Structural Topology Optimization with Smoothly Varying Fiber Orientations

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RESEARCH PAPER

Structural topology optimization with smoothly varying fiber orientations

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Abstract
In recent years, the field of additive manufacturing (AM), often referred to as 3D printing, has seen tremendous growth and radically changed the means by which we describe valid 3D models for production. In particular, it is now conceivable to produce composite structures consisting of smoothly varying oriented anisotropic constitutive materials. In the present work, we propose a sensitivity driven method for the generation of transverse isotropic fiber reinforced structures having smooth spatially varying orientations. Our approach builds upon finite element analysis (FEA) and density-based topology optimization (TO). The local material orientations are formulated as design variables in a stiffness maximization problem, and solved with a non-convex gradient-based optimization scheme. Length-scale control is achieved through the use of filters for regularization. We demonstrate the ability of the proposed approach to handle large-scale 3D problems with synchronous optimization of material densities and orientations yielding millions of design variables on multiple load case scenarios. The method is shown to be compatible with compliant mechanism optimization as well as local volume constraints. Finally, the approach is extended with an additional design variable dictating the ratio of anisotropy for each element, thereby delegating the choice of material type to the optimization scheme.

Keywords Topology optimization · Finite element analysis · Mathematical programming · Anisotropic constitutive material · Smooth fiber orientations · Additive manufacturing

1 Introduction

In the present work, we explore the use of *anisotropic constitutive materials* within the context of topology optimization. The choice of wording is purposefully broad as we aim at proposing a fairly general approach for design optimization considering local material orientations alongside with the topology.

Section 1 presents the motivation toward the optimization of structures using anisotropic materials and reviews existing approaches. Section 2 describes the mathematical model chosen to represent and simulate designs containing the given material models. In Section 3 we address the numerical optimization schemes developed for tailoring material orientation layouts in the context of topology optimization. Finally, Section 4 is dedicated to various numerical experiments, analyzing the results of the present approach and its applicability to industrial design scenarios.

1.1 Motivation

Bakelite was the first artificial fiber reinforced plastic and was synthesized in 1907. However, composite materials have been a staple of mechanical engineering for centuries prior to that. Naturally occurring materials like wood have long been used in construction, specifically for their anisotropic properties granted by continuous fibers. Today, composite materials like reinforced concrete, plywood or fiberglass are commonly used in a wide variety of industrial applications. Advanced synthetic anisotropic materials perform routinely on aircraft and spacecraft in demanding environments.

Producing components consisting of composite materials is typically challenging and multiple manufacturing processes have been developed to tackle this challenge (Gao

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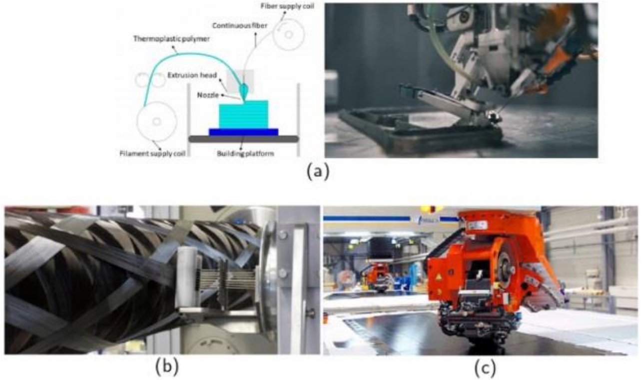
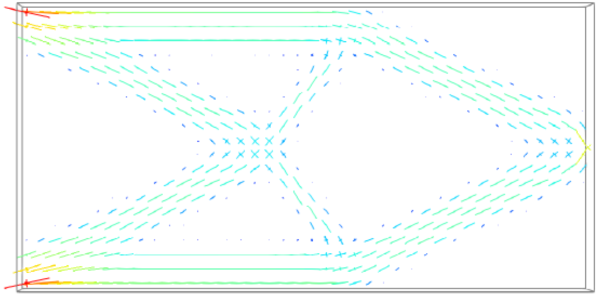
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Goal

- Anisotropic materials have been used in construction and engineering for millennia
- Known to exhibit highly favorable **stiffness-to-mass ratio** when properly oriented
  - Ubiquitous in high-end fields of application like Aeronautics and Aerospace
- Manufacturing processes rapidly progress towards handling composites materials
  - Additive manufacturing (3D Printing) can now produce composite materials with oriented fibers
- Need to handle optimization of oriented fibers in multi load case scenarios
  - Local stress tensor is inappropriate



Fiber-reinforced Additive Manufacturing


Visualization of local stress tensor in optimized cantilever

Kabir, S. M. F., Mathur, K., and Seyam, A.-F. M. (2020). A critical review on 3d printed continuous fiber-reinforced composites: History, mechanism, materials and properties. *Composite Structures*, 232:111476.

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FEA modeling

- Use anisotropic material properties
  - Define element behavior with **orthotropic constitutive law**
  - 9 distinct parameters
    - 3 Young moduli
    - 3 Poisson ratios
    - 3 Shear moduli
  - Reduced to 6 in **Transverse Isotropic** behavior

Material	Y_x	Y_y	Y_z	ν_{xy}	ν_{xz}	ν_{yz}	μ_{xy}	μ_{xz}	μ_{yz}
Iso10	10.0	10.0	10.0	0.25	0.25	0.25	4.0	4.0	4.0
Iso18	18.0	18.0	18.0	0.25	0.25	0.25	7.2	7.2	7.2
Ortho50	10.0	50.0	10.0	0.05	0.25	0.25	3.0	2.0	3.0
Ortho250	10.0	250.0	10.0	0.01	0.25	0.25	5.0	2.0	5.0

Material properties table

Stress vector → $$[\sigma] = C[\varepsilon]$$ ← Strain vector

$$C = \begin{bmatrix} \frac{1-\nu_{23}\nu_{32}}{Y_2Y_3\Delta} & \frac{\nu_{21}+\nu_{23}\nu_{31}}{Y_2Y_3\Delta} & \frac{\nu_{31}+\nu_{32}\nu_{21}}{Y_2Y_3\Delta} & 0 & 0 & 0 \\ \frac{\nu_{12}+\nu_{13}\nu_{32}}{Y_1Y_3\Delta} & \frac{1-\nu_{13}\nu_{31}}{Y_1Y_3\Delta} & \frac{\nu_{32}+\nu_{31}\nu_{12}}{Y_1Y_3\Delta} & 0 & 0 & 0 \\ \frac{\nu_{13}+\nu_{12}\nu_{23}}{Y_1Y_2\Delta} & \frac{\nu_{23}+\nu_{21}\nu_{13}}{Y_1Y_2\Delta} & \frac{1-\nu_{12}\nu_{21}}{Y_1Y_2\Delta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\mu_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\mu_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\mu_{23} \end{bmatrix}$$

Constitutive law matrix in stiffness form

$$\Delta = \frac{1-\nu_{12}\nu_{21}-\nu_{13}\nu_{31}-\nu_{23}\nu_{32}-2\nu_{12}\nu_{13}\nu_{23}}{Y_1Y_2Y_3}$$

Constitutive law matrix determinant

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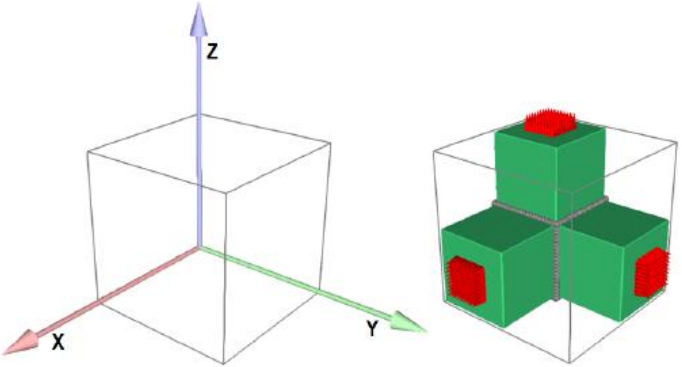
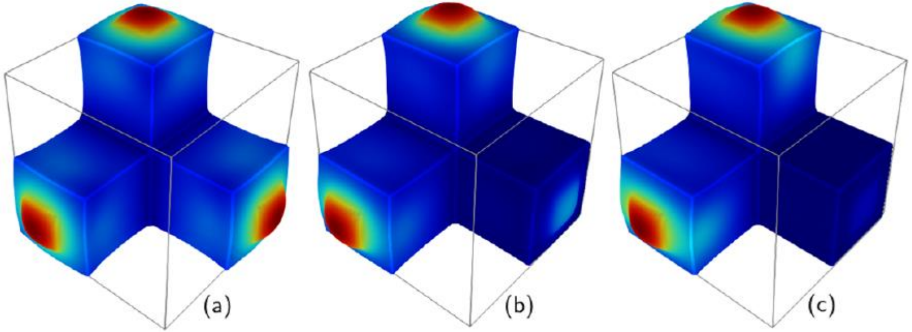


