Efficient and easily accessible Matlab codes for topology optimization

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Motivation of the work

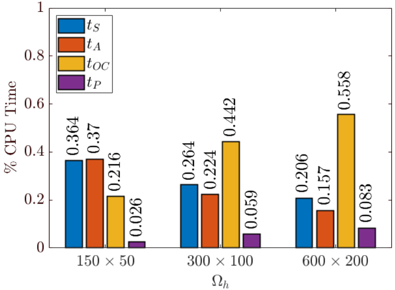
% CPU time distribution in top88[1]

t_S: linear solve

t_A: matrix assembly

t_{OC}: OC update

t_P: setup operations



(a) $r_{\min} = 4, 8, 16$, respectively

t_S will substantially decrease when using highly efficient preconditioned solvers.

Goals

- Cut all the times *other than* t_S, making the code highly efficient for medium-size problems $(10^5 - 10^6$ elements);
- Keep the readability and flexibility of top88

[1] Andreassen et. al. (2011)-*Efficient topology optimization in MATLAB using 88 lines of code*, SMO

Overview of the code and speedups(2)

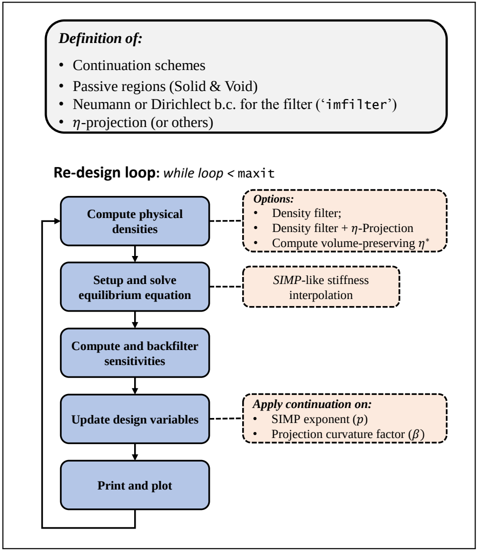
Minimum compliance

$$
\begin{cases}
\min_{\mathbf{x}} c(\hat{\mathbf{x}}) = \mathbf{f}^T \mathbf{u}(\hat{\mathbf{x}}) \\
K(\hat{\mathbf{x}})\mathbf{u}(\hat{\mathbf{x}}) = \mathbf{f} \\
g(\hat{\mathbf{x}}) = \sum_{e=1}^{m} \hat{x}_e v_e - V_f |\Omega_h| \leq 0 \\
0 \leq x_e \leq 1 \,, \quad \forall\, e
\end{cases}
$$

$\hat{x}_e$: physical densities, x_e: design variables

Speedups

- Stiffness matrix assembly;
- Filtering operations and OC update;
- Overall acceleration strategy(3)



(2) Ferrari, Sigmund (2020)-*A new generation 99 line Matlab code for compliance topology optimization and its extension to 3D*, SMO

(3) Li et al. (2020)-*Accelerated fixed-point formulation of Topology Optimization: application to compliance minimization problems*, Mech. Res. Comm.

Testing with the MBB beam example

- $\Omega_h = 600 \times 200$, $V_f = 0.5$, $r_{\min} = 8$
- Test cases
  - T1: Density Fiter, $p = 3$;
  - T2: Density Fiter, $p = 1 \to 3$;
  - T3: Density+Proj. $p = 3$, $\beta = 2 \to 24$;
  - T4: Density+Proj. $p = 1 \to 3$, $\beta = 2 \to 24$;

(a) T1

(b) T2

(c) T3

(d) T4

t_S: linear solve, t_A: matrix assembly,
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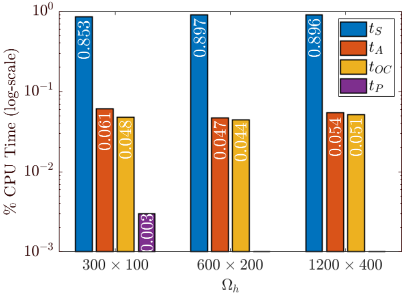


Table: Average iteration time (s) and speedup factors.

