Single variable-based multi-material structural optimization considering interface behavior

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Cheolwoong Kim, Hong Kyoung Seong, Jeonghoon Yoo
Yonsei University, Rep. of Korea
Il Yong Kim
Queen's University, Canada

OUTLINE

1. Introduction
2. Multi-material Topology Optimization using a Single Variable
3. Multi-material Design considering Interfacial Behavior
4. Numerical Examples
5. Conclusion

Based On: Single variable-based multi-material structural optimization considering interface behavior, Cheolwoong Kim, Hong Kyoung Seong, Il Yong Kim, Jeonghoon Yoo, Computer Method in Applied Mechanics and Engineering (2020), 367, 113114
https://doi.org/10.1016/j.cma.2020.113114

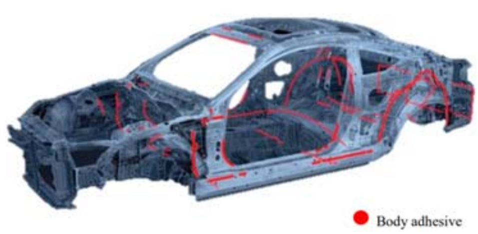
Motivation of this Research

- **Demands on bonding systems (Structural adhesive) for multi-material**

✓ **Technology for weight reduction of the vehicle body for environmental regulation**
➔ Multi material based BIW, etc.

✓ **Adhesive bonding method that can increase structural rigidity while reducing weight**
➔ It can reduce weight and noise compared to mechanical fasteners.
➔ Easy to attach parts that are difficult to weld due to the structure feature.

Volvo's'XC90' chassis with various materials bonded together (from Volvo)



Lexus IS body with structural adhesive (from Lexus.co.uk)

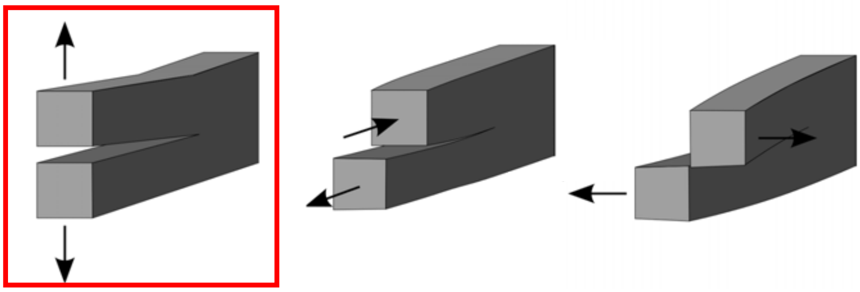
Motivation of this Research

- **Mode of Failure**

✓ **Mode I** : An opening or tensile mode

✓ **Mode II** : A sliding or in-plane shear mode

✓ **Mode III** : A tearing or anti-plane shear mode



Mode 1
Main reason of the failure

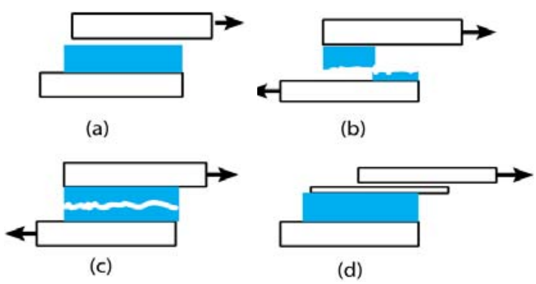
Mode 2
Highest resistance

Mode 3

- **Design of adhesive joints**

✓ The bonded zone is large

✓ **It is mainly loaded in mode 2**

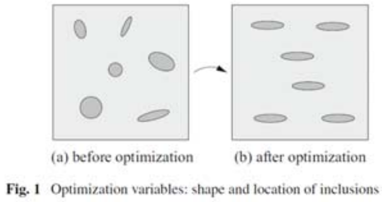
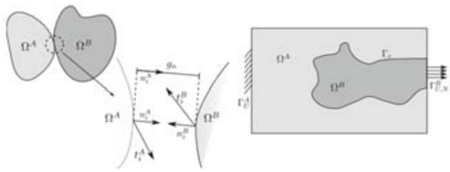
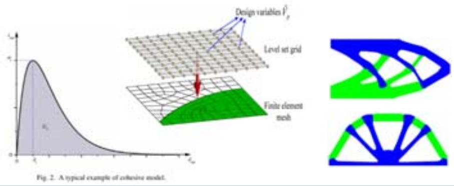


Adhesive Test (Mode 2)
(a) Adhesion Failure (b) Adhesion/Cohesion Failure
(c) Cohesion Failure and (d) Substrate Failure

Previous Researches

▪ **Demands**

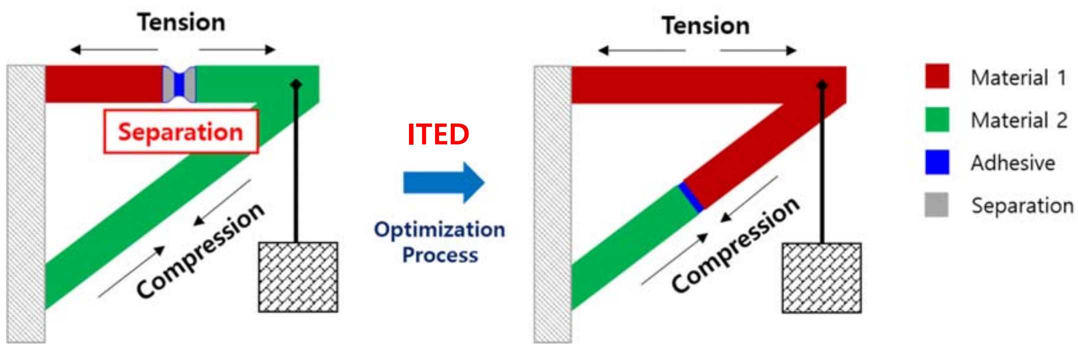
- Non-linear solver
  - Non-linear material
  - Non-linear B.C.
- Extended FEM
- Level set method
- Two component material

Paper	Topic	Keyword	Main Figure
Hilchenbach and Ramn 2015 Structural and Multidisciplinary Optimization	Optimizing the ductility of the composite material structure by designing the shape and location of inclusions in the matrix	➢ Cohesive zone model ➢ XFEM ➢ Two-component material	
Lawry and Maute 2015 Structural and Multidisciplinary Optimization	Optimization design of Multi material structure considering sliding between materials	➢ Sliding contact ➢ Level set method ➢ Two-component material	
Liu et al. 2016 Computer Methods in Applied Mechanics and Engineering	Design a structure to prevent separation of boundaries	➢ Cohesive zone model ➢ XFEM & Level set method ➢ Two-component material	