Test case of material in tension along 3 orthogonal directions



Displacement magnitude for isotropic and orthotropic materials

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FEA modeling

- Material orientation as **additional optimization variables**
  - 1 density variable for each element
  - 2 rotation variables for each element
- Introduction of **rotation matrices** in element stiffness matrix
  - Element stiffness matrix redefined for each element at each iteration
  - Use regular hexahedral elements with linear shape functions

$$\boldsymbol{R_E}(\alpha_E, \theta_E) = \boldsymbol{T_\alpha}(\alpha_E)\boldsymbol{T_\theta}(\theta_E)\boldsymbol{C}\boldsymbol{T_\theta^T}(\theta_E)\boldsymbol{T_\alpha^T}(\alpha_E)$$

Rotated constitutive law

$$\boldsymbol{T_\alpha}(\alpha_E) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c^2 & s^2 & 0 & 0 & -2cs \\ 0 & s^2 & c^2 & 0 & 0 & 2cs \\ 0 & 0 & 0 & c & -s & 0 \\ 0 & 0 & 0 & s & c & 0 \\ 0 & cs & -cs & 0 & 0 & c^2 - s^2 \end{bmatrix}, \text{ with } \begin{cases} c = \cos(\alpha_E) \\ s = \sin(\alpha_E) \end{cases}$$

Transformation matrices for both orientations

$$\boldsymbol{T_\theta}(\theta_E) = \begin{bmatrix} c^2 & s^2 & 0 & -2cs & 0 & 0 \\ s^2 & c^2 & 0 & 2cs & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ cs & -cs & 0 & c^2 - s^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & c & -s \\ 0 & 0 & 0 & 0 & s & c \end{bmatrix}, \text{ with } \begin{cases} c = \cos(\theta_E) \\ s = \sin(\theta_E) \end{cases}$$

$$\boldsymbol{K_E^0} = \iiint \boldsymbol{B_E^T}\boldsymbol{R_E}(\alpha_E, \theta_E)\boldsymbol{B_E}d\boldsymbol{\Omega}$$

Shape functions

New element stiffness matrix

$$\boldsymbol{K_E}(\rho_E, \alpha_E, \theta_E) = \rho_E^\gamma \boldsymbol{K_E^0}(\alpha_E, \theta_E)$$

Material orientation variables

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Sensitivity analysis

- Use the self-adjoint property of the compliance to obtain the sensitivities of the objective function
- The gradient with respect to the densities was derived previously
- The gradient with respect to the material orientation angles is derived using the chain rule as shown

Gradients for the two new design variables

$$\frac{\partial J}{\partial \alpha_E} = -\boldsymbol{\rho}_{\boldsymbol{E}}^{\gamma} \boldsymbol{u}_{\boldsymbol{E}}^{T} \frac{\partial \boldsymbol{K}_{\boldsymbol{E}}^{\boldsymbol{0}}}{\partial \alpha_E} \boldsymbol{u}_{\boldsymbol{E}}, \quad \forall E \in \boldsymbol{\Omega}$$

$$\frac{\partial J}{\partial \theta_E} = -\boldsymbol{\rho}_{\boldsymbol{E}}^{\gamma} \boldsymbol{u}_{\boldsymbol{E}}^{T} \frac{\partial \boldsymbol{K}_{\boldsymbol{E}}^{\boldsymbol{0}}}{\partial \theta_E} \boldsymbol{u}_{\boldsymbol{E}}, \quad \forall E \in \boldsymbol{\Omega}$$

Obtained by adjoint analysis

$$\frac{\partial \boldsymbol{K}_{\boldsymbol{E}}^{\boldsymbol{0}}}{\partial \alpha_E} = \iiint \boldsymbol{B}_{\boldsymbol{E}}^{\boldsymbol{T}} \boldsymbol{T}_{\boldsymbol{\alpha}} \left(\frac{\partial \boldsymbol{T}_{\boldsymbol{\theta}}}{\partial \theta_E} \boldsymbol{C} \boldsymbol{T}_{\boldsymbol{\theta}}^{\boldsymbol{T}} + \boldsymbol{T}_{\boldsymbol{\theta}} \boldsymbol{C} \frac{\partial \boldsymbol{T}_{\boldsymbol{\theta}}^{\boldsymbol{T}}}{\partial \theta_E} \right) \boldsymbol{T}_{\boldsymbol{\alpha}}^{\boldsymbol{T}} \boldsymbol{B}_{\boldsymbol{E}} d\boldsymbol{\Omega}$$

$$\frac{\partial \boldsymbol{K}_{\boldsymbol{E}}^{\boldsymbol{0}}}{\partial \theta_E} = \iiint \boldsymbol{B}_{\boldsymbol{E}}^{\boldsymbol{T}} \left(\frac{\partial \boldsymbol{T}_{\boldsymbol{\alpha}}}{\partial \alpha_E} \boldsymbol{T}_{\boldsymbol{\theta}} \boldsymbol{C} \boldsymbol{T}_{\boldsymbol{\theta}}^{\boldsymbol{T}} \boldsymbol{T}_{\boldsymbol{\alpha}}^{\boldsymbol{T}} + \boldsymbol{T}_{\boldsymbol{\alpha}} \boldsymbol{T}_{\boldsymbol{\theta}} \boldsymbol{C} \boldsymbol{T}_{\boldsymbol{\theta}}^{\boldsymbol{T}} \frac{\partial \boldsymbol{T}_{\boldsymbol{\alpha}}^{\boldsymbol{T}}}{\partial \alpha_E} \right) \boldsymbol{B}_{\boldsymbol{E}} d\boldsymbol{\Omega}$$

Obtained by chain rule

Gradients for the two rotation matrices

$$\frac{\partial \boldsymbol{T}_{\boldsymbol{\alpha}}(\alpha_E)}{\partial \alpha_E} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2cs & 2cs & 0 & 0 & -2(c^2-s^2) \\ 0 & 2cs & -2cs & 0 & 0 & 2(c^2-s^2) \\ 0 & 0 & 0 & -s & -c & 0 \\ 0 & 0 & 0 & c & -s & 0 \\ 0 & c^2-s^2 & s^2-c^2 & 0 & 0 & -4cs \end{bmatrix}, \text{ with } \begin{cases} c = \cos(\alpha_E) \\ s = \sin(\alpha_E) \end{cases}$$

$$\frac{\partial \boldsymbol{T}_{\boldsymbol{\theta}}(\theta_E)}{\partial \theta_E} = \begin{bmatrix} -2cs & 2cs & 0 & -2(c^2-s^2) & 0 & 0 \\ 2cs & -2cs & 0 & 2(c^2-s^2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ c^2-s^2 & s^2-c^2 & 0 & -4cs & 0 & 0 \\ 0 & 0 & 0 & 0 & -s & -c \\ 0 & 0 & 0 & 0 & c & -s \end{bmatrix}, \text{ with } \begin{cases} c = \cos(\theta_E) \\ s = \sin(\theta_E) \end{cases}$$

