$$\boldsymbol{F}_{\text{int}} = \underbrace{\left(\sum_{e=1}^{m} \int_{A_e}\int_{L_e} \boldsymbol{B}_e^T D_e \boldsymbol{B}_e dAdL \right) \boldsymbol{\lambda}^k + \sum_{e=1}^{m} \int_{A_e}\int_{L_e} \boldsymbol{B}_e^T \sigma_e^{*k} dAdL}_{②}$$

$$\boldsymbol{R} = \boldsymbol{F}_{\text{int}} - \boldsymbol{F}_{\text{ext}} = \boldsymbol{0}$$

Structural end-compliance

$$J(\boldsymbol{A}, \boldsymbol{u}) = \sum_{e=1}^{m} \int_{A_e}\int_{L_e} (\underbrace{\sigma_e^K}_{④})^T \underbrace{\varepsilon_e^K}_{③} dAdL$$

$$= \sum_{e=1}^{m} \int_{A_e}\int_{L_e} (D\boldsymbol{B}_e \boldsymbol{\lambda}^K + \sigma_e^{*K})^T (\boldsymbol{B}_e \boldsymbol{u}^K) dAdL$$

$$= \underbrace{(\boldsymbol{K}\boldsymbol{\lambda}^K + \sum_{e=1}^{m} \int_{A_e}\int_{L_e} \boldsymbol{B}_e^T \sigma_e^* dAdL)^T}_{②} \boldsymbol{u}^K$$

$$= \boldsymbol{F}_{\text{ext}}^T \boldsymbol{u}^K$$

Optimization problem

$$\min_{\boldsymbol{A}} \quad J(\boldsymbol{A}, \boldsymbol{u}) = \boldsymbol{F}_{\text{ext}}^T \boldsymbol{u}$$

$$\text{s.t.} \quad \boldsymbol{R}(\boldsymbol{A}, \boldsymbol{u}) = \boldsymbol{F}_{\text{ext}} - \boldsymbol{F}_{\text{int}} = \boldsymbol{0}$$

$$V(\boldsymbol{A}) = \sum_{e=1}^{m} A_e L_e \le V_{\max}$$

$$A_e \ge 0, \quad e = 1, 2, \cdots, m$$

2. Design methodology

Sensitivity analysis with discrete material dataset

Adjoint method $\boldsymbol{R} = \boldsymbol{0}$

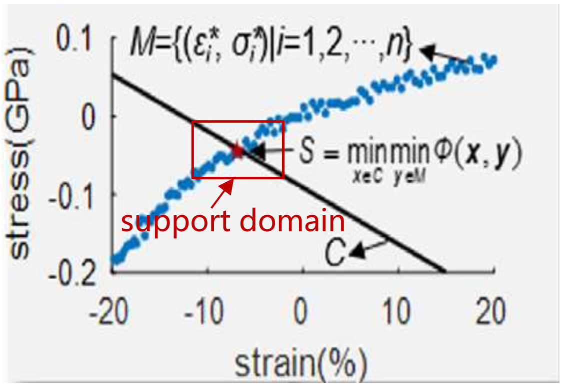
$$\frac{\partial \boldsymbol{F}_{\text{int}}}{\partial \boldsymbol{u}^K} = \sum_{e=1}^{m} \int_{A_e} \int_{L_e} \boldsymbol{B}_e^T \frac{\partial \sigma_e^K}{\partial \boldsymbol{u}^K} dAdL = \sum_{e=1}^{m} \int_{A_e} \int_{L_e} \boldsymbol{B}_e^T \frac{\partial \sigma_e^K}{\partial \varepsilon_e^K} \boldsymbol{B}_e dAdL$$

?

$$\tilde{J} = \boldsymbol{F}_{\text{ext}}^T \boldsymbol{u} + \boldsymbol{\beta}^T \boldsymbol{R}$$

$$\frac{\partial \tilde{J}}{\partial A_e} = \boldsymbol{F}_{\text{ext}}^T \frac{\partial \boldsymbol{u}^K}{\partial A_e} + \boldsymbol{\beta}^T \left(\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}^K} \frac{\partial \boldsymbol{u}^K}{\partial A_e} + \frac{\partial \boldsymbol{R}}{\partial A_e} \right) = \left(\boldsymbol{F}_{\text{ext}}^T - \boldsymbol{\beta}^T \frac{\partial \boldsymbol{F}_{\text{int}}}{\partial \boldsymbol{u}^K} \right) \frac{\partial \boldsymbol{u}^K}{\partial A_e} - \boldsymbol{\beta}^T \frac{\partial \boldsymbol{F}_{\text{int}}}{\partial A_e}$$

$$\frac{\partial J}{\partial A_e} = -\boldsymbol{\beta}^T \frac{\partial \boldsymbol{F}_{\text{int}}}{\partial A_e}$$



