Cellular Level Set in B-Splines (CLIBS)

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Publications:

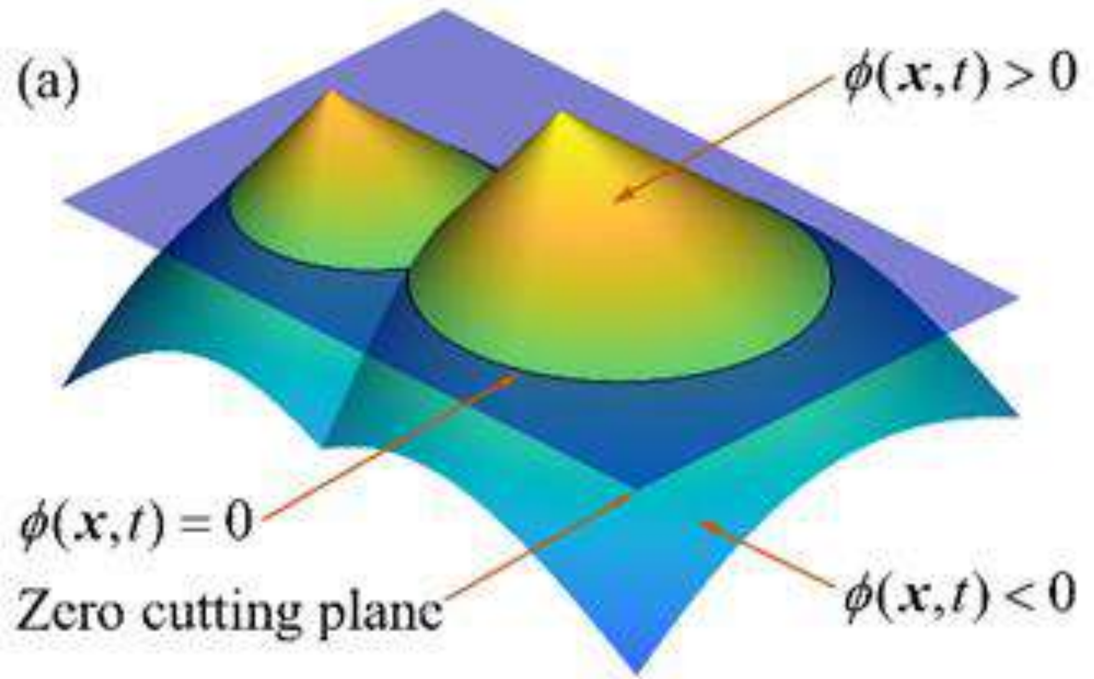
MY Wang, H Zong, Q Ma, Y Tian, M Zhou,
Cellular Level Set in B-Splines (CLIBS): A Method for Modeling and Topology Optimization of Cellular Structures,
Computer Methods in Applied Mechanics and Engineering,
DOI: 10.1016/j.cma.2019.02.026, 2019

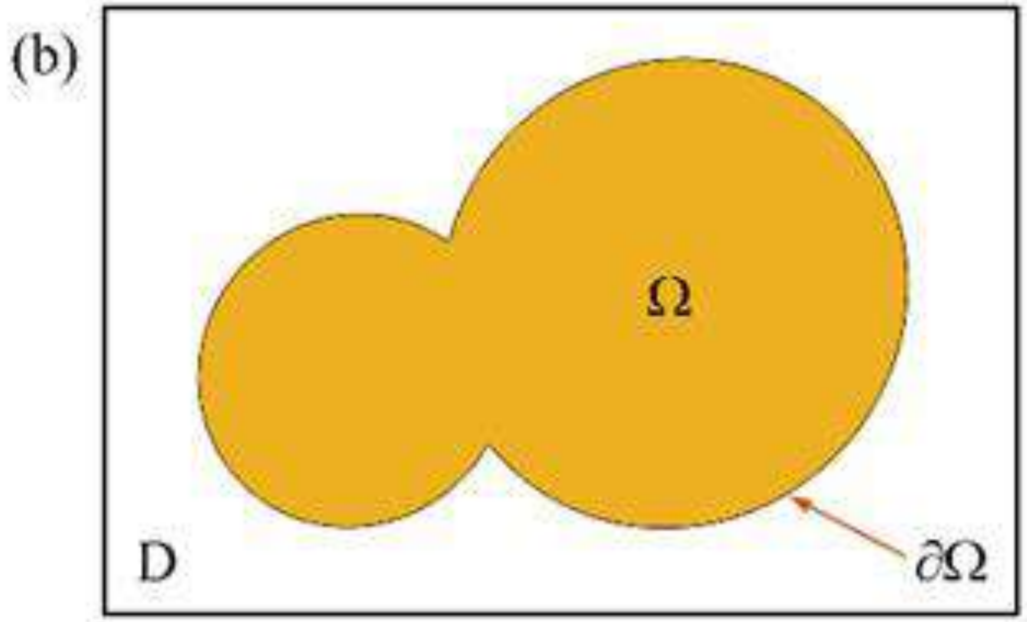
H Liu, H Zong, Y Tian, Q Ma, MY Wang,
A novel subdomain level set method for structural topology optimization and its application in graded cellular structure design,
Structural and Multidisciplinary Optimization 60 (6), 2221-2247, 2019.