Goal of this Study

▪ **Multi-material topology optimization considering adhesive material behavior**

- Through Interfacial Tension Energy Density (ITED), the boundary, which is considered an adhesive, is designed to be placed in a non-tensile area.
- Stiffness is also secured simultaneously by combining with conventional compliance optimization



▪ **Main features**

Linear Analysis | Single Mesh | 2 or more materials

OUTLINE

Topology Optimization with Reaction Diffusion Equation

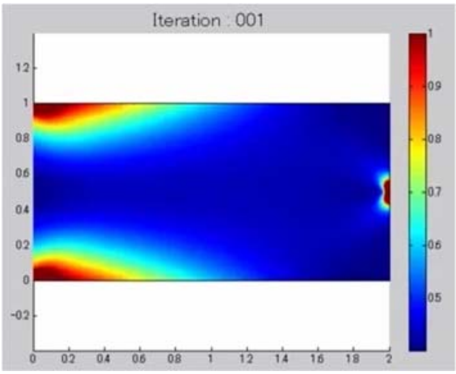
- RDE is used to update the design variable(element density) ϕ

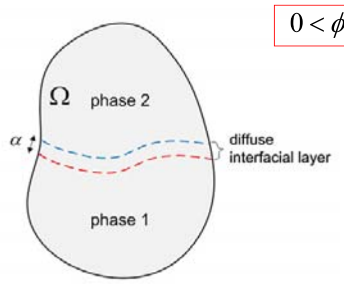
➔ The design complexity can be controlled through the diffusion coefficient, and it has the feature of convergence stability by the Laplacian term.

Reaction-diffusion Equation ➔

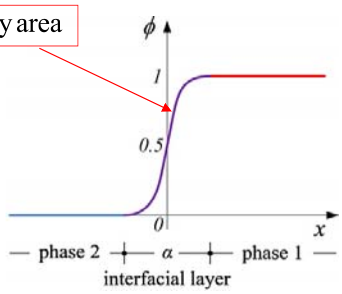
$$\begin{cases} \dfrac{\partial \phi(\mathbf{x},\, t)}{\partial t} = \alpha \nabla^2 \phi(\mathbf{x},\, t) - \dfrac{\partial \tilde{\mathcal{L}}(\phi,\, \lambda,\, \mathbf{x})}{\partial \phi} \quad \text{in } \mathbf{x} \in \Omega,\; 0 < t \le T \\ \dfrac{\partial \phi(\mathbf{x},\, t)}{\partial \hat{\mathbf{n}}} = 0 \quad \text{on } \partial\Omega \end{cases}$$

ϕ = Design variable (Element Density)
$\tilde{\mathcal{L}}$ = Augmented Lagrangian
λ = Lagrange multiplier
α = Diffusion Coefficient





$0 < \phi < 1$: Gray area



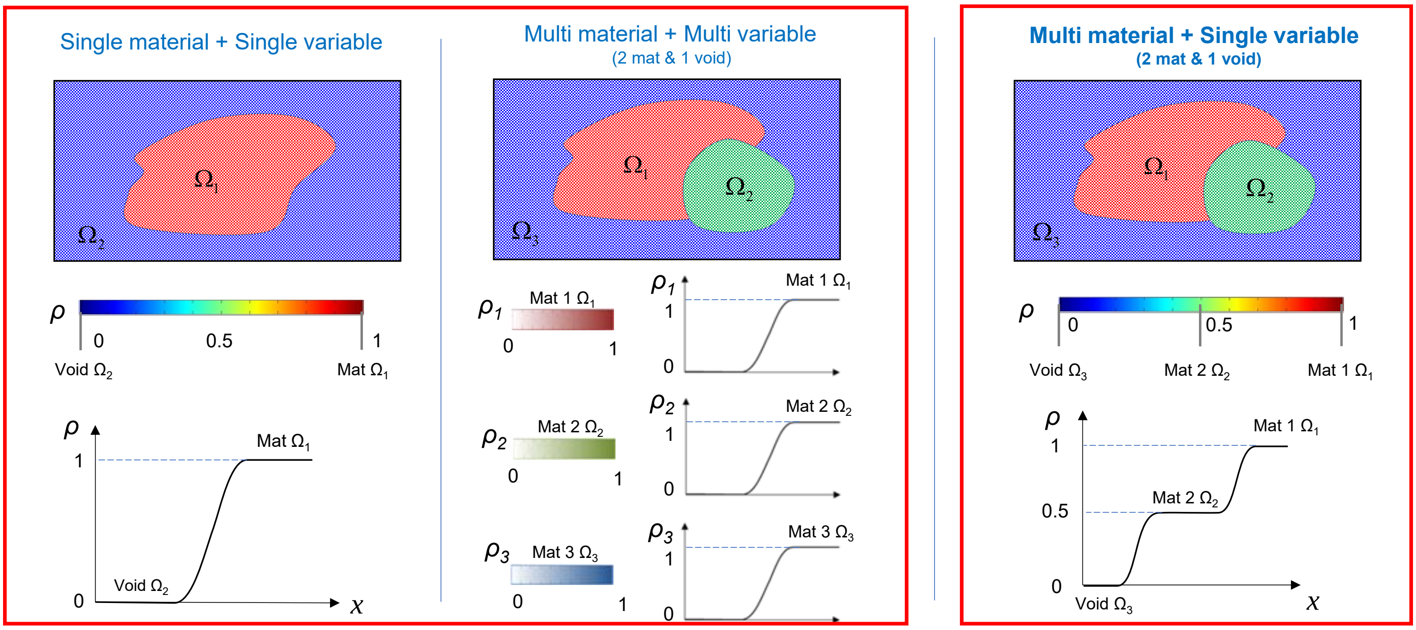
- Developed to express phase transition
- Clear boundaries by adjusting coefficient.
- Combining filtering + update scheme

"Topology Optimization Using a Reaction-Diffusion Equation", Jae Seok Choi, Takayuki Yamada, Kazuhiro Izui, Shinji Nishiwaki, Jeonghoon Yoo, Computer Methods in Applied Mechanics and Engineering, Vol. 200, Issues 29-32, pp. 2407-2420, 2011.

Phase Section Method: Single variable-based topology optimization (1)

- In a multi-material design, *m-1* variables are typically required to represent *m* materials.

➔ Phase Section Method is a method of expressing *m* substances with just one variable



Phase Section Method: Single variable-based topology optimization (2)

- Design variables should converge at different points between 0 and 1.

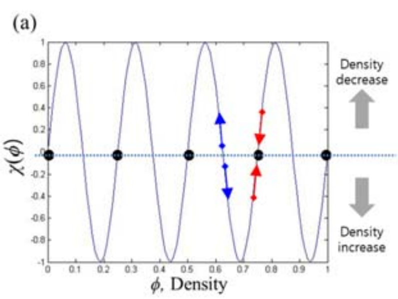
➔ It can be achieved by $\chi_n(\phi)$ and $\psi_n(\phi)$.