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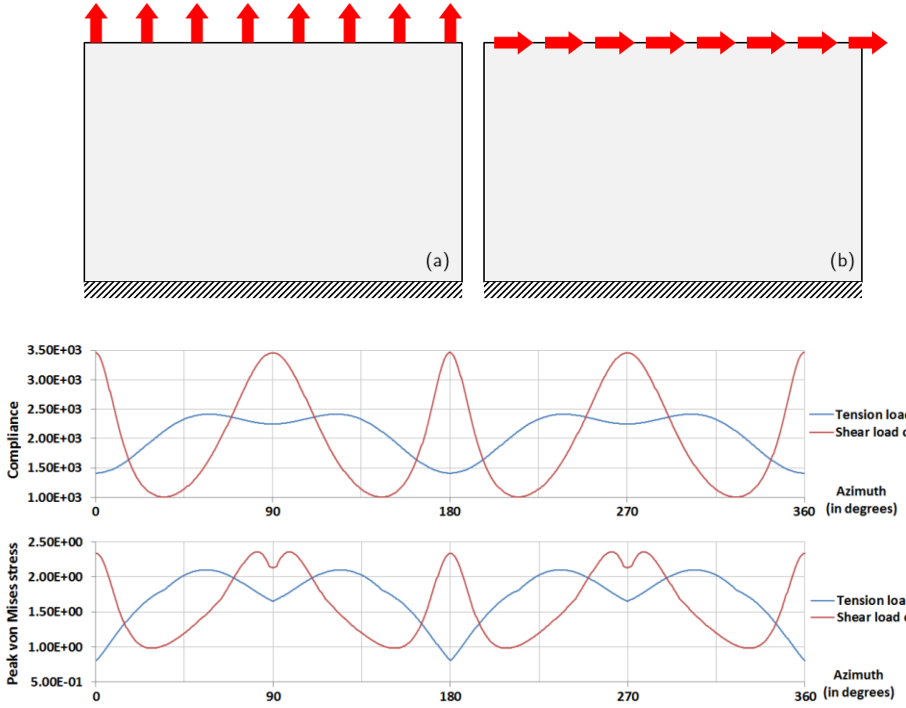
Non-convexity

- The solution-space for the material orientation optimization is extremely **non-convex**
- Orientation of orthotropic materials has multiple **fundamental local minima**
- Number of local minima increases exponentially with the number of design variables



Solution spaces for 1 orientation variable

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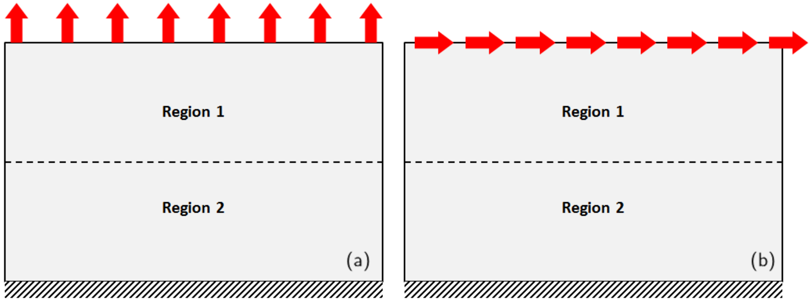
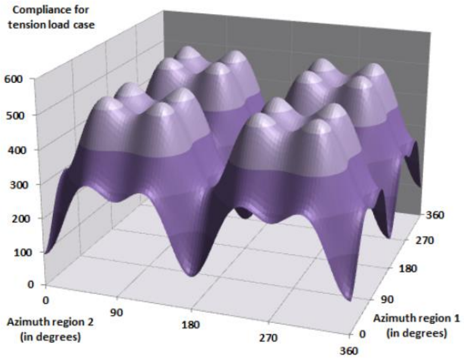
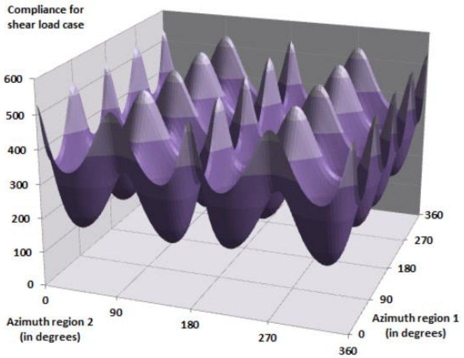




Solution space for tension load case with 2 orientation variables



Solution space for shear load case with 2 orientation variables

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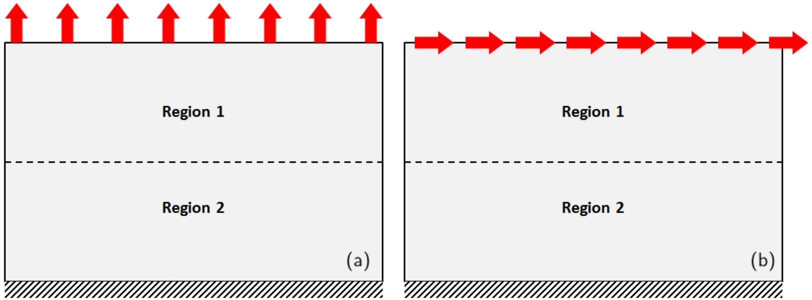
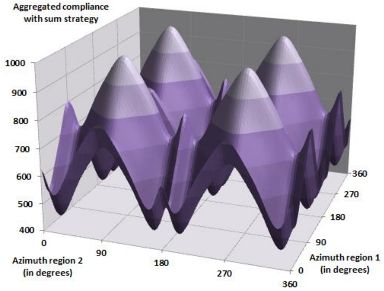
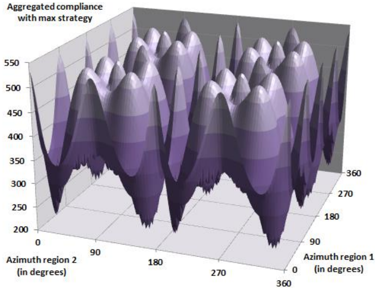
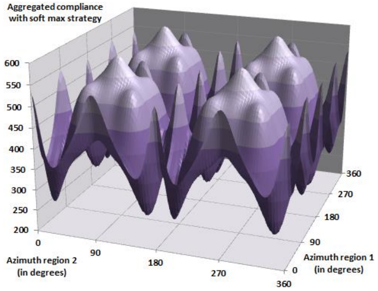




Solution space for sum aggregation



Solution space for max aggregation



Solution space for soft-max aggregation

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Structural Topology Optimization with Smoothly Varying Fiber Orientations

Optimization scheme

- Pure gradient-based optimization schemes are ill suited for this problem
- **Hybrid scheme**
  - Simulated annealing
    - Continuation scheme on move limits
    - Initial bounds at 45° and exponential decay at a rate of 0.9
  - Gradient descent
- No need for explicit box constraints due to the orientation periodicity

$$\alpha_E \leftarrow \alpha_E - \frac{\partial J_{sm}}{\partial \alpha_E}, \quad \forall E \in \mathbf{\Omega}$$

$$\theta_E \leftarrow \theta_E - \frac{\partial J_{sm}}{\partial \theta_E}, \quad \forall E \in \mathbf{\Omega}$$

Pure gradient descent update rules

Objective function gradient

Simulated annealing “energy” term

Move limits

$$\alpha_E \leftarrow \min\left(\max\left(\alpha_E - \frac{\zeta}{J_{sm}}\frac{\partial J_{sm}}{\partial \alpha_E}, \alpha_E - m_\alpha\right), \alpha_E + m_\alpha\right), \quad \forall E \in \mathbf{\Omega}$$

$$\theta_E \leftarrow \min\left(\max\left(\theta_E - \frac{\zeta}{J_{sm}}\frac{\partial J_{sm}}{\partial \theta_E}, \theta_E - m_\theta\right), \theta_E + m_\theta\right), \quad \forall E \in \mathbf{\Omega}$$

Hybrid design update rules

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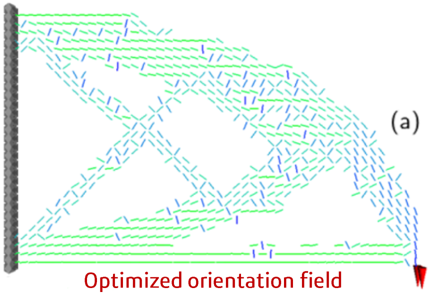
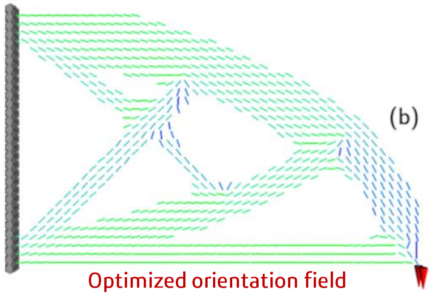
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Orientation regularization and length-scale control

- Orientation field **lacks length-scale control**
  - Regularization scheme to ensure continuity of material orientation field
  - Filtering in vector space
  - Applied after each design update iteration