Hui Liu, Ye Tian, Hongming Zong, Qingping Ma, Michael Yu Wang,
Fully parallel level set method for large-scale structure topology optimization,
Computers & Structures 221, 13-27, 2019

Y Song, Q Ma, Y He, M Zhou, MY Wang,
Stress-based shape and topology optimization with cellular level set in B-splines,
Structural and Multidisciplinary Optimization, 2020

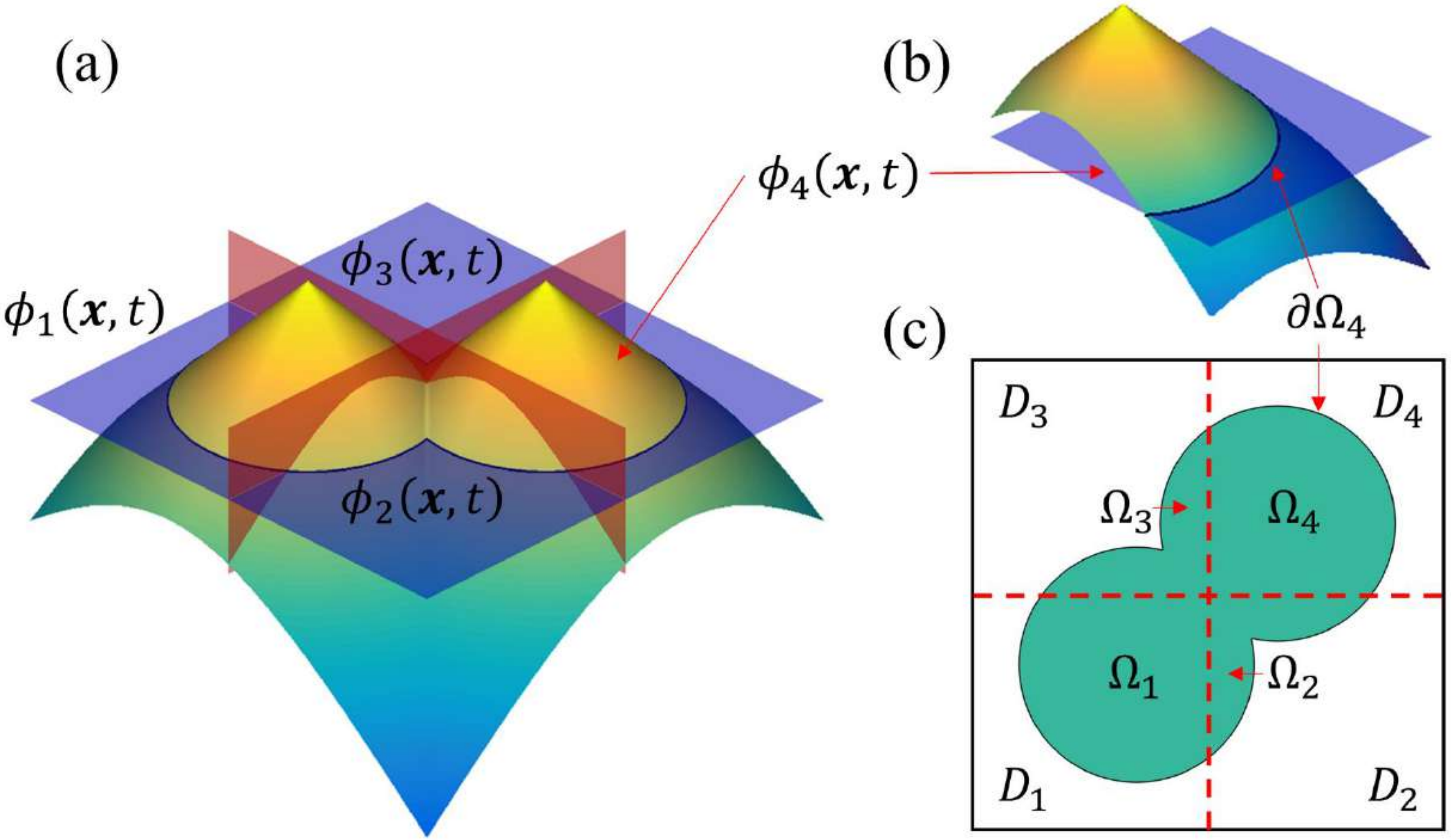
Global Level Set (Wang, Wang, Guo, 2003)