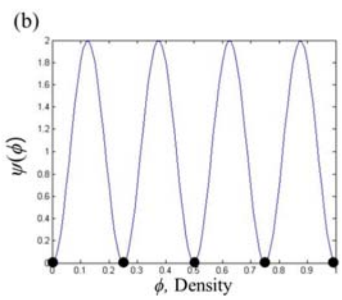
$$\frac{\phi-\phi'}{dt}=\kappa\nabla^2\phi-\left[\frac{\partial\tilde{\mathcal{L}}}{\partial\phi}(\phi')\right]$$

Original RD based Top opt

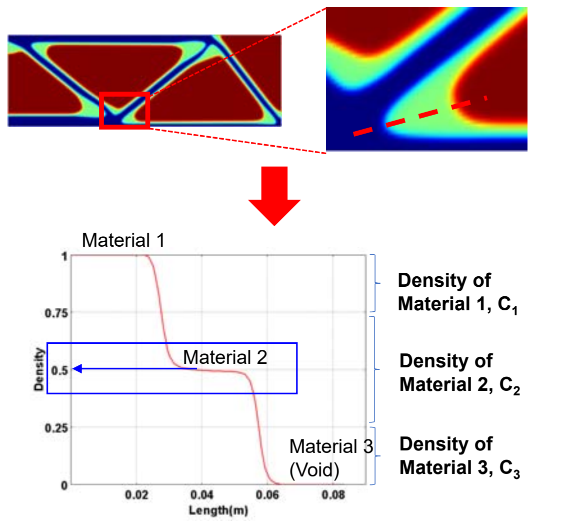
$$\frac{\phi-\phi'}{dt}=\kappa\nabla^2\phi-\left[q\times\boxed{\chi(\phi')}+\left(\boxed{\psi(\phi')}+1\right)\frac{\partial\tilde{\mathcal{L}}}{\partial\phi}(\phi')\right]$$

Phase Section method





$$\chi_n(\phi)=-\sin\left((n-1)\left(2\pi\phi-\frac{\pi}{(n-1)}\right)\right)\qquad \psi_n(\phi)=\cos\left((n-1)\left(2\pi\phi-\frac{\pi}{(n-1)}\right)\right)+1$$



"Multi-phase topology optimization with a single variable using the phase field design method", Hong Kyoung Seong, Cheol Woong Kim, Jeonghoon Yoo, Jaewook Lee, International Journal for Numerical Methods in Engineering, Vol. 119, Issue 5, pp. 334-360, 2019.

OUTLINE

Main idea - Interface Tension Energy Density (ITED)

- **Tension energy stored at the boundaries between materials**

ITED : $J_2 = \int_{\Omega_{\text{interface}}} \frac{1}{2} t_{tens} \varepsilon_{tens} d\Omega$

$$\mathbf{t} = \mathbf{S}\mathbf{n}$$

$$t_n = \mathbf{t} \cdot \mathbf{n}, \ \ where \ \ \mathbf{n} = \frac{\nabla \phi}{\|\nabla \phi\|}$$

$$t_n = \begin{cases} t_{tens} & \text{if } \ t_n > 0 : \text{tension} \\ t_{comp} & \text{if } \ t_n < 0 : \text{compression} \end{cases}$$

$$\boldsymbol{\varepsilon} = \mathbf{E}\mathbf{n}$$

$$\varepsilon_n = \boldsymbol{\varepsilon} \cdot \mathbf{n}, \ \ where \ \ \mathbf{n} = \frac{\nabla \phi}{\|\nabla \phi\|}$$

$$\varepsilon_n = \begin{cases} \varepsilon_{tens} & \text{if } \ \varepsilon_n > 0 : \text{tension} \\ \varepsilon_{comp} & \text{if } \ \varepsilon_n < 0 : \text{compression} \end{cases}$$

Example)

$\text{if } t_n > 0 : tension$ $\qquad$ $\text{if } t_n < 0 : Compression$

ITED > 0 $\qquad$ **ITED = 0**

Interface Tension Energy Density (ITED)

- **Minimize the Interface tension energy to avoid the delamination in the interface of the materials**

$$\underset{\phi}{\text{Min}}\ J(x,\phi(x)) = \eta_1 J_1 + \eta_2 J_2$$

where $J(x,\phi(x)) =$ Objective function

Compliance $J_1 = \int_{\Gamma_N} \mathbf{f}^{\mathrm{T}}\mathbf{u}(\phi)ds$ → **Stiffness**

ITED $J_2 = \int_{\Omega_{\text{interface}}} \frac{1}{2}\boxed{t_{tens}}\boxed{\varepsilon_{tens}}d\Omega$ → **Separation**

$$\eta_i = \frac{\int_\Omega dx}{\left\|\partial J_i / \partial\phi\right\|_{L^2}} \quad i = 1,2$$

$$\mathbf{t} = \mathbf{S}\mathbf{n}$$

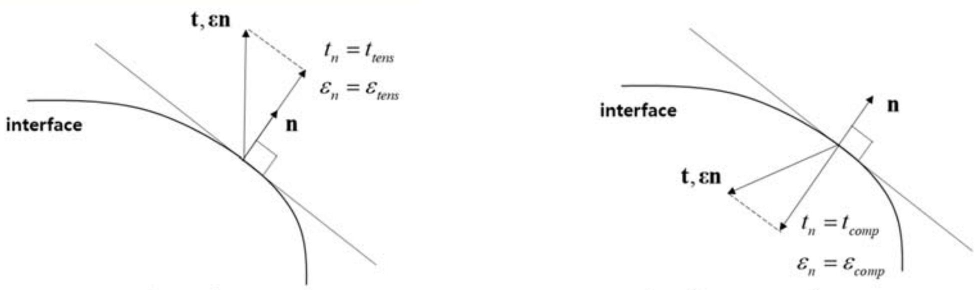
$$t_n = \mathbf{t}\cdot\mathbf{n},\ \ where\ \ \mathbf{n} = \frac{\nabla\phi}{\|\nabla\phi\|}$$

$$t_n = \begin{cases} t_{tens} & \text{if } t_n > 0 : \text{tension} \\ t_{comp} & \text{if } t_n < 0 : \text{compression} \end{cases}$$

$$\boldsymbol{\varepsilon} = \mathbf{E}\mathbf{n}$$

$$\varepsilon_n = \boldsymbol{\varepsilon}\cdot\mathbf{n},\ \ where\ \ \mathbf{n} = \frac{\nabla\phi}{\|\nabla\phi\|}$$

$$\varepsilon_n = \begin{cases} \varepsilon_{tens} & \text{if } \varepsilon_n > 0 : \text{tension} \\ \varepsilon_{comp} & \text{if } \varepsilon_n < 0 : \text{compression} \end{cases}$$