Ω_h	300×100 $r_{\min} = 4$	600×200 $r_{\min} = 8$	1200×400 $r_{\min} = 16$
top99neo(s)	0.231	1.19	5.69
top88U[4]	1.55	1.57	1.78
top88	2.66	4.09	5.51

[4] Use of sparse2 for assembly and conv2 for filtering

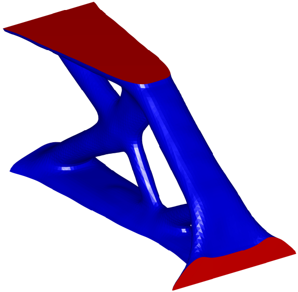
Extension to 3D domains (`top3D125`)

- change elemental stiffness matrix (8-nodes hexahedron element);
- change reshape operations (12 lines total);

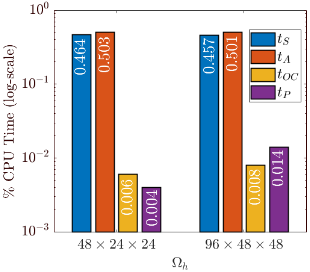
Table: Average iteration time (s) and speedup factors.

Ω_h	$48 \times 24 \times 24$ $r_{\min} = \sqrt{3}$	$96 \times 48 \times 48$ $r_{\min} = 2\sqrt{3}$
`top3D125`(s)	1.79	14.20
`top3Dmgcg`[(5)]	1.78	1.92

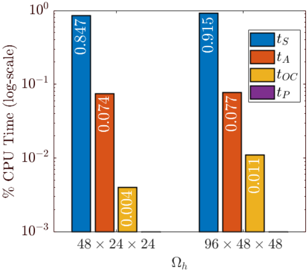
$\Omega_h = 96 \times 48 \times 48$, $V_f = 0.12$



t_S: linear solve, t_A: matrix assembly, t_{OC}: OC update, t_P: setup operations



(a) `top3Dmgcg`

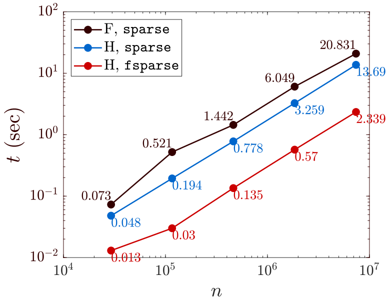


(b) `top3D125`

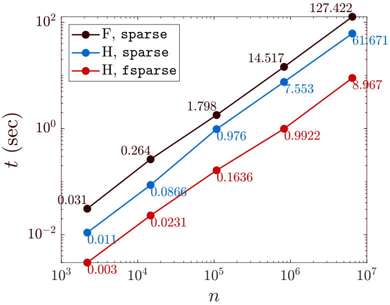
[(5)] Amir et.al. (2014)-*On multigrid-CG for efficient topology optimization*, SMO

Improving the stiffness matrix assembly

- Assemble only lower half $K^{(s)} \rightarrow$ cut $\approx 45\%$ of CPU time and RAM;
- Define mesh-related indices as `'int32'`[6] $\rightarrow$ RAM is cut to $\approx 1/4$ and CPU time to $\approx 1/10$;
- `chol` and similar work with $K^{(s)}$ (not "\"!!), (CG, MINRES can be adapted);



(a) 2D discretization



(b) 3D discretization

[6] `https://github.com/stefanengblom/stenglib`, Engblom, Lukarski (2015)-*Fast Matlab compatible sparse assembly on multicore computers*, Parallel Computing

Improving the efficiency of the OC update

$\delta_- = \max\{0, x_{k,e} - \mu\}$, $\delta_+ = \min\{1, x_{k,e} + \mu\}$ (local lower/upper bounds).

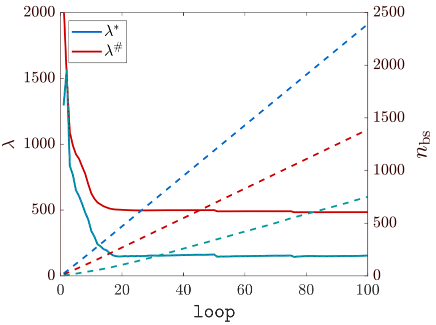
Primal & dual variables updates

Compute $(\mathbf{x}_{k+1}, \lambda_k^*)$ by iterations

$$x_{k+1,e} = \max\{\delta_-, \min\{\delta_+, x_{k,e}\left(-\frac{\partial_e c(\mathbf{x}_k)}{\lambda \partial_e V(\mathbf{x}_k)}\right)^{\frac{1}{2}}\}\}$$

$$\lambda = \left(\frac{\sum_{e\in\mathcal{M}} x_{k,e}(-\partial_e c(\mathbf{x}_k)/\partial_e V(\mathbf{x}_k))^{1/2}}{g(\mathbf{x}_k)/\partial_e V(\mathbf{x}_k) - |\mathcal{L}|\delta_- - |\mathcal{U}|\delta_+}\right)^2$$

$$\mathcal{M} = \{e \mid \delta_- < x_e < \delta_+\}$$
$$\mathcal{L} = \{e \mid x_e = \delta_-\}$$
$$\mathcal{U} = \{e \mid x_e = \delta_+\}$$



n_{bs}: cumulative number of "bisections"