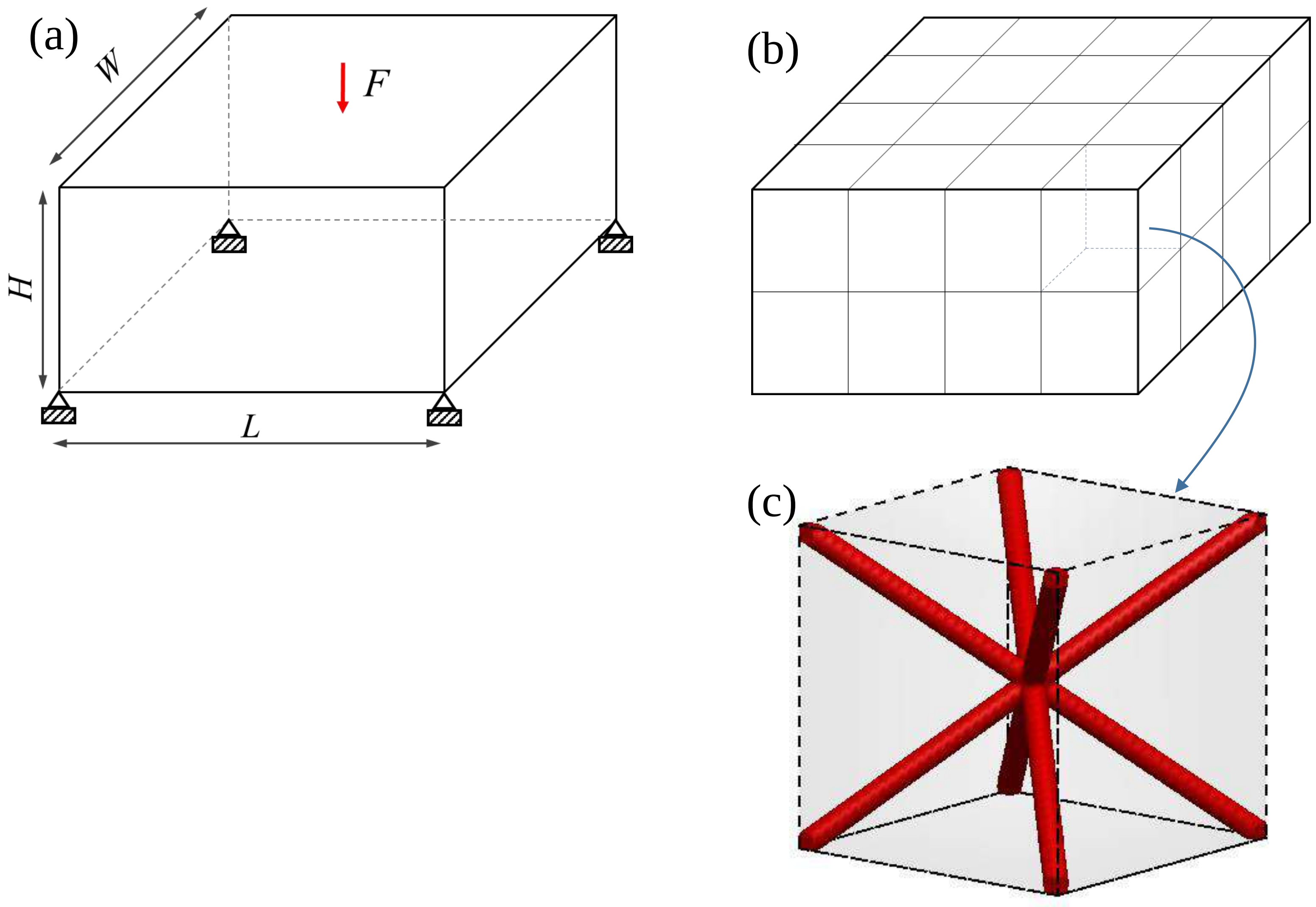




Modeling	Optimization
Global-support function	Boundary shape velocity
H-J Equation	Shape sensitivity
Re-Initialization	Velocity extension

Cellular Level Sets (Wang et. al, 2019)