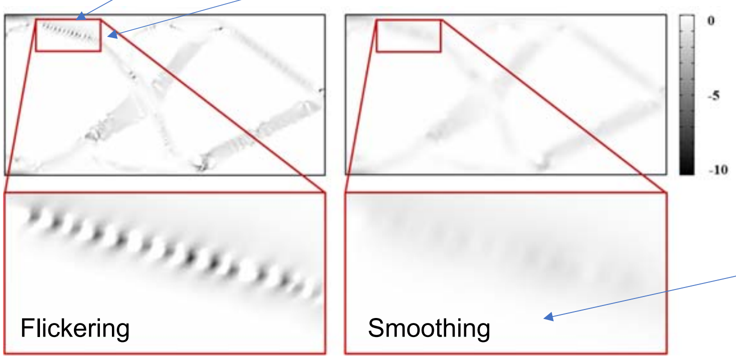
$t_n > 0$: tension $\qquad$ $t_n < 0$: compression

Helmholtz Filtering for Sensitivity

- **Singularity**

$$\frac{\partial j_2}{\partial\phi} = \frac{1}{2}\left(\mathbf{n}\cdot(\frac{\partial\mathbf{C}}{\partial\phi}\boldsymbol{\varepsilon})\mathbf{n}\right)\left(\mathbf{n}\cdot(\boldsymbol{\varepsilon}\mathbf{n})\right) + \left(\boxed{\frac{\partial\mathbf{n}}{\partial\nabla\phi}}\cdot(\mathbf{C}\boldsymbol{\varepsilon})\mathbf{n}\right)\left(\mathbf{n}\cdot(\boldsymbol{\varepsilon}\mathbf{n})\right) + \left(\mathbf{n}\cdot(\mathbf{C}\boldsymbol{\varepsilon})\mathbf{n}\right)\left(\boxed{\frac{\partial\mathbf{n}}{\partial\nabla\phi}}\cdot(\boldsymbol{\varepsilon}\mathbf{n})\right)$$

on $\Omega_{\text{interface}} = \{\phi \mid \nabla\phi > \alpha\}$



$\frac{dJ}{d\phi}$ $\qquad$ $\frac{d\tilde{J}}{d\phi}$

- **Alleviate singularity using Helmholtz filtering**

$$\boxed{\frac{dJ_1}{d\phi} = -\left(\frac{\partial\mathbf{C}(\phi)}{\partial\phi}\boldsymbol{\varepsilon}(\mathbf{u})\right):\boldsymbol{\varepsilon}(\mathbf{u})}$$

$$\boxed{\frac{dJ_2}{d\phi} = \int_\Omega \frac{\partial j_2}{\partial\phi}d\Omega - \int_\Omega\left(\frac{\partial\mathbf{C}}{\partial\phi}\boldsymbol{\varepsilon}(\mathbf{u})\right):\boldsymbol{\varepsilon}(\mathbf{p})d\Omega}$$

(Lazarov and Sigmund, 2011)

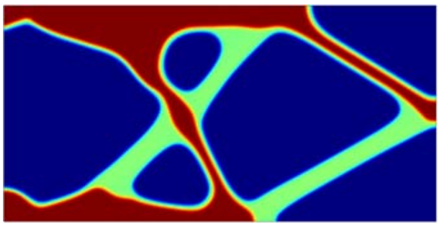
$$\boxed{\frac{dJ}{d\phi}} = \eta_1\boxed{\frac{dJ_1}{d\phi}} + \eta_2\boxed{\frac{dJ_2}{d\phi}}$$

$$\begin{cases} -r^2\nabla^2\left(\frac{d\tilde{J}}{d\phi}\right) + \frac{d\tilde{J}}{d\phi} = \boxed{\frac{dJ}{d\phi}} \\ \frac{\partial(dJ/d\phi)}{\partial\mathbf{n}} = 0 \end{cases}$$

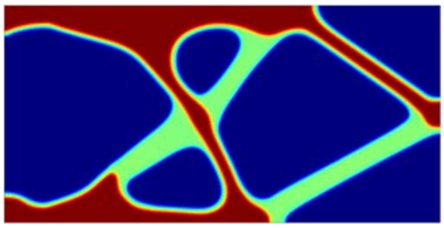
Effect of the Helmholtz Sensitivity Filtering

- **Singularity**

No filtering



Filtering r = 0.0026



- **Alleviate singularity using Helmholtz filtering**

Formulation

- **Phase section method**

$$\underset{\phi}{\textbf{minimize}} \quad J(x,\phi(x)) = \eta_1 J_1 + \eta_2 J_2$$

$$\textbf{subject to} \quad G(\phi) = \int_{\Omega_D} \phi\, dx - V_{req}\int_{\Omega_D} dx \le 0 \quad \text{and} \quad \frac{\sigma}{\exp(-0.5)}\left\{C_m(\phi) - C_{req\_m}\right\} = 0,$$

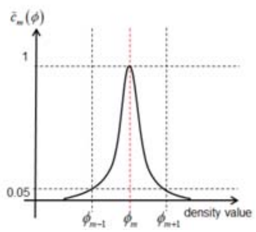
$$\text{where} \quad 0 < \phi_{\min} \le \phi \le 1, \quad m = 1, 2, 3, \ldots, n$$

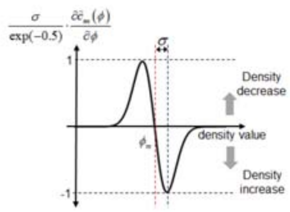
$$\text{and} \quad C_m(\phi) = \int_{\Omega_D} \exp\left(-\frac{(\phi-\phi_m)^2}{2\sigma^2}\right) dx \Big/ \int_{\Omega_D} dx = \int_{\Omega_D} \tilde{c}_m(\phi)dx \Big/ \int_{\Omega_D} dx$$

$F(\phi, \mathbf{u}(\phi))$: design objective
$G(\phi)$: volume fraction
V_{req} : volume requirement
$C_m(\phi)$: composition ratio
C_{req_m} : composition ratio requirement
$\tilde{c}_m$: density range
σ : bandwidth of $\tilde{c}_m$

	Target density	Area (2D) & Volume (3D)	Weight
Single-material	0 (void), 1 (material)	$V = \int_{\Omega_D} \phi\, dx \Big/ \int_{\Omega_D} dx$	V
Single variable based Multi-material	$\phi_m \ (m = 1, 2, \ldots, n)$	$C_m = \int_{\Omega_D} \tilde{c}_m(\phi)dx \Big/ \int_{\Omega_D} dx$	$\sum_{m=2}^{n} C_m \cdot \phi_m = \sum_{m=2}^{n} V_m = V$