Optimized orientation field without regularization



Optimized orientation field with regularization

$$\widetilde{\boldsymbol{\phi}_{\boldsymbol{E}}} = \frac{\sum_{e \in \boldsymbol{\omega}} v_e w_{E,e} \overline{\boldsymbol{\phi}_{\boldsymbol{e}}}}{\sum_{e \in \boldsymbol{\omega}} v_e w_{E,e}}, \quad \forall E \in \boldsymbol{\Omega}$$

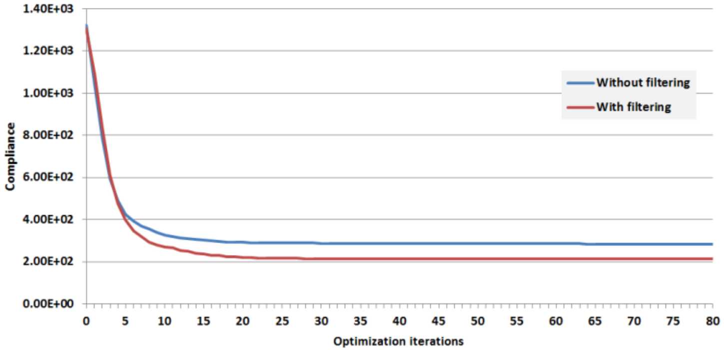
Orientation filtering in vector space

$$\text{where } \overline{\boldsymbol{\phi}_{\boldsymbol{e}}} = \begin{cases} -\boldsymbol{\phi}_{\boldsymbol{e}} \text{ if } \boldsymbol{\phi}_{\boldsymbol{E}} \cdot \boldsymbol{\phi}_{\boldsymbol{e}} \leq 0 \\ \boldsymbol{\phi}_{\boldsymbol{e}} \text{ otherwise} \end{cases}$$

Correction for periodicity

$$w_{E,e} = R - \|x_E - x_e\|$$

Weight filter with linear decay



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Structural Topology Optimization with Smoothly Varying Fiber Orientations

Orientation regularization and length-scale control

- **Regularization scheme** prevents rapid changes of local material orientation
- The filter radius controls the maximum curvature of the material orientation field
- **Length scale control and mesh independence** is achieved by tying the regularization parameters to the mesh resolution

$$\widetilde{\phi_E} = \frac{\sum_{e\in\omega} v_e w_{E,e} \overline{\phi_e}}{\sum_{e\in\omega} v_e w_{E,e}}, \quad \forall E \in \Omega$$

Orientation filtering in vector space

$$\text{where } \overline{\phi_e} = \begin{cases} -\phi_e & \text{if } \phi_E \cdot \phi_e \leq 0 \\ \phi_e & \text{otherwise} \end{cases}$$

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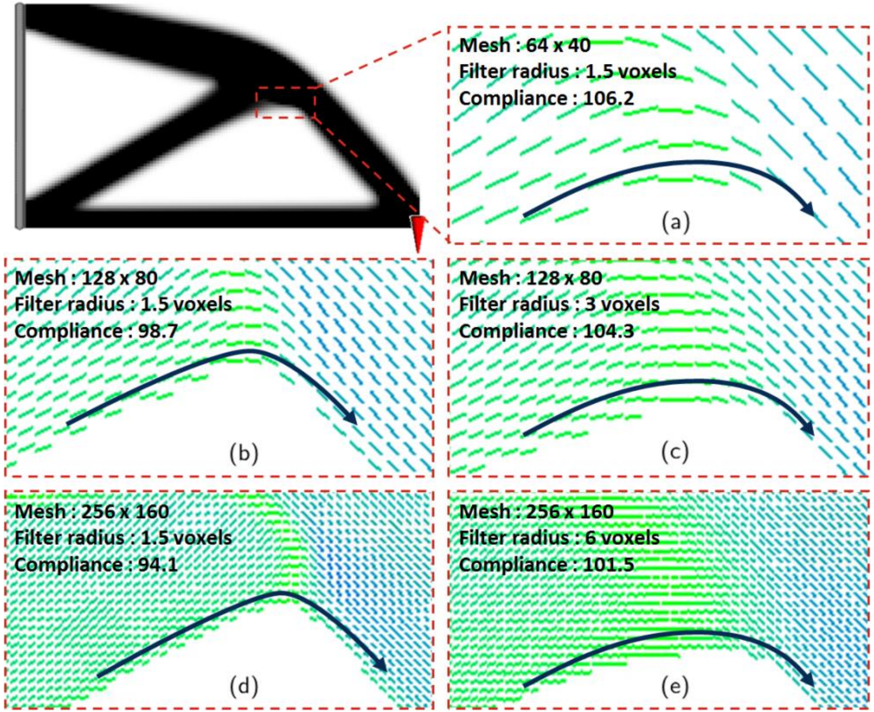

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Structural Topology Optimization with Smoothly Varying Fiber Orientations

Optimization problem formulation

- Final formulation for topology optimization
  - 3D on multiple load cases
  - Sensitivity driven
  - Synchronous optimization of local material density and orientation
- The resulting designs have an **optimized topology and material orientation** field smoothly varying throughout the design space

$$\arg\min_{\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta}} \quad J_{sm}(\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta}) = \left(\sum_{i\in \boldsymbol{LC}} \left(\boldsymbol{f_i}^T\boldsymbol{u_i}\right)^n\right)^{1/n}$$ Objective function

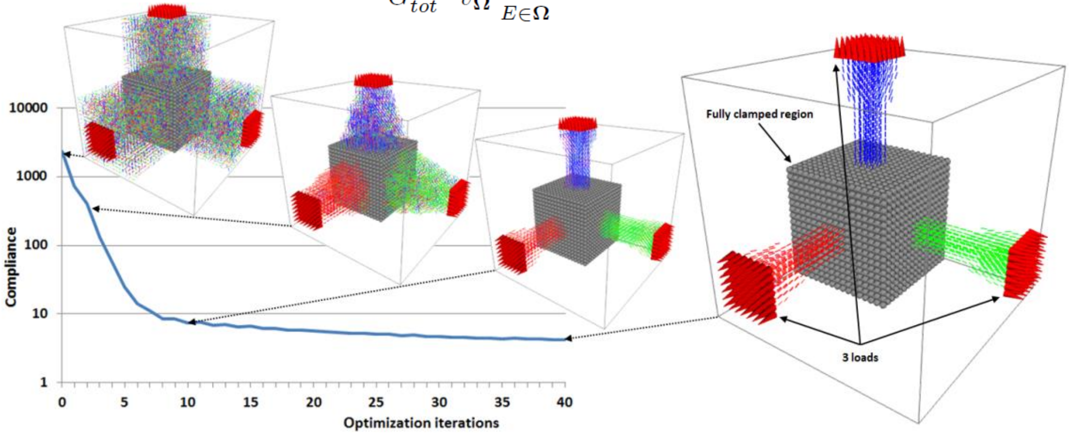
$$s.t. \quad \boldsymbol{K_i}(\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta})\boldsymbol{u_i} = \boldsymbol{f_i}, \quad \forall i \in \boldsymbol{LC}$$ Equality Constraint

$$0 \leq \rho_E \leq 1, \quad \forall E \in \boldsymbol{\Omega}$$ Box Constraint

$$-\pi \leq \alpha_E \leq \pi, \quad \forall E \in \boldsymbol{\Omega}$$ Box Constraint

$$-\pi/2 \leq \theta_E \leq \pi/2, \quad \forall E \in \boldsymbol{\Omega}$$ Box Constraint

$$G_{tot}(\boldsymbol{\rho}) = \frac{1}{G^\star_{tot}\cdot v_{\Omega}} \sum_{E\in\boldsymbol{\Omega}} (\rho_E v_E) - 1 \leq 0$$ Inequality Constraint