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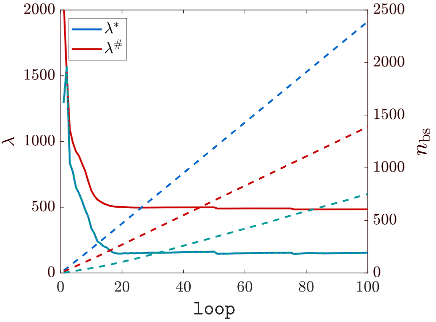
Repeated filtering operations are avoided if we adopt volume-preserving filters.



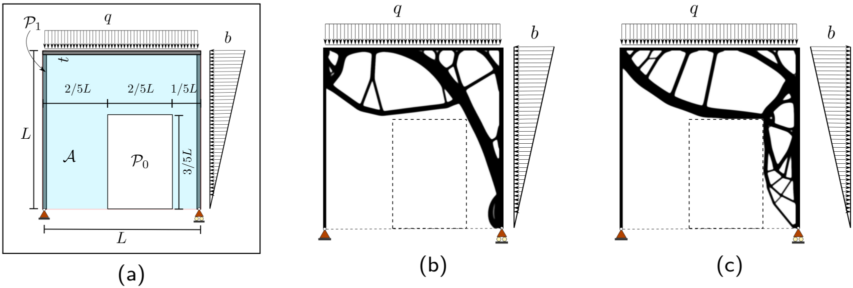
n_{bs}: cumulative number of “bisections”

	$\Lambda = [0, 10^9]$	Est.	PD
n_{bs}	2390	1389	750
$t_{\mathrm{stf}}(s)$	1.18	0.65	0.27
$t_{\mathrm{vpf}}(s)$	0.04	0.03	0.03

Introduction of passive (solid & void) elements

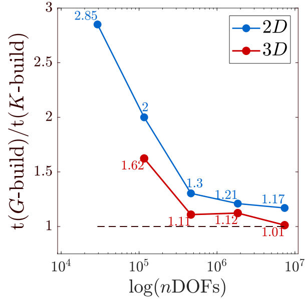
Reinforcement of a solid frame while keeping a void region

- $\Omega_h = 900 \times 900$ elements, $V_f = 0.2$;
- Average cost per iteration: $10.8s$ ($1.62 \cdot 10^6$ DOFs)



Extension to linearized buckling: topBuck250

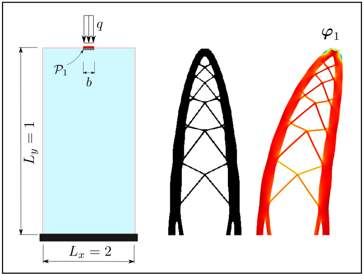
set up buckling analysis ≈ with the same cost as linear analysis



Topology Optimization with linearized buckling criteria in 250 lines of Matlab

Federico Ferrari · Ole Sigmund · James K. Guest

- Fully vectorized setup of the buckling eigenproblem and buckling load factors (BLFs) sensitivity analysis;
- Includes 4 design problems by default
  - (a) $\max \mathrm{BLF}$, s.t. $\{\mathrm{vol}, \mathrm{compliance}\}$ constraints
  - (b) $\min \mathrm{vol}$, s.t. $\{\mathrm{BLF}, \mathrm{compliance}\}$ constraints
  - (c) $\min \mathrm{Compliance}$, s.t. vol constraint
  - (d) $\min \mathrm{vol}$, s.t. $\mathrm{compliance}$ constraint
- Explicit OC update based on MMA-like approximations;



..COMING SOON!

Thank you for your attention

`top99neo` and `top3D125` codes can be found at `www.topopt.dtu.dk`

Visit also `https://www.ce.jhu.edu/topopt` for upcoming news and codes