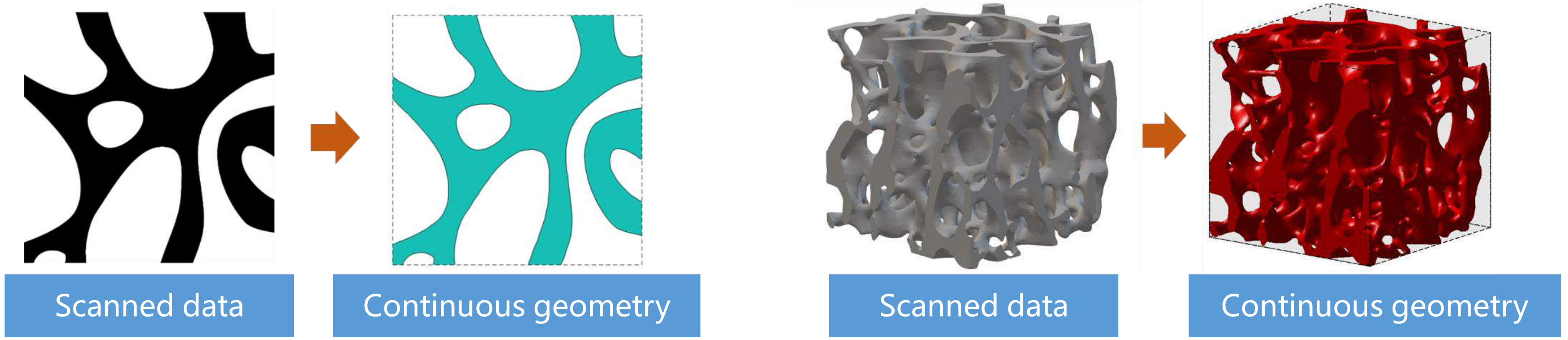



(a)
W
F
H
L
(b)
(c)

High Definition Cellular Level Set in B-Splines

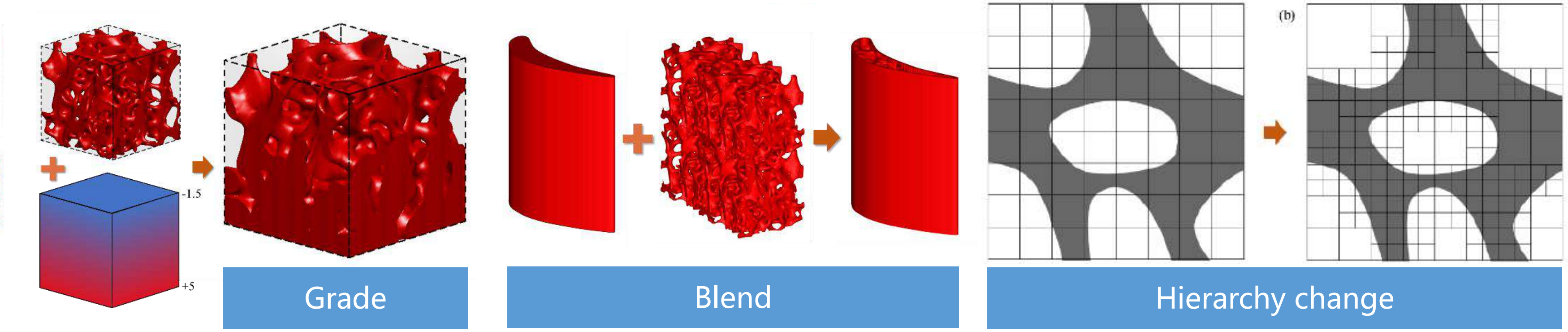
B-Splines

- Efficient reverse engineering
- Easy modeling of large-scale solid/cellular structures
- Compatible with standard CAD systems



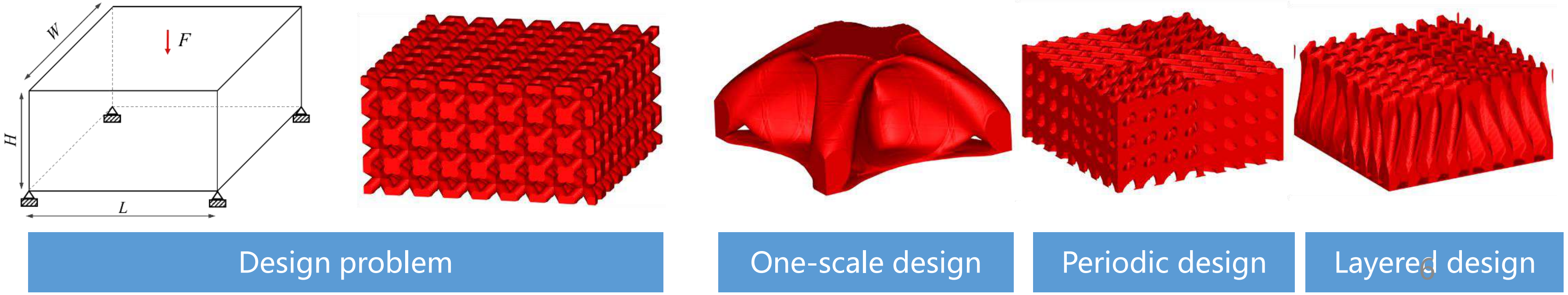
HD Cellular

- Porous and lightweight
- Powerful manipulation functions: divide, connect, periodic, dilate, erode, grade, blend, hierarchy change, etc.