Rrepresentation of $(\varepsilon_e^K, \sigma_e^K)$ with a data cluster in *M*

Data cluster $(\varepsilon_{i_e}^*, \sigma_{i_e}^*) \in M \quad i_e = 1, 2, \cdots, N_e$ within a support domain of $(\varepsilon_e^K, \sigma_e^K)$

where $d(\varepsilon_e^K, \varepsilon_{i_e}^*) \le d_I$ with d_I denoting the radius of the support domain

Define the relevance of each data $(\varepsilon_{i_e}^*, \sigma_{i_e}^*) \in M \quad i_e = 1, 2, \cdots, N_e$ to $(\varepsilon_e^K, \sigma_e^K)$

$p_{i_e}(\varepsilon_e^K) \quad i_e = 1, 2, \cdots, N_e$

?

Probabilistic representation $\sigma_e^K = \sum_{i_e=1}^{N_e} p_{i_e}(\varepsilon_e^K) \sigma_{i_e}^* \Rightarrow \frac{\partial \sigma_e^K}{\partial \varepsilon_e^K} = \sum_{i_e=1}^{N_e} \frac{\partial p_{i_e}(\varepsilon_e^K)}{\partial \varepsilon_e^K} \sigma_{i_e}^*$

2. Design methodology

Sensitivity analysis with discrete material dataset

$$\frac{\partial \sigma_e^K}{\partial \varepsilon_e^K} = \sum_{i_e=1}^{N_e} \frac{\partial p_{i_e}(\varepsilon_e^K)}{\partial \varepsilon_e^K} \sigma_{i_e}^*$$

Moving Least Square (MLS) $\boldsymbol{p}(\varepsilon_e^K) = \left[p_1(\varepsilon_e^K), p_2(\varepsilon_e^K), \cdots, p_{N_e}(\varepsilon_e^K) \right]^T$

$$\boldsymbol{p}(\varepsilon_e^K) = \boldsymbol{N}^T(\varepsilon_e^K)\boldsymbol{A}^{-1}(\varepsilon_e^K)\boldsymbol{B}^{-1}(\varepsilon_e^K)$$

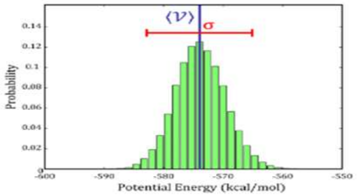
$$\boldsymbol{A}(\varepsilon_e^K) = \sum_{i_e=1}^{N_e} w(\varepsilon_e^K - \varepsilon_{i_e}^*)\boldsymbol{N}(\varepsilon_{i_e}^*)\boldsymbol{N}^T(\varepsilon_{i_e}^*)$$

$$\boldsymbol{B}(\varepsilon_e^K) = \left[w(\varepsilon_e^K - \varepsilon_1^*)\boldsymbol{N}(\varepsilon_1^*) \cdots w(\varepsilon_e^K - \varepsilon_{N_e}^*)\boldsymbol{N}(\varepsilon_{N_e}^*) \right]$$

$\boldsymbol{N}^T(\varepsilon_e^K)$ - basis function (independent of the material dataset)

$w(\varepsilon_e^K - \varepsilon_{i_e}^*)$ - weight function

Gauss function $w(\varepsilon_e^K - \varepsilon_{i_e}^*) = \dfrac{e^{-\eta^2(\varepsilon_e^K - \varepsilon_{i_e}^*)^2/d_I^2} - e^{-\eta^2}}{1 - e^{-\eta^2}}$



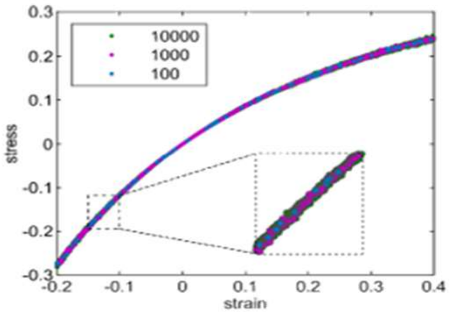
Henky model

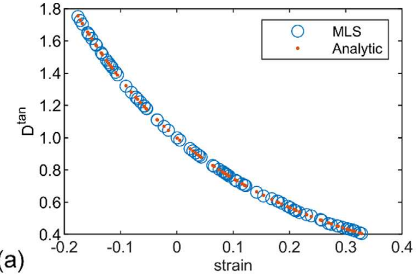
$\sigma = \frac{E}{1+\varepsilon}\ln(1+\varepsilon)$

