Structural Topology Optimization with Smoothly Varying Fiber Orientations

Martin-Pierre SCHMIDT, Laura COURET, Christian GOUT, Claus B.W. PEDERSEN

Structural and Multidisciplinary Optimization (2020)
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Goal

- Anisotropic materials have been used in construction and engineering for millennia
- Known to exhibit highly favorable stiffness-to-mass ratio when properly oriented
  - Ubiquitous in high-end fields of application like Aeronautics and Aerospace
- Manufacturing processes rapidly progress towards handling composites materials
  - Additive manufacturing (3D Printing) can now produce composite materials with oriented fibers
- Need to handle optimization of oriented fibers in multi load case scenarios
  - Local stress tensor is inappropriate
Structural Topology Optimization with Smoothly Varying Fiber Orientations

FEA modeling

- Use anisotropic material properties
  - Define element behavior with **orthotropic constitutive law**
  - 9 distinct parameters
    - 3 Young moduli
    - 3 Poisson ratios
    - 3 Shear moduli
  - Reduced to 6 in **Transverse Isotropic** behavior

\[
\begin{align*}
\varepsilon & = \left[ \begin{array}{cccccc}
1 - \nu_{32}^2 & \nu_{21} + \nu_{23}^2 \varepsilon_{31} & \nu_{21} + \nu_{23}^2 \varepsilon_{31} & 0 & 0 & 0 \\
1 - \nu_{13}^2 & \frac{1 - \nu_{13}^2 \varepsilon_{31}}{Y_1 Y_3} & \frac{1 - \nu_{13}^2 \varepsilon_{31}}{Y_1 Y_3} & 0 & 0 & 0 \\
\frac{\nu_{12}}{Y_1 Y_3} & \frac{1 - \nu_{13}^2 \varepsilon_{31}}{Y_1 Y_3} & \frac{1 - \nu_{13}^2 \varepsilon_{31}}{Y_1 Y_3} & 0 & 0 & 0 \\
\frac{1 - \nu_{12}^2 \varepsilon_{31}}{Y_1 Y_3} & \frac{1 - \nu_{13}^2 \varepsilon_{31}}{Y_1 Y_3} & \frac{1 - \nu_{13}^2 \varepsilon_{31}}{Y_1 Y_3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\mu_{12} & 0 \\
0 & 0 & 0 & 0 & 0 & 2\mu_{13} \\
0 & 0 & 0 & 0 & 0 & 2\mu_{23}
\end{array} \right]
\end{align*}
\]

Constitutive law matrix in stiffness form

\[
\Delta = \frac{1 - \nu_{12}^2 \varepsilon_{31} - \nu_{13}^2 \varepsilon_{31} - \nu_{23}^2 \varepsilon_{31} - 2\nu_{12} \nu_{13} \varepsilon_{23} Y_1 Y_3}{Y_1 Y_2 Y_3}
\]

Constitutive law matrix determinant

---

Material properties table

<table>
<thead>
<tr>
<th>Material</th>
<th>$Y_x$</th>
<th>$Y_y$</th>
<th>$Y_z$</th>
<th>$\nu_{xy}$</th>
<th>$\nu_{yx}$</th>
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<th>$\mu_{xy}$</th>
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<td>10.0</td>
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<tr>
<td>Ortho50</td>
<td>10.0</td>
<td>50.0</td>
<td>10.0</td>
<td>0.05</td>
<td>0.25</td>
<td>0.25</td>
<td>3.0</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Ortho250</td>
<td>10.0</td>
<td>250.0</td>
<td>10.0</td>
<td>0.01</td>
<td>0.25</td>
<td>0.25</td>
<td>5.0</td>
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Test case of material in tension along 3 orthogonal directions

Displacement magnitude for isotropic and orthotropic materials
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FEA modeling