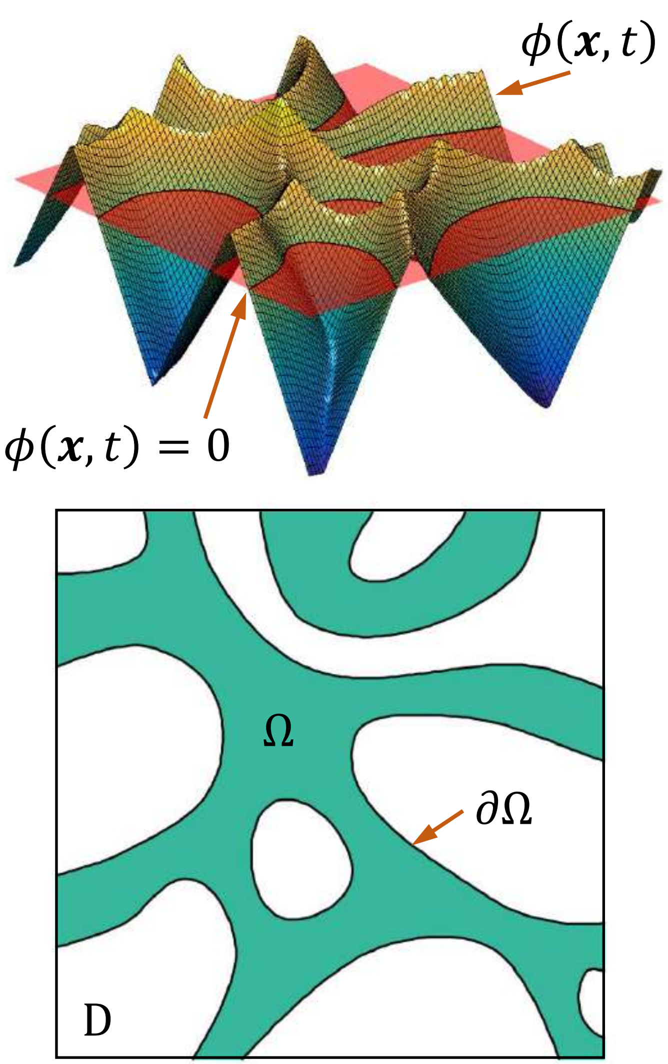
Level Set

- Continuous and smooth boundary representation
- Topology optimization with functional requirements
- Automatic workflow