- **Composition ratio constraint**





$$C_m(\phi) = \int_{\Omega_D} \exp\left(-\frac{(\phi-\phi_m)^2}{2\sigma^2}\right) dx \Big/ \int_{\Omega_D} dx = \int_{\Omega_D} \tilde{c}_m(\phi)dx \Big/ \int_{\Omega_D} dx$$

$$\frac{\partial C_m(\phi)}{\partial \phi} = \int_{\Omega_D} \left\{-\frac{(\phi-\phi_m)}{\sigma^2}\right\} \times \exp\left(-\frac{(\phi-\phi_m)^2}{2\sigma^2}\right) dx = \int_{\Omega_D} \frac{\partial \tilde{c}_m(\phi)}{\partial \phi} dx$$

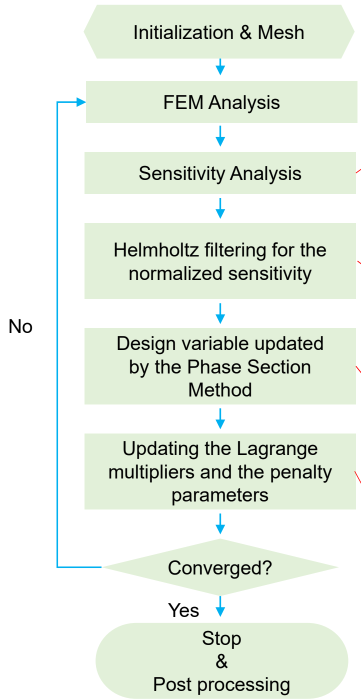
- **Optimizer**

$$\frac{\phi - \phi'}{dt} = \kappa \nabla^2 \phi - \left[q \times \chi(\phi') + \left(\psi(\phi') + 1\right) \frac{\partial \tilde{\mathcal{L}}}{\partial \phi'} \right]$$

$$\chi_n(\phi) = -\sin\left((n-1)\left(2\pi\phi - \frac{\pi}{(n-1)}\right)\right)$$

$$\psi_n(\phi) = \cos\left((n-1)\left(2\pi\phi - \frac{\pi}{(n-1)}\right)\right) + 1$$

Flowchart of the Optimization Process



$$\underset{\phi}{\textbf{minimize}}\quad J(\phi,\ \mathbf{u}(\phi))=\eta_1 J_1+\eta_2 J_2$$

$$\textbf{subject to}\quad G(\phi)=\int_{\Omega_D}\phi\,dx-V_{req}\int_{\Omega_D}dx\le 0\quad\text{and}\quad\frac{\sigma}{\exp(-0.5)}\left\{C_m(\phi)-C_{req\_m}\right\}=0$$

$$\frac{dJ}{d\phi}=\eta_1\frac{dJ_1}{d\phi}+\eta_2\frac{dJ_2}{d\phi}$$

$$-r^2\nabla^2\left(\frac{\partial\tilde{J}}{\partial\phi}\right)+\frac{\partial\tilde{J}}{\partial\phi}=\frac{\partial J}{\partial\phi}$$

$$\frac{\phi-\phi'}{dt}=\kappa\nabla^2\phi-\left[q\times\chi(\phi')+\left(\psi(\phi')+1\right)\frac{\partial\tilde{\mathcal{L}}}{\partial\phi}\right]$$

$$\lambda_0^{t_2}=\lambda_0^{t_1}+\xi_0^{t_1}\hat{G}\left(\phi_{t_1}\right),\quad \xi_0^{t_2}=\max(1,1.01\times\xi_0^{t_1})$$

$$\lambda_m^{t_2}=\lambda_m^{t_1}+\xi_m^{t_1}\hat{G}_m\left(\phi_{t_1}\right),\quad \xi_m^{t_2}=\max(20,\ 1.01\times\xi_m^{t_1})$$

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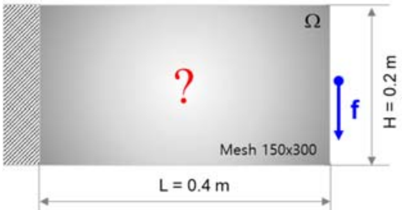
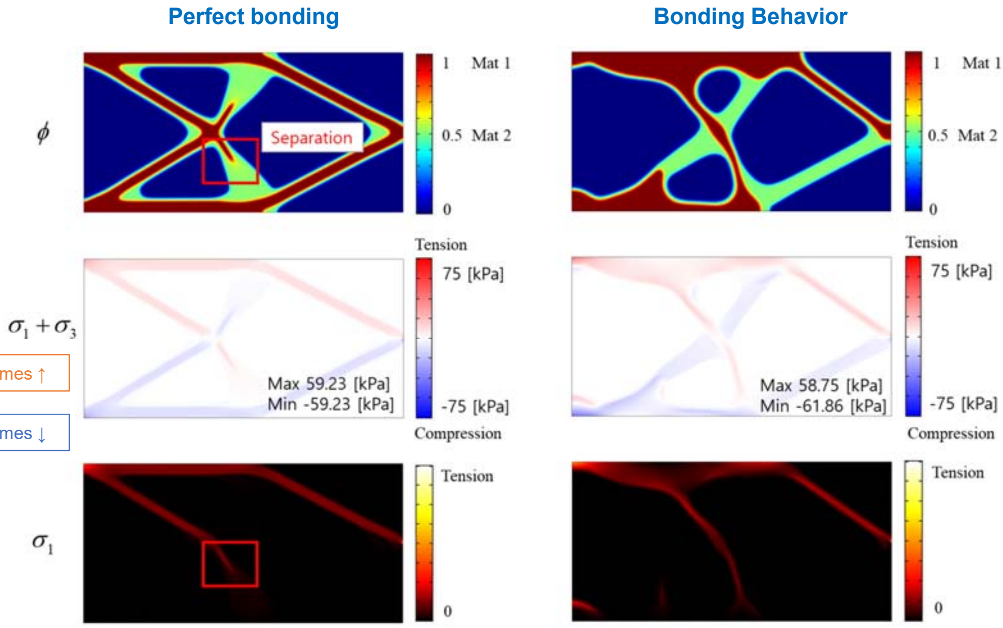
Cantilever Beam: Comparison with perfect bonding results

- With the ITED method, boundary separation can be avoided even if the stiffness is reduced
- Three-phase material