- Material orientation as additional optimization variables
  - 1 density variable for each element
  - 2 rotation variables for each element
- Introduction of rotation matrices in element stiffness matrix
  - Element stiffness matrix redefined for each element at each iteration
  - Use regular hexahedral elements with linear shape functions

$$R_E(\alpha_E, \theta_E) = T_\alpha(\alpha_E)T_\theta(\theta_E)CT_\theta^T(\theta_E)T_\alpha^T(\alpha_E)$$

$$T_\alpha(\alpha_E) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & c & 0 & 0 & -2c & 0 \\
0 & s & 0 & 0 & 2c & 0 \\
0 & 0 & c & -s & 0 & 0 \\
0 & cs & -cs & 0 & c^2 & -s^2 \\
\end{bmatrix}$$, with $$\begin{cases}
c = \cos(\alpha_E) \\
s = \sin(\alpha_E) \end{cases}$$

$$T_\theta(\theta_E) = \begin{bmatrix}
c^2 & s^2 & 0 & -2cs & 0 & 0 \\
s^2 & c^2 & 0 & 2cs & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
cs & -cs & 0 & c^2 & -s^2 & 0 \\
0 & 0 & 0 & c & -s & 0 \\
0 & 0 & 0 & s & c & 0 \\
\end{bmatrix}$$, with $$\begin{cases}
c = \cos(\theta_E) \\
s = \sin(\theta_E) \end{cases}$$

$$K_E^0 = \iint \int B_E^T R_E(\alpha_E, \theta_E) B_E d\Omega$$

New element stiffness matrix

$$K_E(\rho_E, \alpha_E, \theta_E) = \rho_E^2 K_E^0(\alpha_E, \theta_E)$$

Shape functions

Material orientation variables
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Sensitivity analysis

- Use the self-adjoint property of the compliance to obtain the sensitivities of the objective function.

- The gradient with respect to the densities was derived previously.

- The gradient with respect to the material orientation angles is derived using the chain rule as shown.

\[
\frac{\partial J}{\partial \alpha_E} = -\rho_E u_E^T \frac{\partial K^0_E}{\partial \alpha_E} u_E, \quad \forall E \in \Omega
\]

\[
\frac{\partial J}{\partial \theta_E} = -\rho_E u_E^T \frac{\partial K^0_E}{\partial \theta_E} u_E, \quad \forall E \in \Omega
\]

\[
\frac{\partial K^0_E}{\partial \alpha_E} = \int \int B_E^T T_\alpha \left( \frac{\partial T_\theta}{\partial \alpha_E} C T_\theta^T + T_\theta C \frac{\partial T_\theta}{\partial \theta_E} \right) T_\alpha^T B_E d\Omega
\]

\[
\frac{\partial K^0_E}{\partial \theta_E} = \int \int B_E^T \left( \frac{\partial T_\alpha}{\partial \alpha_E} T_\theta C T_\theta^T T_\alpha + T_\alpha T_\theta C T_\theta \frac{\partial T_\alpha}{\partial \alpha_E} \right) B_E d\Omega
\]

Gradients for the two new design variables obtained by adjoint analysis.

Gradients for the two rotation matrices obtained by chain rule.
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Non-convexity

- The solution-space for the material orientation optimization is extremely **non-convex**

- Orientation of orthotropic materials has multiple **fundamental local minima**

- Number of local minima increases exponentially with the number of design variables
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Optimization scheme

- Pure gradient-based optimization schemes are ill suited for this problem

- **Hybrid scheme**
  - Simulated annealing
    - Continuation scheme on move limits
    - Initial bounds at 45° and exponential decay at a rate of 0.9
  - Gradient descent

- No need for explicit box constraints due to the orientation periodicity

\[
\alpha_E \leftarrow \alpha_E - \frac{\partial J_{sm}}{\partial \alpha_E}, \quad \forall E \in \Omega
\]

\[
\theta_E \leftarrow \theta_E - \frac{\partial J_{sm}}{\partial \theta_E}, \quad \forall E \in \Omega
\]

Pure gradient descent update rules

\[
\alpha_E \leftarrow \min \left( \max \left( \alpha_E - \frac{\zeta \partial J_{sm}}{J_{sm}} \alpha_E - m_\alpha, \alpha_E + m_\alpha \right), \forall E \in \Omega \right)
\]

\[
\theta_E \leftarrow \min \left( \max \left( \theta_E - \frac{\zeta \partial J_{sm}}{J_{sm}} \theta_E - m_\theta, \theta_E + m_\theta \right), \forall E \in \Omega \right)
\]

Objective function gradient

Simulated annealing “energy” term

Move limits

Hybrid design update rules
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Orientation regularization and length-scale control