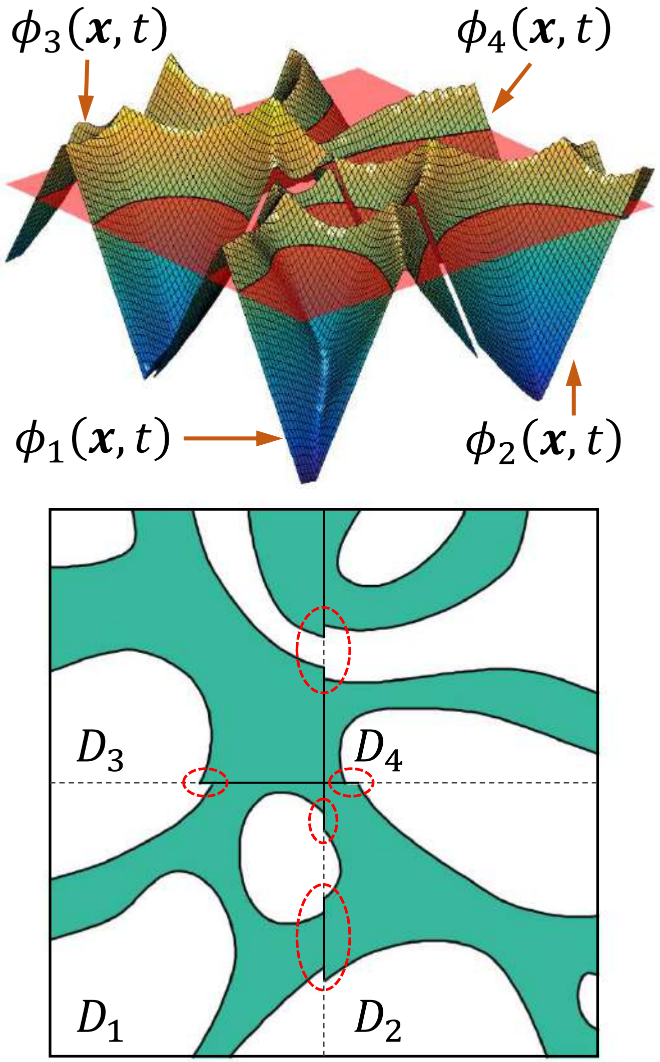


Cellular Level Set Model

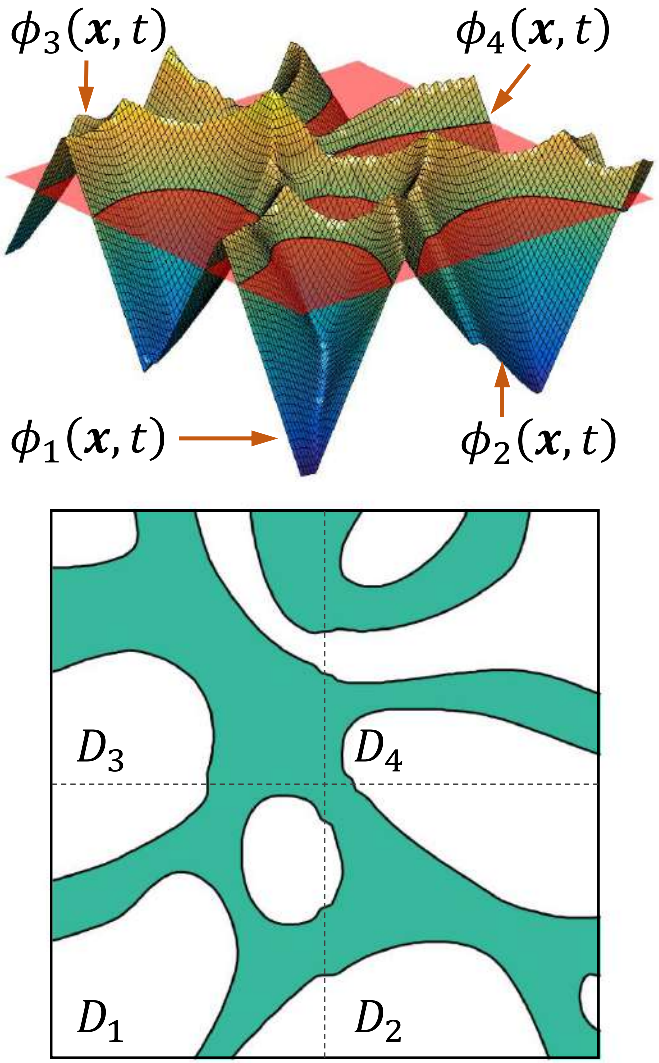
Global Level Set



Cellular Level Set



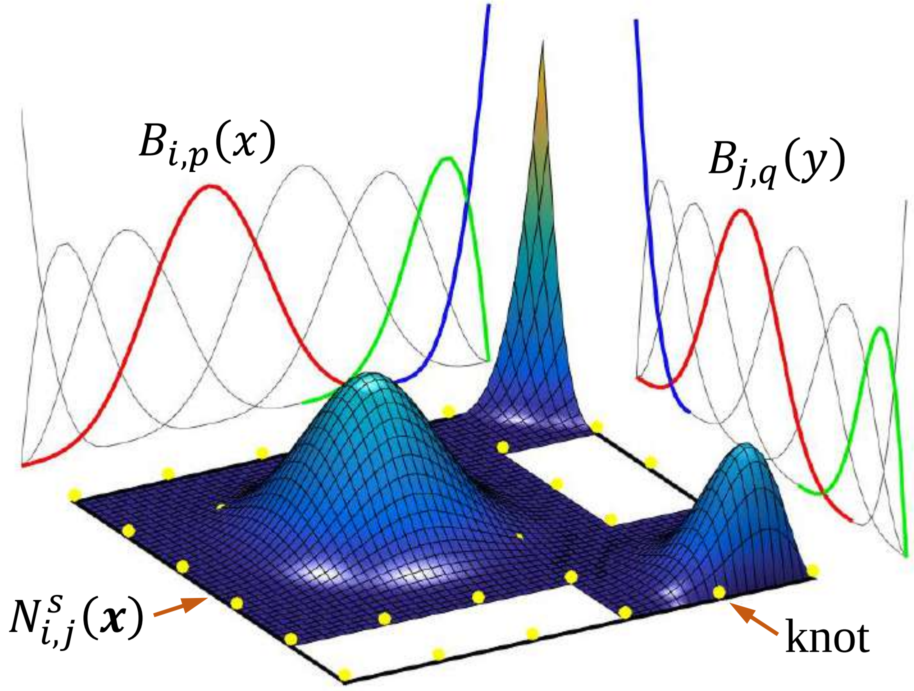
Cellular Level Set in B-Splines



Level Set in Trivariate B-Splines

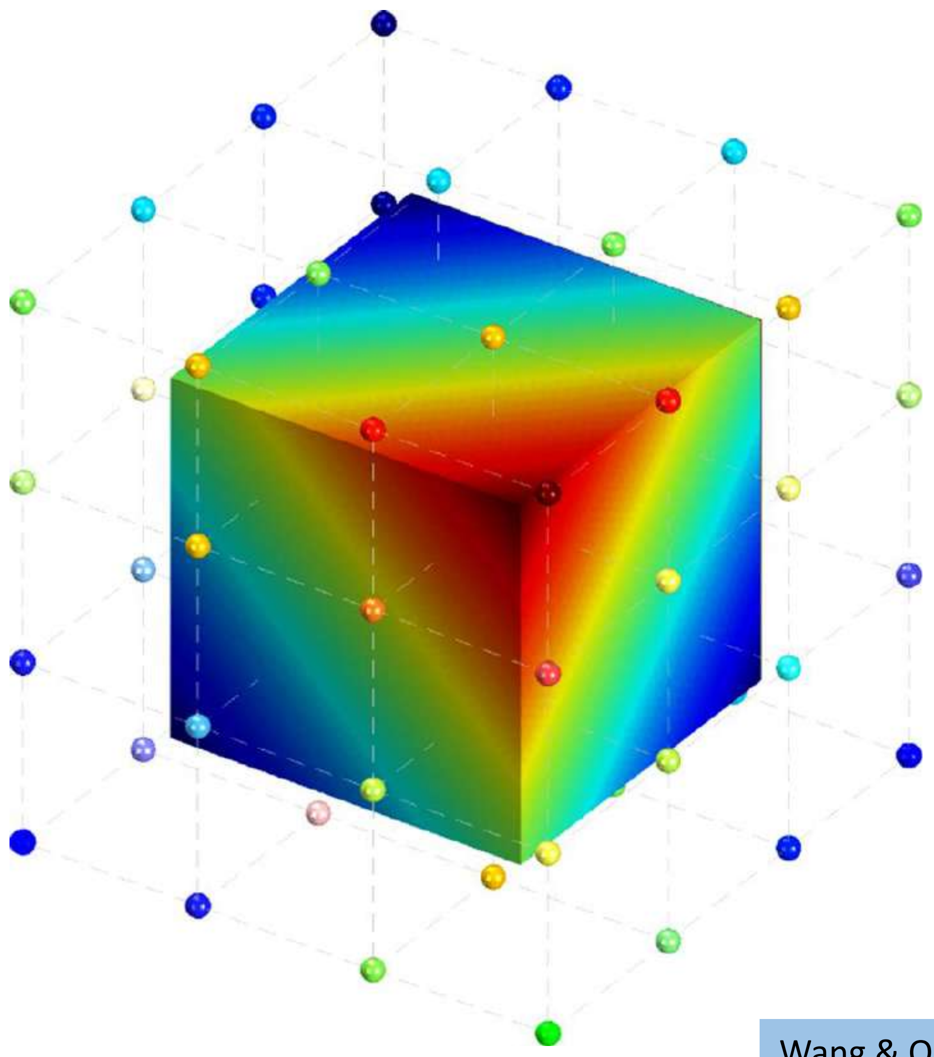
A Level Set Function in B-Splines

$$\phi_s(\boldsymbol{x},t)=\sum_{i=0}^{n}\sum_{j=0}^{m}c_{i,j}^{s}(t)N_{i,j}^{s}(\boldsymbol{x})=\sum_{i=0}^{n}\sum_{j=0}^{m}c_{i,j}^{s}(t)B_{i,p}(x)B_{j,q}(y)$$