Compliance history on a test scenario using material *Ortho50*

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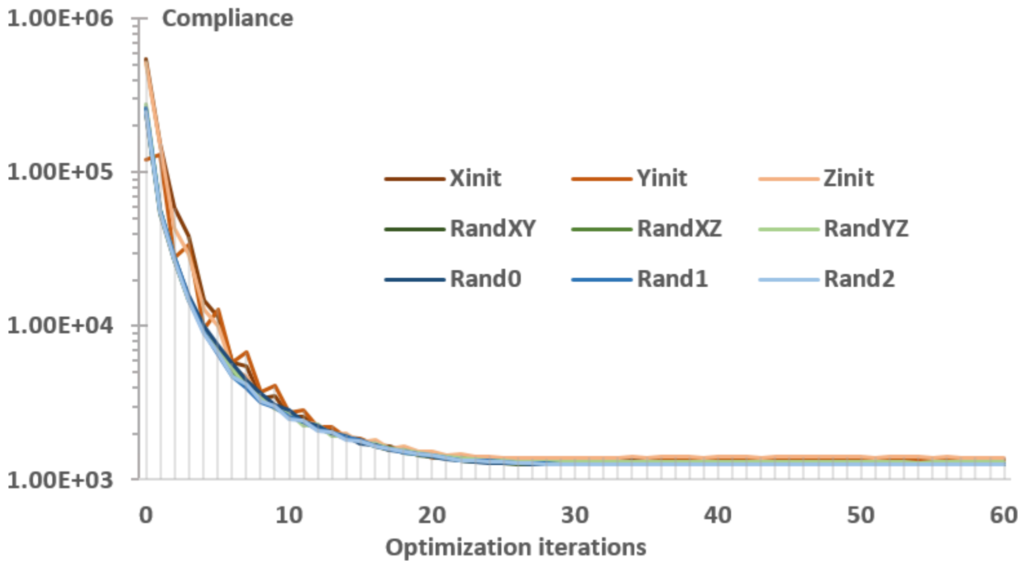
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Global convergence

- Classic SIMP already has no theoretical guarantee of global convergence
  - Nevertheless “grey” initialization and length scale control usually considered good enough
- Non-convexity more pronounced with anisotropic material
- Numerical experiments allows evaluating global convergence
  - Different 2D and 3D cases
  - Different material orientation initialization strategies



	Xinit	Yinit	Zinit	RandXY	RandXZ	RandYZ	Rand0	Rand1	Rand2
Compliance	135	133	140	130	127	132	126	128	126

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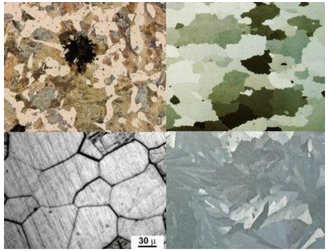
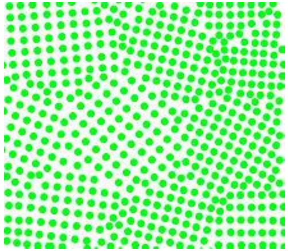
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Mechanical performance of isotropic and orthotropic materials

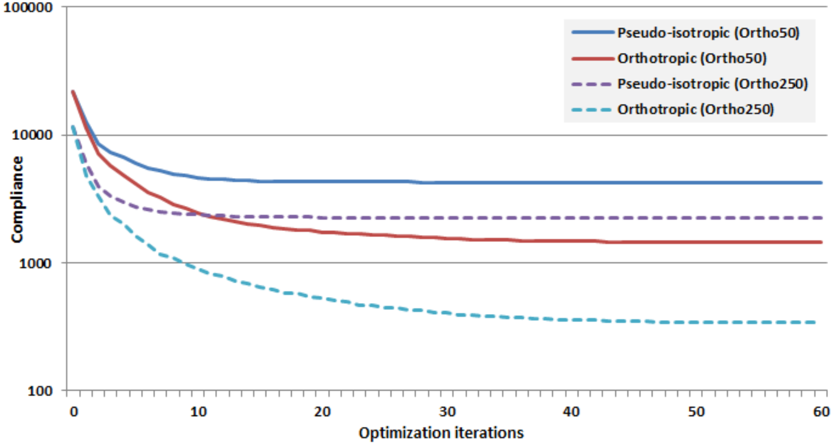
- **Goal**: Comparative numerical experiment to determine the benefits of optimizing the orientation of orthotropic materials
- **Solution**: Emulate pseudo-isotropic behavior of polycrystalline structures
  - Cantilever optimization on 512 x 256 grid with high volume fraction
- **Results**: Material orientation optimization increases the stiffness of the resulting structure
  - 2.9x higher stiffness with *Ortho50*
  - 6.5x higher stiffness with *Ortho250*



Microscopic polycrystalline structure of iron, steel and zinc



Differently oriented crystallites in a polycrystalline material



Compliance history (log scale) comparing the performance of pseudo-isotropic and orthotropic constitutive materials

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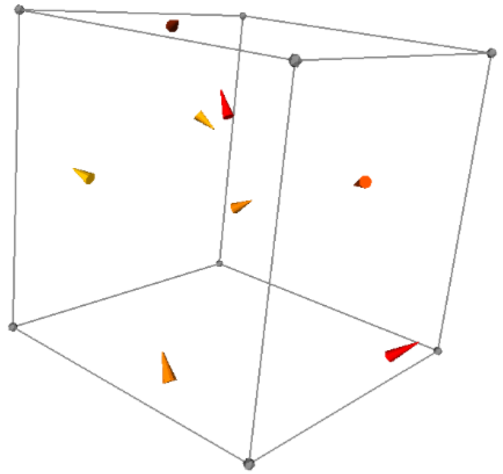
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Performances on high resolution mesh

- Procedurally generated **high-resolution** test case with **multiple load cases**
- Synchronous optimization of material densities and orientations
- Volume fraction constraint set to 10%
- Hardware
  - FEM generation and optimization steps executed on *20 Cores 2.40GHz Intel Xeon CPU*
  - FEA step with Jacobi CG solving for equilibrium executed on a NVIDIA *Quadro RTX6000 GPU*

Mesh resolution	$64 \times 64 \times 64$	$128 \times 128 \times 128$	$160 \times 160 \times 160$
Number of load cases	5	5	8
Number of displacement variables	823 851	6 440 043	12 519 819
Number of design variables	792 432	6 291 456	12 288 000
Time per optimization iteration	$\approx 90s - 160s$	$\approx 330s - 400s$	$\approx 1200s - 1400s$

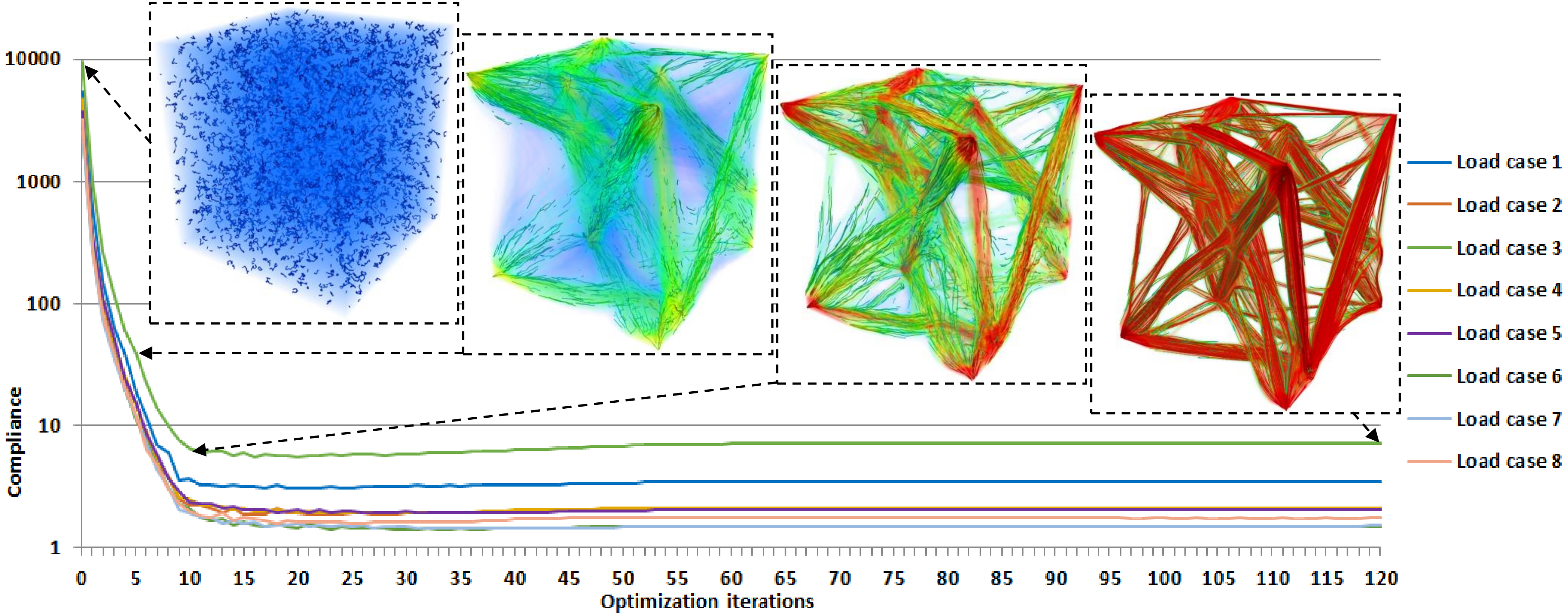
Result of 3 runs at varying resolution and number of load cases using material *Ortho250*