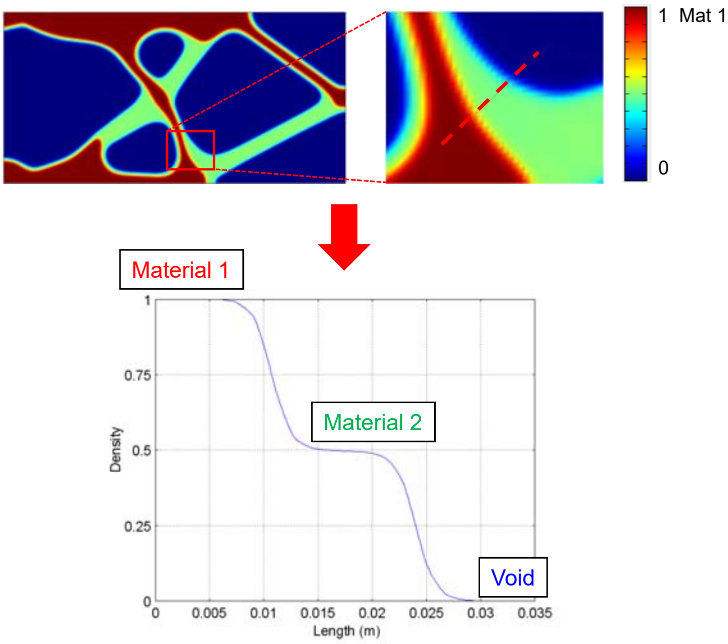


	Perfect bonding (Without the ITED)	Bonding Behavior (With the ITED)	
Compliance (J)	3.42 e-8	6.33 e-8	1.85 times ↑
ITED (J)	4.24 e-8	1.18 e-8	3.59 times ↓
Vol	0.273	0.272	
Mat.1	0.198	0.198	
Mat.2	0.198	0.198	

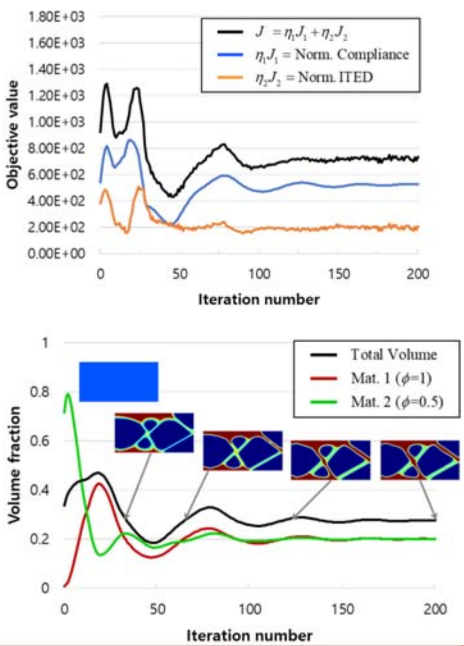


Cantilever Beam: Design variable & convergence graph

- Design variable

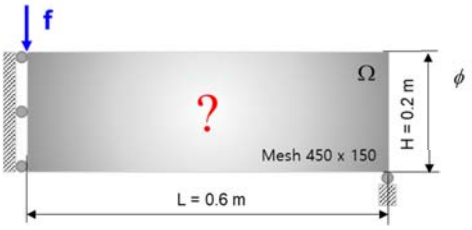


- Convergence graph



Half MBB: Comparison with perfect bonding results

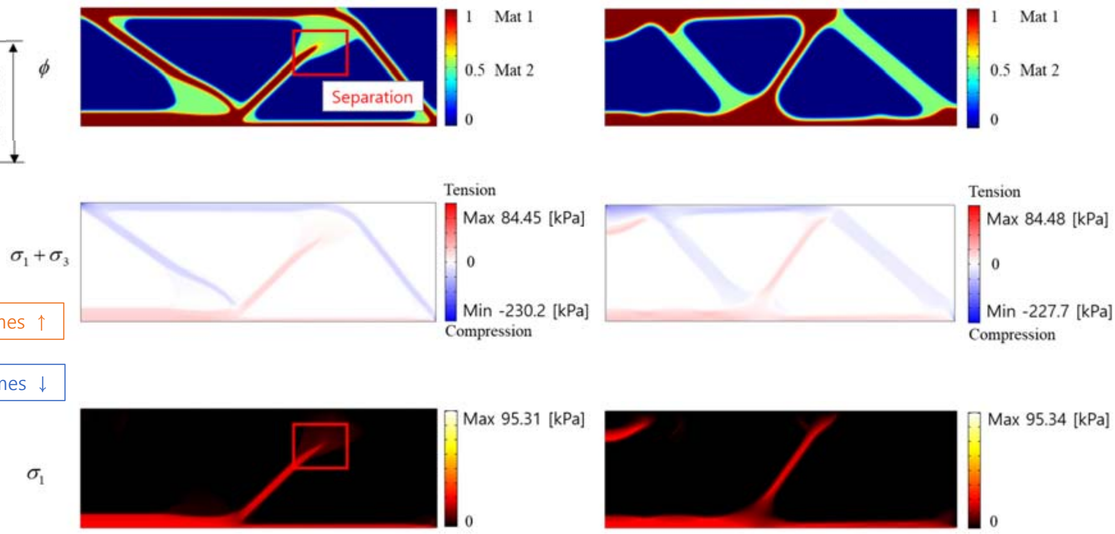
- With the ITED method, boundary separation can be avoided even if the stiffness is reduced
- Three-phase material



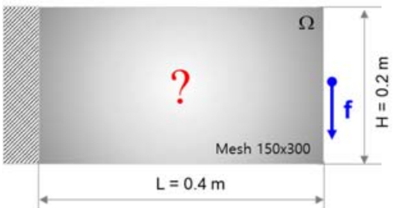
	Perfect bonding (Without the ITED)	Bonding Behavior (With the ITED)
Compliance (J)	9.90 E-8	1.53 E-7
ITED (J)	1.26 E-7	7.51 E-8
Vol	0.2765	0.275
Mat.1	0.198	0.198
Mat.2	0.198	0.198