$$\phi_s(\boldsymbol{x}, t) = \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} c_{i,j,k}^{s}(t)\, B_{i,p}^{s}(x) B_{j,q}^{s}(y) B_{k,r}^{s}(z) = 0$$

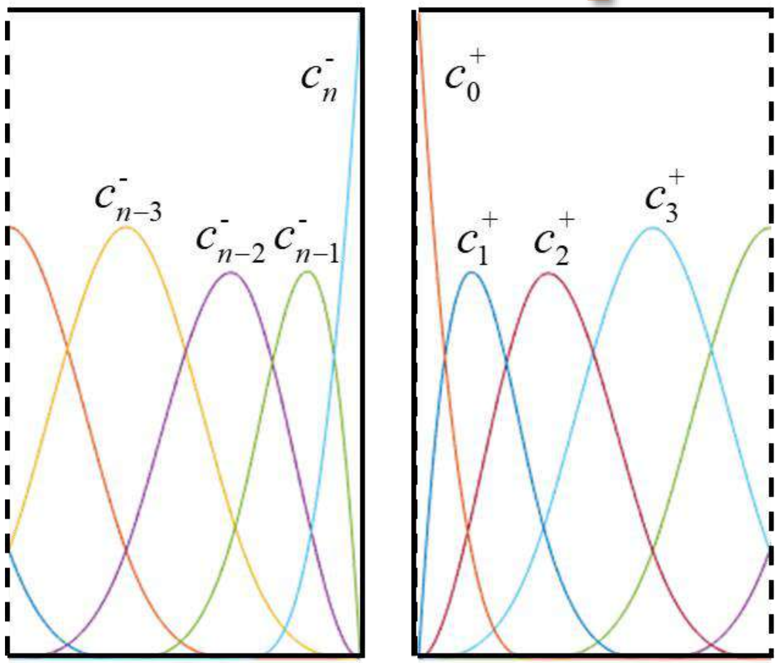
Trivariate B-spline Level Set Function



Wang & Qian 2015

- **Equality constraints**

$$f^+(x) = \left(-\frac{11}{12}c_2^+ + \frac{7}{4}c_1^+ - c_0^+\right)x^3 + \left(\frac{3}{2}c_2^+ - \frac{9}{2}c_1^+ + 3c_0^+\right)x^2 + (3c_1^+ - 3c_0^+)x + c_0^+,\quad \text{for } 0 \le x < 1$$