$E = 1$

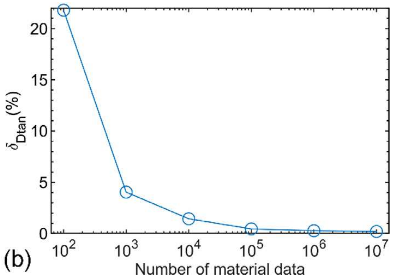
Material dataset

$M = \{(\varepsilon_i^*, \sigma_i^*) \mid i = 1, 2, \cdots, n\}$





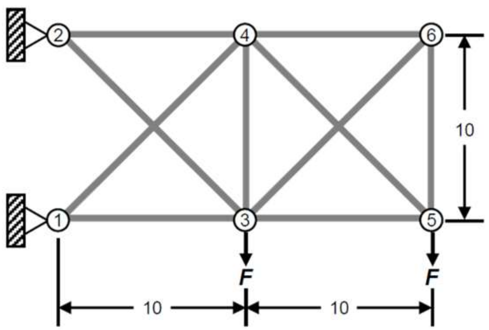
(a)



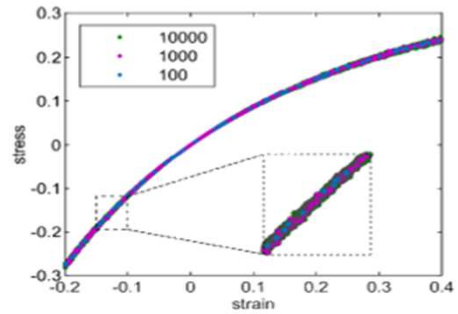
(b)