1.91 times ↑

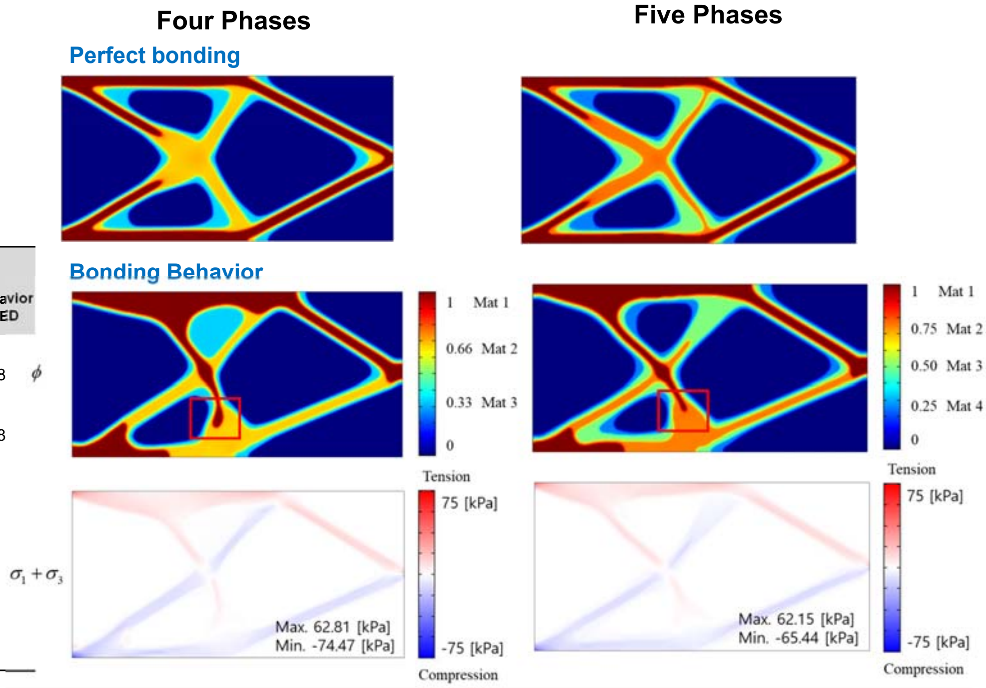
1.67 times ↓



Four & Five Phase Material



	4 material phases		5 material phases	
	Perfect bonding Without the ITED	Bonding behavior With the ITED	Perfect bonding Without the ITED	Bonding behavior With the ITED
Compliance [J]	3.662 e-8	5.797 e-8	3.829 e-8	5.104 e-8
ITED [J]	7.413 e-8	1.229 e-8	6.771 e-8	1.347 e-8
Vol.	0.2758	0.2745	0.2776	0.2764
Mat.1	0.1485	0.1485	0.1188	0.1188
Mat.2	0.1485	0.1485	0.1188	0.1188
Mat.3	0.1485	0.1485	0.1188	0.1189
Mat.4	-	-	0.1188	0.1189

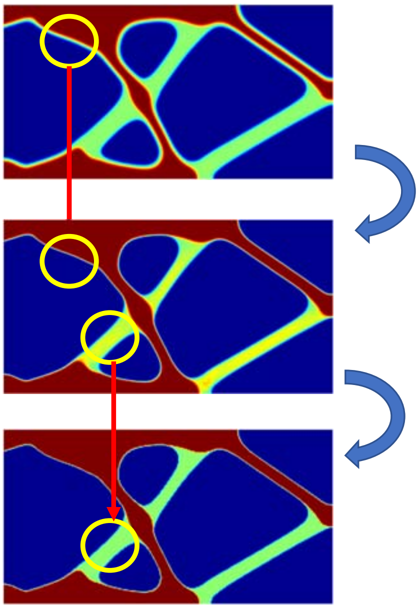


Post Processing: Using two steps of the Heaviside projection

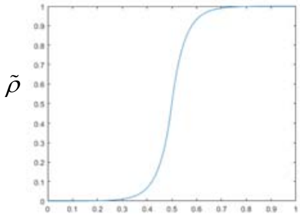
- After completing the initial concept design, gray area where it is not material are removed using Heaviside projection.

Optimal result

Post processing



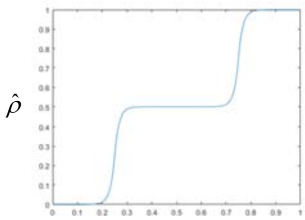
Distinguished by 0 and 1



$$\tilde{\rho} = \begin{cases} \eta\left(e^{-\beta(1-\rho/\eta)} + e^{-\beta}\left(\rho/\eta - 1\right)\right) & \text{if } 0 \le \rho \le \eta \\ \eta + \eta\left[1 - e^{-\beta(\tilde{\rho}-\eta)/\eta} + e^{-\beta}(\tilde{\rho}-\eta)/\eta\right] & \text{if } \eta < \rho \le 2\eta \end{cases}$$

where,
$\beta = 10$
$\eta = 0.5$

Reducing the medium density value



$$\hat{\rho} = \begin{cases} \eta\left[e^{-\beta(1-\tilde{\rho}/\eta)} + e^{-\beta}\left(\tilde{\rho}/\eta - 1\right)\right] & \text{if } 0 \le \tilde{\rho} \le \eta \\ \eta + \eta\left[1 - e^{-\beta(\tilde{\rho}-\eta)/\eta} + e^{-\beta}(\tilde{\rho}-\eta)/\eta\right] & \text{if } \eta < \tilde{\rho} \le 2\eta \\ 2\eta + \eta\left[e^{-\beta(1-(\tilde{\rho}-2\eta)/\eta)} - e^{-\beta}(1-(\tilde{\rho}-2\eta)/\eta)\right] & \text{if } 2\eta < \tilde{\rho} \le 3\eta \\ 3\eta + \eta\left[1 - e^{-\beta(\tilde{\rho}-3\eta)/\eta} + e^{-\beta}\left(\tilde{\rho}-3\eta\right)/\eta\right] & \text{if } 3\eta < \tilde{\rho} \le 1 \end{cases}$$

where,
$\beta = 15$
$\eta = 0.25$

OUTLINE

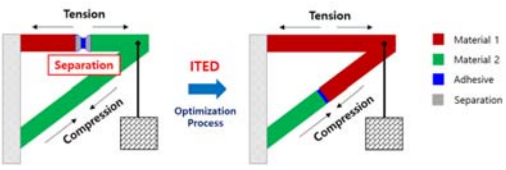
1. Introduction
2. Multi-material Topology Optimization using a Single Variable
3. Multi-material Design considering Interfacial Behavior
4. Numerical Examples
5. Conclusion

Conclusion

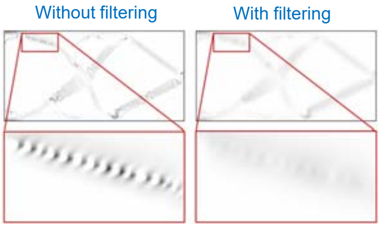
- ITED defined as the product of tensile stress and tensile strain is proposed.
- By combining ITED and Compliance, it simultaneously achieves boundary separation prevention and high rigidity design.
- By Applying Helmholtz filtering to the sensitivity, the singularity problem arising from ITED can be alleviated.
- Various numerical examples demonstrate the effectiveness of the method proposed in this study.