Optimization scenario: cube design space
Fully clamped corners
Multiple distinct load cases

Structural Topology Optimization with Smoothly Varying Fiber Orientations

Performances on high resolution mesh



Compliance history for the 8 load cases on the 160 x 160 x 160 mesh

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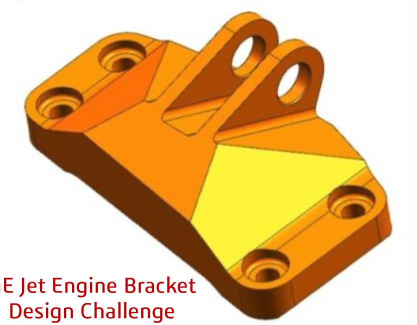
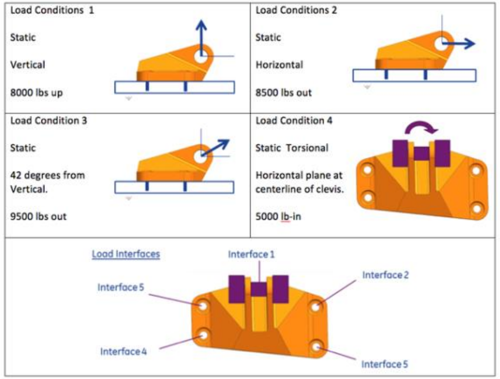
Structural Topology Optimization with Smoothly Varying Fiber Orientations

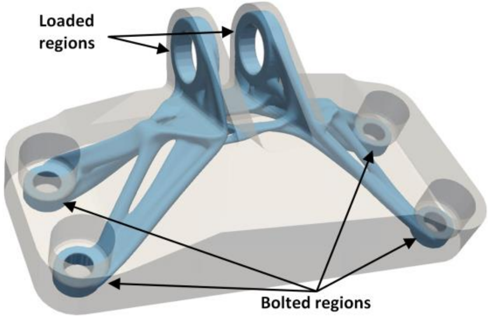
Application

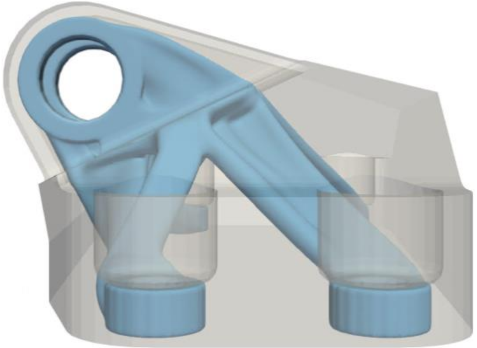


GE Jet Engine Bracket
Design Challenge

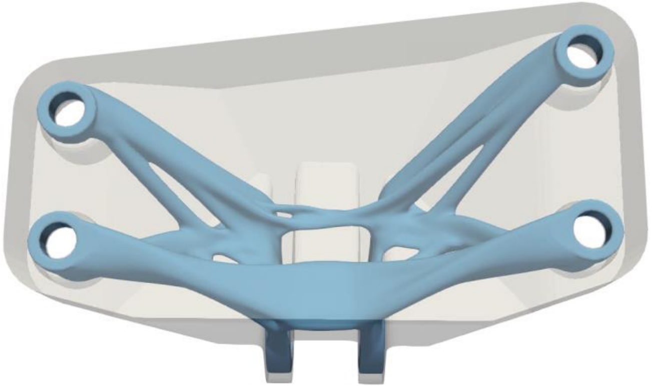


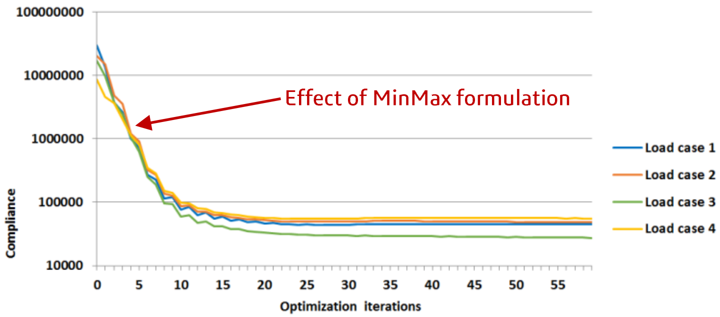
https://grabcad.com/challenges/ge-jet-engine-bracket-challenge





Raw isosurface of the optimized design
2.7M optimized hexahedral elements
8.1M design variables

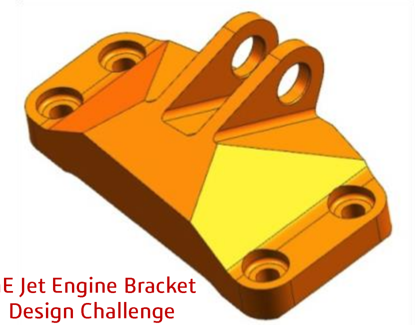




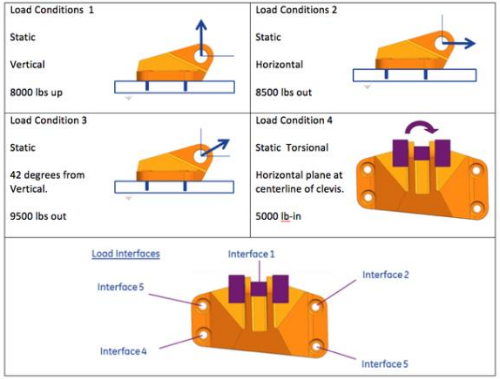
Compliance history for the 4 load cases using material *Ortho250*

Structural Topology Optimization with Smoothly Varying Fiber Orientations

Application



GE Jet Engine Bracket
Design Challenge



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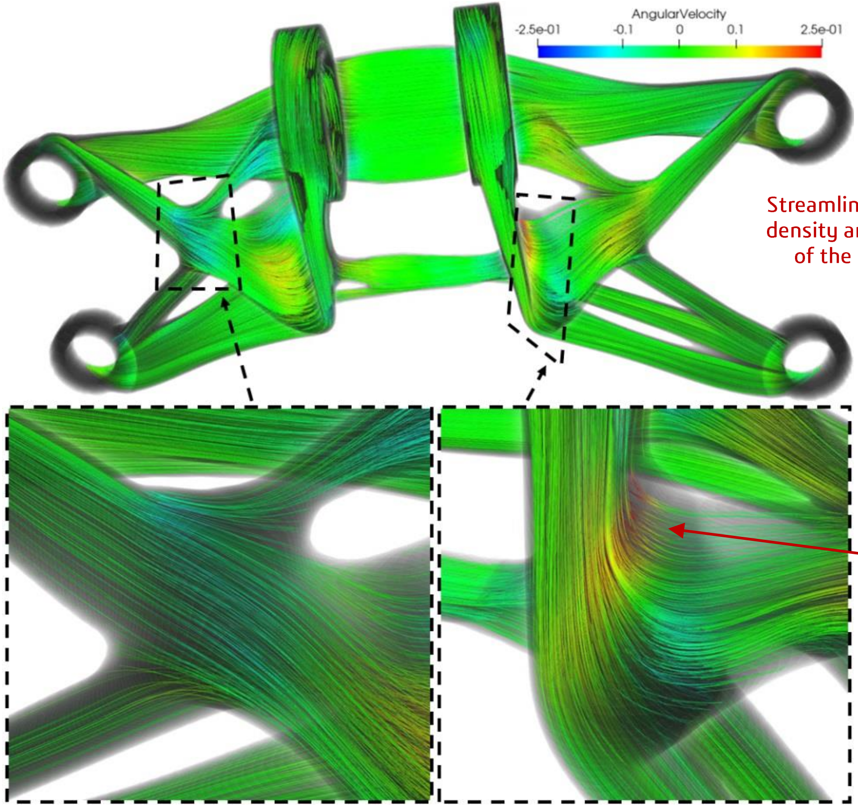