3. Numerical validation

10-bar plane truss



Material dataset I

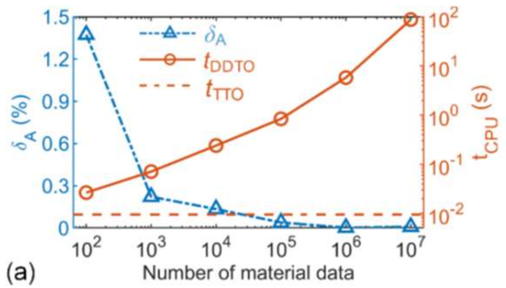


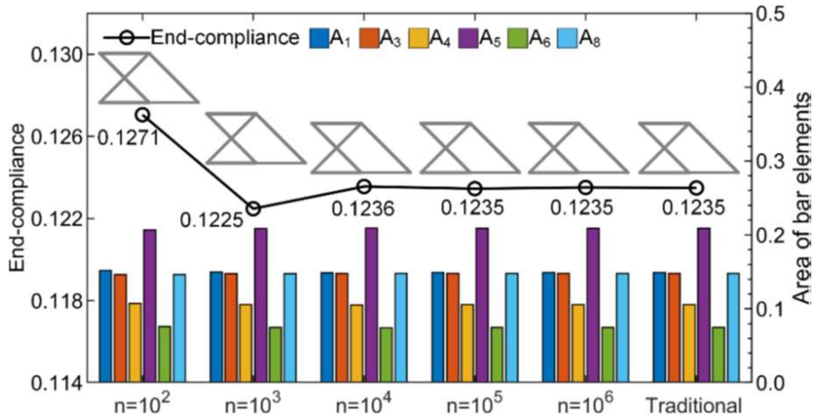
Henky model

$$\sigma = \frac{E}{1+\varepsilon}\ln(1+\varepsilon)$$

$$E = 1$$

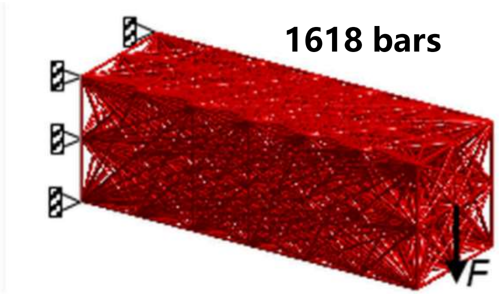
Discrepancy vs. Computational cost



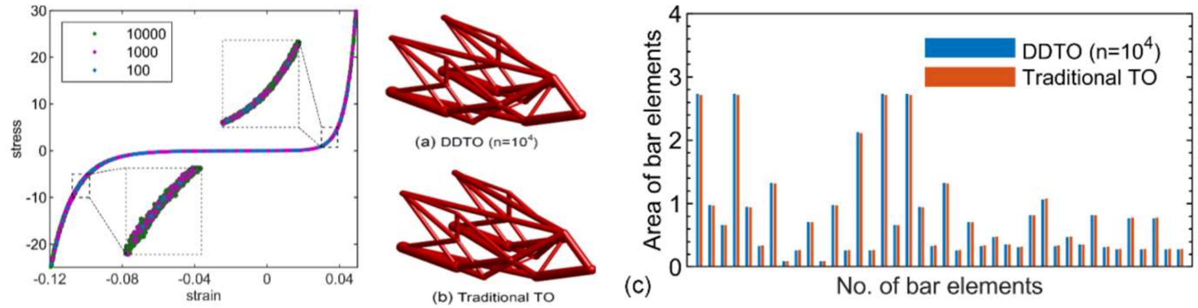


3. Numerical validation

Cantilevered space truss



Material I



Ogden model

$$\sigma = \sum_{i=1}^{I} \frac{\gamma_i}{\beta_i}\left((1+\varepsilon)^{\beta_i} - 1\right)$$

$$I = 2, \gamma_i = -\gamma_i = E/(\beta_1 - \beta_2)$$

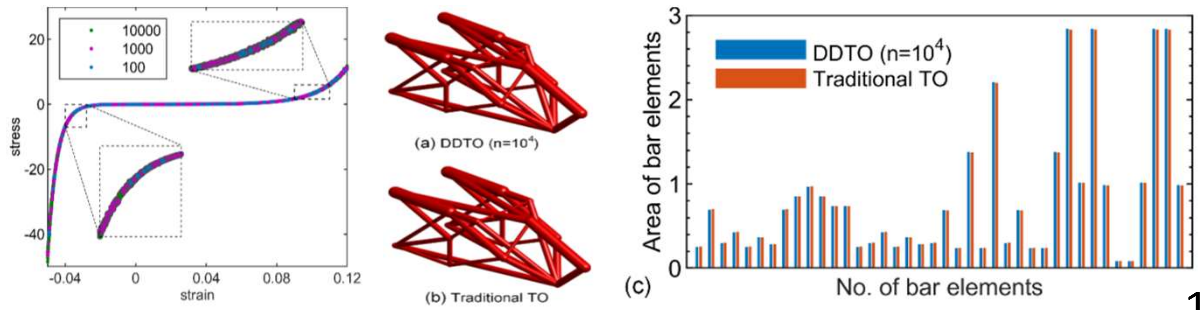
Material I

$$E = 1, \beta_1 = 187.6, \beta_1 = -67.9$$

Material II

$$E = 1, \beta_1 = 71.34, \beta_1 = -181.89$$

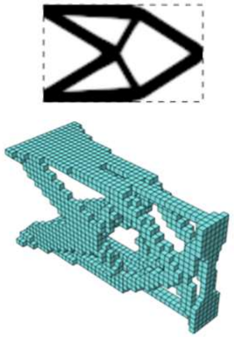
Material II



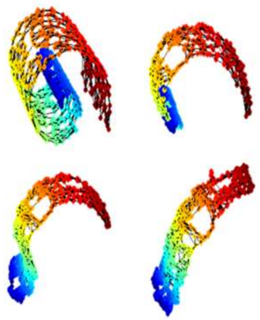
4. Conclusions & perspectives

- **First and preliminary studies are made using datasets for the description of material behavior in truss topology optimization**
- **Data-driven structural analysis employed for structural response analysis and sensitivity analysis are developed based on MLS approach**
- **The proposed DDTO framework is appealing in engineering applications with emerging new materials**

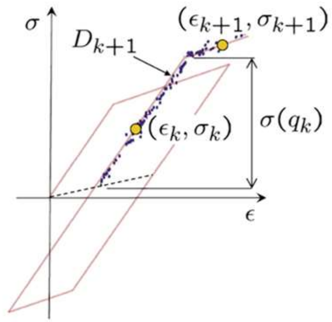
Continuum structure



Dimensionality reduction



Different material datasets



High efficiency

THANKS