< ITED >

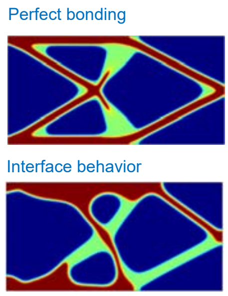
$$J_2 = \int_{\Omega_{interface}} \frac{1}{2} t_{tens} \varepsilon_{tens} d\Omega$$



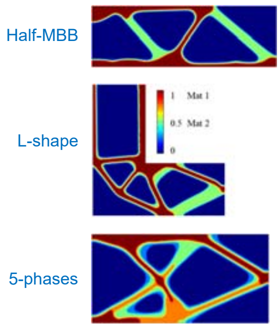
< Effect of Helmholtz filtering>



< Comparison >



< Examples >



References

[1] M.P. Bendsøe, N. Kikuchi, Generating optimal topologies in structural design using a homogenization method, Comput. Methods Appl. Mech. Engrg., 71 (1988) 197-224.
[2] K. Suzuki, N. Kikuchi, A homogenization method for shape and topology optimization, Comput. Methods Appl. Mech. Engrg., 93 (1991) 291-318.
[3] M.P. Bendsøe, O. Sigmund, Topology Optimization: Theory, Methods and Applications., Springer, 2003.
[4] M.P. Bendsøe, O. Sigmund, Material interpolation schemes in topology optimization, Arch. Appl. Mech., 69 (1999) 635-654.
[5] M.Y. Wang, X. Wang, D. Guo, A level set method for structural topology optimization, Comput. Methods Appl. Mech. Engrg., 192 (2003) 227-246.
[6] G. Allaire, F. Jouve, A.-M. Toader, Structural optimization using sensitivity analysis and a level-set method, J. Comput. Phys., 194 (2004) 363-393.
[7] G. Allaire, F. De Gournay, F. Jouve, A.-M. Toader, Structural optimization using topological and shape sensitivity via a level set method, Control Cybern., 34 (2005) 59.
[8] T. Yamada, K. Izui, S. Nishiwaki, A. Takezawa, A topology optimization method based on the level set method incorporating a fictitious interface energy, Comput. Methods Appl. Mech. Engrg., 199 (2010) 2876-2891.
[9] O. Sigmund, Morphology-based black and white filters for topology optimization, Struct. Multidiscip. Optim., 33 (2007) 401-424.
[10] B. Bourdin, Filters in topology optimization, Int. J. Numer. Methods Engrg., 50 (2001) 2143-2158.
[11] Z. Luo, L. Tong, J. Luo, P. Wei, M.Y. Wang, Design of piezoelectric actuators using a multiphase level set method of piecewise constants, J. Comput. Phys., 228 (2009) 2643-2659.
[12] H. Ghasemi, H.S. Park, T. Rabczuk, A multi-material level set-based topology optimization of flexoelectric composites, Comput. Methods Appl. Mech. Engrg., 332 (2018) 47-62.
[13] X. Wang, Y. Mei, M.Y. Wang, Level-set method for design of multi-phase elastic and thermoelastic materials, Int. J. Mech. Mater. Des., 1 (2004) 213-239.
[14] A. Faure, G. Michailidis, G. Parry, N. Vermaak, R. Estevez, Design of thermoelastic multi-material structures with graded interfaces using topology optimization, Struct. Multidiscip. Optim., 56 (2017) 823-837.
[15] N. Kishimoto, K. Izui, S. Nishiwaki, T. Yamada, Optimal design of electromagnetic cloaks with multiple dielectric materials by topology optimization, Appl. Phys. Lett., 110 (2017) 201104.
[16] C.F. Hvejsel, E. Lund, Material interpolation schemes for unified topology and multi-material optimization, Struct. Multidiscip. Optim., 43 (2011) 811-825.
[17] D. Li, I.Y. Kim, Multi-material topology optimization for practical lightweight design, Struct. Multidiscip. Optim., 58 (2018) 1081-1094.

References

[18] M.Y. Wang, X. Wang, “Color” level sets: a multi-phase method for structural topology optimization with multiple materials, Comput. Methods Appl. Mech. Engrg., 193 (2004) 469-496.
[19] L. Yin, G. Ananthasuresh, Topology optimization of compliant mechanisms with multiple materials using a peak function material interpolation scheme, Struct. Multidiscip. Optim., 23 (2001) 49-62.
[20] M. Bruyneel, SFP—a new parameterization based on shape functions for optimal material selection: application to conventional composite plies, Struct. Multidiscip. Optim., 43 (2011) 17-27.
[21] T. Gao, W. Zhang, P. Duysinx, A bi-value coding parameterization scheme for the discrete optimal orientation design of the composite laminate, Int. J. Numer. Methods Engrg., 91 (2012) 98-114.
[22] W. Zuo, K. Saitou, Multi-material topology optimization using ordered SIMP interpolation, Struct. Multidiscip. Optim., 55 (2017) 477-491.
[23] J. Liu, Y. Ma, A new multi-material level set topology optimization method with the length scale control capability, Comput. Methods Appl. Mech. Engrg., 329 (2018) 444-463.
[24] H.K. Seong, C.W. Kim, J. Yoo, J. Lee, Multiphase topology optimization with a single variable using the phase-field design method, Int. J. Numer. Methods Engrg., (2019).
[25] N. Vermaak, G. Michailidis, G. Parry, R. Estevez, G. Allaire, Y. Bréchet, Material interface effects on the topology optimizationof multi-phase structures using a level set method, Struct. Multidiscip. Optim., 50 (2014) 623-644.
[26] C.F. Hilchenbach, E. Ramm, Optimization of multiphase structures considering damage, Struct. Multidiscip. Optim., 51 (2015) 1083-1096.
[27] M. Lawry, K. Maute, Level set topology optimization of problems with sliding contact interfaces, Struct. Multidiscip. Optim., 52 (2015) 1107-1119.
[28] P. Liu, Y. Luo, Z. Kang, Multi-material topology optimization considering interface behavior via XFEM and level set method, Comput. Methods Appl. Mech. Engrg., 308 (2016) 113-133.
[29] P. Liu, Z. Kang, Integrated topology optimization of multi-component structures considering connecting interface behavior, Comput. Methods Appl. Mech. Engrg., 341 (2018) 851-887.
[30] D. Da, J. Yvonnet, L. Xia, G. Li, Topology optimization of particle-matrix composites for optimal fracture resistance taking into account interfacial damage, Int. J. Numer. Methods Engrg., 115 (2018) 604-626.
[31] T.T. Nguyen, J. Yvonnet, Q.-Z. Zhu, M. Bornert, C. Chateau, A phase-field method for computational modeling of interfacial damage interacting with crack propagation in realistic microstructures obtained by microtomography, Comput. Methods Appl. Mech. Engrg., 312 (2016) 567-595.
[32] B.S. Lazarov, O. Sigmund, Filters in topology optimization based on Helmholtz-type differential equations, Int. J. Numer. Methods Engrg., 86 (2011) 765-781.

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NRF National Research Foundation of Korea

KETEP Korea Institute of Energy Technology Evaluation and Planning