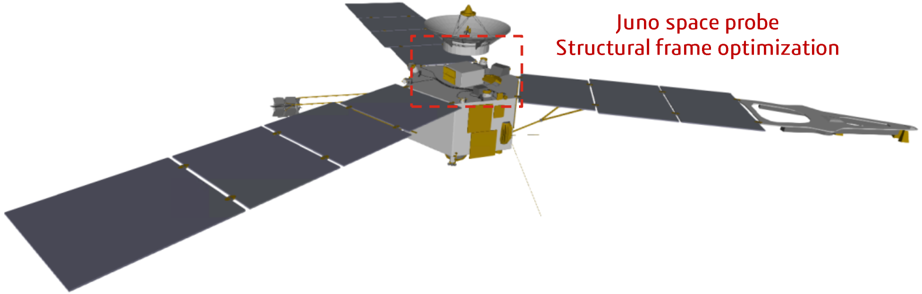
Streamlines calculated on the density and orientation fields of the optimized design

Angular velocity display help locate and check the smoothness of the material orientation field even in the highest curvature regions

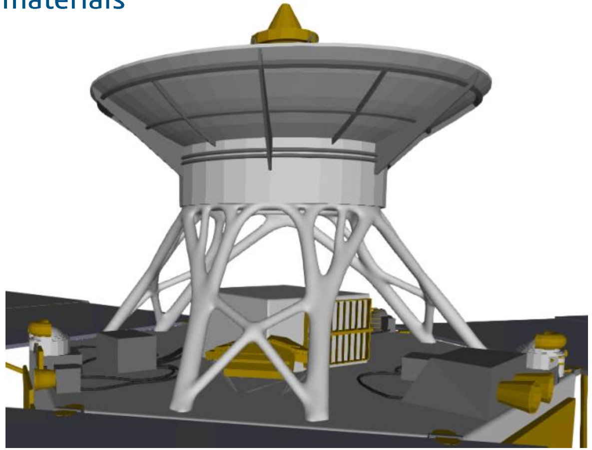
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Application

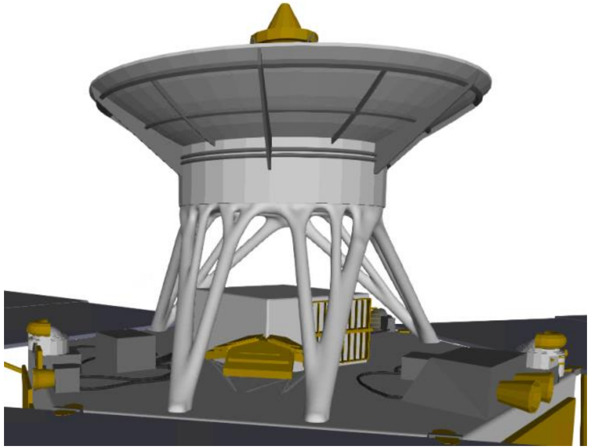
- Scenario based on **Juno space probe**
  - Optimization of High-Gain Antenna (HGA) **support frame**
  - **Multiple load cases** based on assembly in VAB and launch peak G-Forces
  - Comparison of optimized topologies achieved using **isotropic** and **oriented orthotropic** constitutive materials



Juno space probe
Structural frame optimization



Optimized structural frame using isotropic material



Optimized structural frame using oriented orthotropic material

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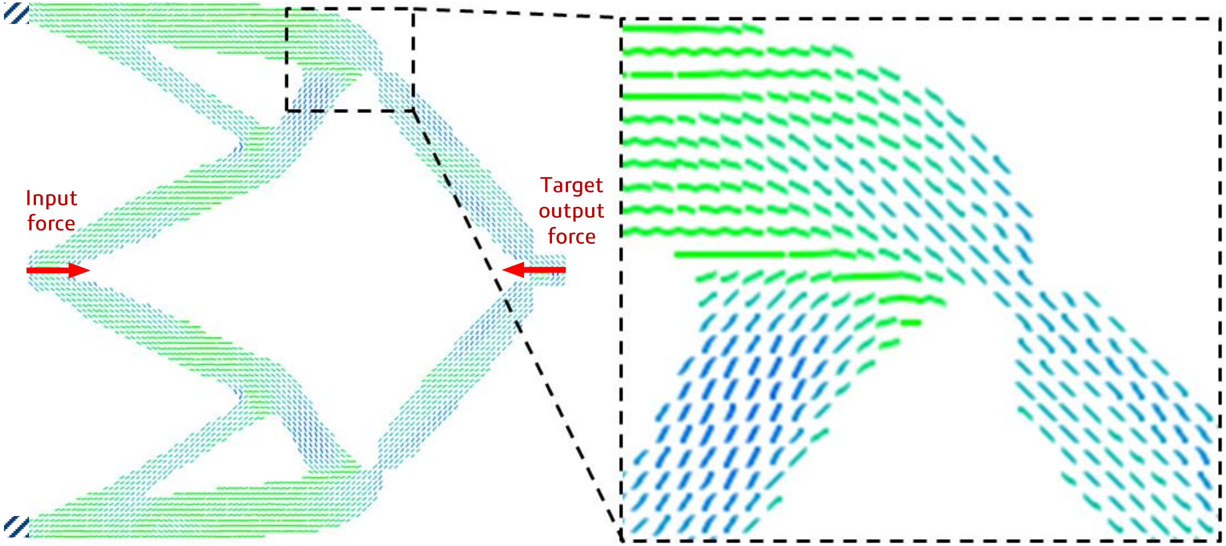
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Compliant mechanism optimization

- Compliant mechanism optimization is a type of problem where optimal material orientation may not be aligned with local stress tensors regardless of the number of load cases



Optimized force inverter on a 100 x 100 mesh

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Structural Topology Optimization with Smoothly Varying Fiber Orientations

Application

- Use porosity constraint in combination with material orientation optimization on setup with multiple load cases
  - Adds new trivial derivatives
  - MMA with continuation scheme

$$\underset{\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta}}{\arg\min} \quad J_{sm}(\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta}) = \left(\sum_{i \in \boldsymbol{LC}} (\boldsymbol{f}_i^T \boldsymbol{u}_i)^n \right)^{1/n}$$

$$\text{s.t.} \quad \boldsymbol{K}_i(\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta})\boldsymbol{u}_i = \boldsymbol{f}_i \quad \forall i \in \boldsymbol{LC}$$

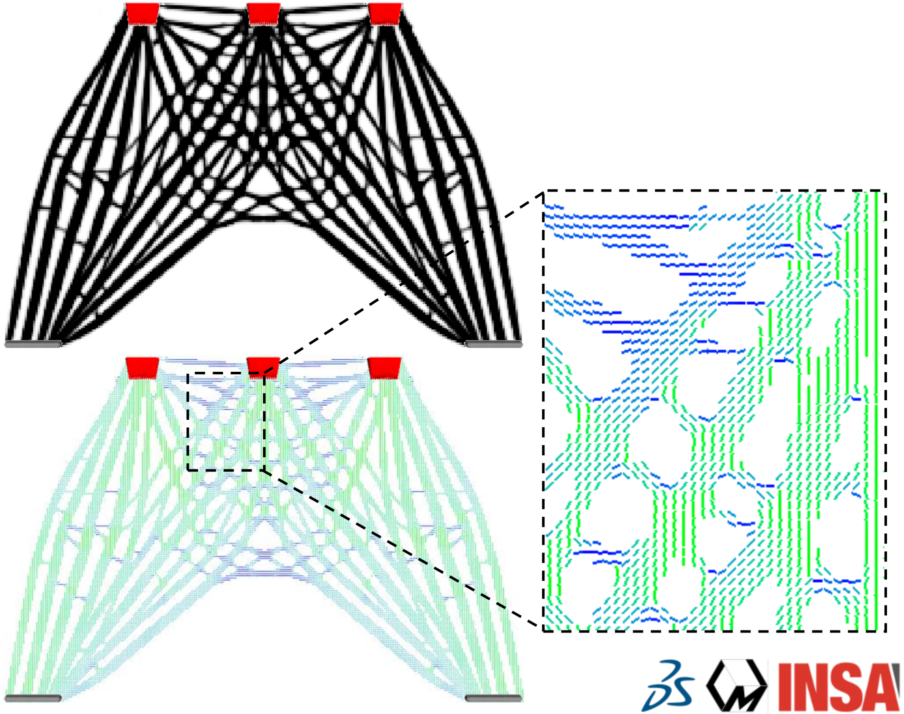
$$0 \leq \rho_E \leq 1, \quad \forall E \in \boldsymbol{\Omega}$$