- Orientation field **lacks length-scale control**
  - Regularization scheme to ensure continuity of material orientation field
  - Filtering in vector space
  - Applied after each design update iteration

\[
\tilde{\phi}_E = \frac{\sum_{e \in \omega} v_e w_{E,e} \phi_e}{\sum_{e \in \omega} v_e w_{E,e}}, \quad \forall E \in \Omega
\]

where \( \phi_e = \begin{cases} 
-\phi_e & \text{if } \phi_E \cdot \phi_e \leq 0 \\
\phi_e & \text{otherwise}
\end{cases} \)

- Orientation filtering in vector space
- Correction for periodicity
- Weight filter with linear decay

Optimized orientation field without regularization

Optimized orientation field with regularization
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Orientation regularization and length-scale control

- **Regularization scheme** prevents rapid changes of local material orientation
- The filter radius controls the maximum curvature of the material orientation field
- **Length scale control and mesh independence** is achieved by tying the regularization parameters to the mesh resolution

\[
\tilde{\phi}_E = \sum_{e \in \omega} v_e w_{E,e} \bar{\phi}_e, \quad \forall E \in \Omega
\]

where \( \bar{\phi}_e = \begin{cases} -\phi_e & \text{if } \phi_E \cdot \phi_e \leq 0 \\ \phi_e & \text{otherwise} \end{cases} \)

\[
w_{E,e} = R - \| x_E - x_e \|
\]

Orientation filtering in vector space

Correction for periodicity

Weight filter with linear decay
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Optimization problem formulation

- Final formulation for topology optimization
  - 3D on multiple load cases
  - Sensitivity driven
  - Synchronous optimization of local material density and orientation

- The resulting designs have an optimized topology and material orientation field smoothly varying throughout the design space

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Objective function
Equality Constraint
Box Constraint
Box Constraint
Box Constraint
Inequality Constraint

Compliance history on a test scenario using material Ortho50
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Global convergence

- Classic SIMP already has no theoretical guarantee of global convergence
  - Nevertheless “grey” initialization and length scale control usually considered good enough
- Non-convexity more pronounced with anisotropic material
- Numerical experiments allows evaluating global convergence
  - Different 2D and 3D cases
  - Different material orientation initialization strategies

<table>
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<tr>
<th></th>
<th>Xinit</th>
<th>Yinit</th>
<th>Zinit</th>
<th>RandXY</th>
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<td>128</td>
<td>126</td>
</tr>
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Mechanical performance of isotropic and orthotropic materials

- **Goal**: Comparative numerical experiment to determine the benefits of optimizing the orientation of orthotropic materials

- **Solution**: Emulate pseudo-isotropic behavior of polycrystalline structures
  - Cantilever optimization on 512 x 256 grid with high volume fraction

- **Results**: Material orientation optimization increases the stiffness of the resulting structure
  - 2.9x higher stiffness with Ortho50
  - 6.5x higher stiffness with Ortho250

---

Compliance history (log scale) comparing the performance of pseudo-isotropic and orthotropic constitutive materials
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Performances on high resolution mesh

- Procedurally generated **high-resolution** test case with **multiple load cases**
- Synchronous optimization of material densities and orientations
- Volume fraction constraint set to 10%
- Hardware
  - FEM generation and optimization steps executed on **20 Cores 2.40GHz Intel Xeon CPU**
  - FEA step with Jacobi CG solving for equilibrium executed on a **NVIDIA Quadro RTX6000 GPU**

<table>
<thead>
<tr>
<th>Mesh resolution</th>
<th>64 × 64 × 64</th>
<th>128 × 128 × 128</th>
<th>160 × 160 × 160</th>
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<tbody>
<tr>
<td>Number of load cases</td>
<td>5</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Number of displacement variables</td>
<td>823 851</td>
<td>6 440 043</td>
<td>12 519 819</td>
</tr>
<tr>
<td>Number of design variables</td>
<td>792 432</td>
<td>6 291 456</td>
<td>12 288 000</td>
</tr>
<tr>
<td>Time per optimization iteration</td>
<td>≈ 90s – 160s</td>
<td>≈ 330s – 400s</td>
<td>≈ 1200s – 1400s</td>
</tr>
</tbody>
</table>

Result of 3 runs at varying resolution and number of load cases using material **Ortho250**
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Performances on high resolution mesh

Compliance history for the 8 load cases on the 160 x 160 x 160 mesh
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Application