$$-\pi \leq \alpha_E \leq \pi, \quad \forall E \in \boldsymbol{\Omega}$$

$$-\pi/2 \leq \theta_E \leq \pi/2, \quad \forall E \in \boldsymbol{\Omega}$$

$$G_{tot}(\boldsymbol{\rho}) = \frac{1}{G_{tot}^\star \cdot |\Omega|} \sum_{E \in \Omega} \rho_E - 1 \leq 0$$

$$G_{dyn}(\boldsymbol{\rho}) = \left(\frac{1}{|\Omega|} \sum_{E \in \Omega} \frac{\overline{\rho_E}^p}{G_{dyn,E}^{\star\ p}} \right)^{1/p} - 1 \leq 0$$



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Structural Topology Optimization with Smoothly Varying Fiber Orientations

Extension to varying degree of orthotropy

- Introduce an **additional optimization variable** for the ratio of orthotropy

$$\boldsymbol{C_E}(\boldsymbol{\tau}) = \left(\tau_E^q \boldsymbol{C_{ortho}} + (1-\tau_E)^q \boldsymbol{C_{iso}}\right)$$

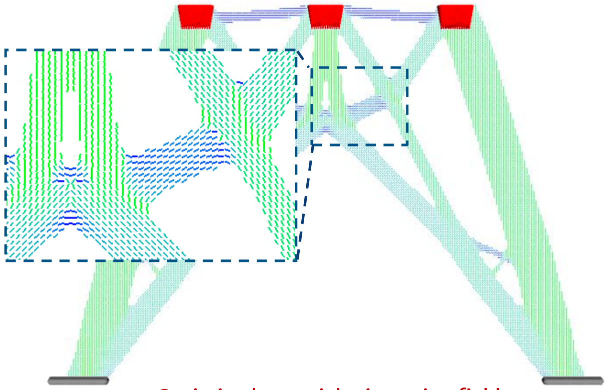
Ratio of orthotropy — Orthotropic material — Isotropic material

Gradient for the new design variable

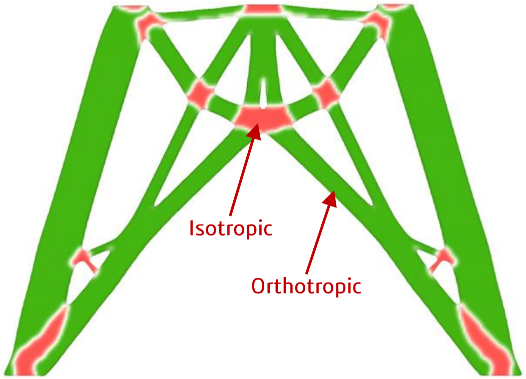
$$\frac{\partial \boldsymbol{C_E}}{\partial \tau_E} = q\left(\tau_E^{q-1} \boldsymbol{C_{ortho}} - (1-\tau_E)^{q-1} \boldsymbol{C_{iso}}\right)$$

$$\begin{aligned}
\underset{\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta}}{\arg\min} \quad & J_{sm}(\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta},\boldsymbol{\tau}) = \left(\sum_{i \in \boldsymbol{LC}} \left(\boldsymbol{f_i^T u_i}\right)^n\right)^{1/n} \\
s.t. \quad & \boldsymbol{K_i}(\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta},\boldsymbol{\tau})\boldsymbol{u_i} = \boldsymbol{f_i}, \quad \forall i \in \boldsymbol{LC} \\
& 0 \leq \rho_E \leq 1, \quad \forall E \in \boldsymbol{\Omega} \\
& -\pi \leq \alpha_E \leq \pi, \quad \forall E \in \boldsymbol{\Omega} \\
& -\pi/2 \leq \theta_E \leq \pi/2, \quad \forall E \in \boldsymbol{\Omega} \\
& 0 \leq \tau_E \leq 1, \quad \forall E \in \boldsymbol{\Omega} \\
& G_{tot}(\boldsymbol{\rho}) = \frac{1}{G_{tot}^{\star} \cdot |\boldsymbol{\Omega}|} \sum_{E \in \boldsymbol{\Omega}} \rho_E - 1 \leq 0
\end{aligned}$$

Optimization formulation with additional design variable controlling the ratio of orthotropy



Optimized material orientation field



Optimized ratio of orthotropy field

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Structural Topology Optimization with Smoothly Varying Fiber Orientations

Extension to varying degree of orthotropy

▶ Introduce an **additional optimization variable** for the ratio of orthotropy

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Ratio of orthotropy — Orthotropic material — Isotropic material

Gradient for the new design variable

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\underset{\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta}}{\arg\min} \quad & J_{sm}(\boldsymbol{\rho},\boldsymbol{\alpha},\boldsymbol{\theta},\boldsymbol{\tau}) = \left(\sum_{i\in \boldsymbol{LC}} \left(\boldsymbol{f_i^T u_i}\right)^n\right)^{1/n} \\
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& 0 \le \rho_E \le 1, \quad \forall E \in \boldsymbol{\Omega} \\
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& -\pi/2 \le \theta_E \le \pi/2, \quad \forall E \in \boldsymbol{\Omega} \\
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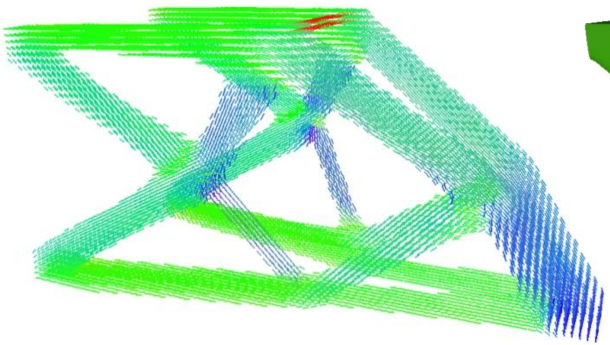
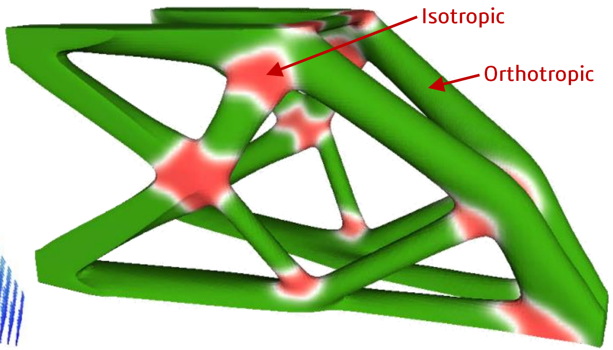
& G_{tot}(\boldsymbol{\rho}) = \frac{1}{G_{tot}^{\star} \cdot |\boldsymbol{\Omega}|} \sum_{E\in\boldsymbol{\Omega}} \rho_E - 1 \le 0
\end{aligned}$$

Optimization formulation with additional design variable controlling the ratio of orthotropy



Optimized material orientation field



Optimized ratio of orthotropy field

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Structural Topology Optimization with Smoothly Varying Fiber Orientations

Conclusion

- Mathematical model incorporating **oriented orthotropic material** in density based topology optimization
- Use of a **hybrid optimization scheme** to handle the **non-convexity** of the material orientation optimization problem
- Convergence and performance analysis of the proposed method in **high resolution multi load case optimization scenarios**
- Demonstration of **compatibility** with other constraints like the **porosity control** and other problem formulations like **compliant mechanisms**
- Extension to introduce another additional design variable allowing to optimization scheme to choose which **material type** to apply while simultaneously optimizing its **density and orientation**