GE Jet Engine Bracket Design Challenge

https://grabcad.com/challenges/ge-jet-engine-bracket-challenge

Loaded regions

Bolted regions

Raw isosurface of the optimized design
2.7M optimized hexahedral elements
8.1M design variables

Effect of MinMax formulation

Compliance history for the 4 load cases using material Ortho250
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Application

GE Jet Engine Bracket Design Challenge

Streamlines calculated on the density and orientation fields of the optimized design

https://grabcad.com/challenges/ge-jet-engine-bracket-challenge

Angular velocity display help locate and check the smoothness of the material orientation field even in the highest curvature regions
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Application

- Scenario based on Juno space probe
  - Optimization of High-Gain Antenna (HGA) support frame
  - Multiple load cases based on assembly in VAB and launch peak G-Forces
  - Comparison of optimized topologies achieved using *isotropic* and *oriented orthotropic* constitutive materials

Optimized structural frame using isotropic material

Optimized structural frame using oriented orthotropic material
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Compliant mechanism optimization

Compliant mechanism optimization is a type of problem where optimal material orientation may not be aligned with local stress tensors regardless of the number of load cases.
Structural Topology Optimization with Smoothly Varying Fiber Orientations

Application

- Use porosity constraint in combination with material orientation optimization on setup with multiple load cases
- Adds new trivial derivatives
- MMA with continuation scheme

\[
\begin{align*}
\arg \min_{\rho, \alpha, \theta} \quad & J_{sm}(\rho, \alpha, \theta) = \left( \sum_{i \in LC} \left( f_i^T u_i \right)^n \right)^{1/n} \\
\text{s.t.} \quad & K_i(\rho, \alpha, \theta) u_i = f_i, \quad \forall i \in LC \\
& 0 \leq \rho_E \leq 1, \quad \forall E \in \Omega \\
& -\pi \leq \alpha_E \leq \pi, \quad \forall E \in \Omega \\
& -\pi/2 \leq \theta_E \leq \pi/2, \quad \forall E \in \Omega \\
& G_{tot}(\rho) = \frac{1}{G_{tot}^* |\Omega|} \sum_{E \in \Omega} \rho_E - 1 \leq 0 \\
& G_{dyn}(\rho) = \left( \frac{1}{|\Omega|} \sum_{E \in \Omega} \frac{\rho_E}{G_{dyn,E}^p} \right)^{1/p} - 1 \leq 0
\end{align*}
\]
**Structural Topology Optimization with Smoothly Varying Fiber Orientations**

**Extension to varying degree of orthotropy**

- Introduce an **additional optimization variable** for the ratio of orthotropy

\[ C_E(\tau) = (\tau_E^q C_{\text{ortho}} + (1 - \tau_E)^q C_{\text{iso}}) \]

\[ \frac{\partial C_E}{\partial \tau E} = q \left( \tau_E^{q-1} C_{\text{ortho}} - (1 - \tau_E)^{q-1} C_{\text{iso}} \right) \]

**Optimization formulation with additional design variable controlling the ratio of orthotropy**

\[ \arg \min_{\rho, \alpha, \theta} J_{sm}(\rho, \alpha, \theta, \tau) = \left( \sum_{i \in LC} (f_i^T u_i)^n \right)^{1/n} \]

\[ s.t. \quad K_i(\rho, \alpha, \theta, \tau) u_i = f_i, \quad \forall i \in LC \]
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\[ 0 \leq \tau_E \leq 1, \quad \forall E \in \Omega \]

\[ G_{tot}(\rho) = \frac{1}{G_{tot} \cdot |\Omega|} \sum_{E \in \Omega} \rho_E - 1 \leq 0 \]
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G_{tot}(\rho) = \frac{1}{G_{tot} \cdot |\Omega|} \sum_{E \in \Omega} \rho_E - 1 \leq 0
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Optimization formulation with additional design variable controlling the ratio of orthotropy

Optimized material orientation field

Optimized ratio of orthotropy field
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Conclusion

- Mathematical model incorporating oriented orthotropic material in density based topology optimization

- Use of a hybrid optimization scheme to handle the non-convexity of the material orientation optimization problem

- Convergence and performance analysis of the proposed method in high resolution multi load case optimization scenarios

- Demonstration of compatibility with other constraints like the porosity control and other problem formulations like compliant mechanisms

- Extension to introduce another additional design variable allowing to optimization scheme to choose which material type to apply while simultaneously optimizing its density and orientation