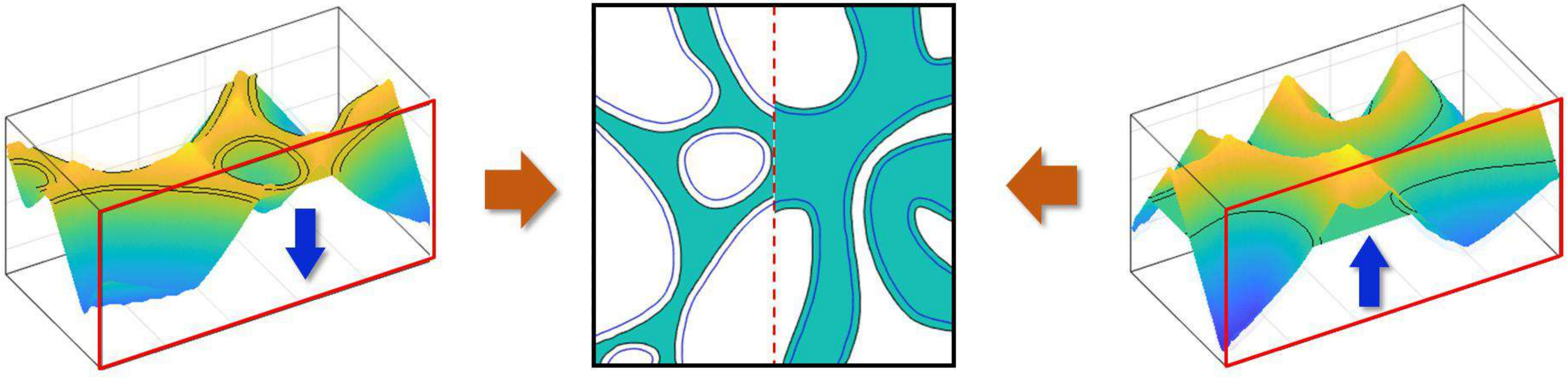


C^0 continuity	$c_0^+ = c_n^-$
C^1 continuity	$c_0^+ = c_n^-$ $c_1^+ - c_0^+ = -c_{n-1}^- + c_n^-$
C^2 continuity	$c_0^+ = c_n^-$ $c_1^+ - c_0^+ = -c_{n-1}^- + c_n^-$ $c_2^+ - 3c_1^+ + 2c_0^+ = c_{n-2}^- - 3c_{n-1}^- + 2c_n^-$

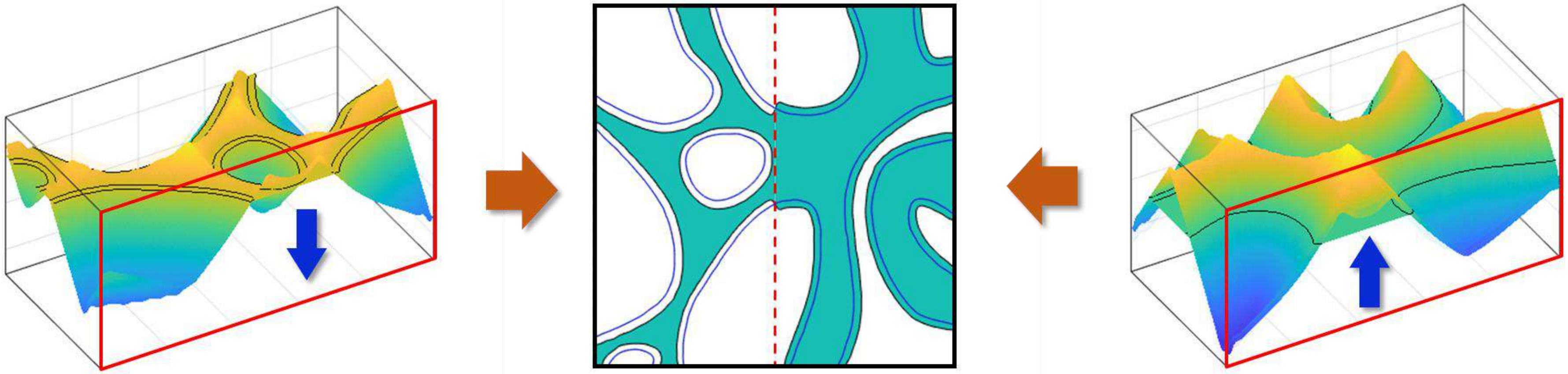
$$f^-(x) = \left(\frac{11}{12}c_{n-2}^- - \frac{7}{4}c_{n-1}^- + c_n^-\right)x^3 + \left(\frac{3}{2}c_{n-2}^- - \frac{9}{2}c_{n-1}^- + 3c_n^-\right)x^2 + (-3c_{n-1}^- + 3c_n^-)x + c_n^-,\quad \text{for } -1 \le x \le 0$$

Cell Connectivity

- **Directly connect**



- **C^0 continuity** $(c_0^+)^* = (c_n^-)^* = (c_0^+ + c_n^-)/2$



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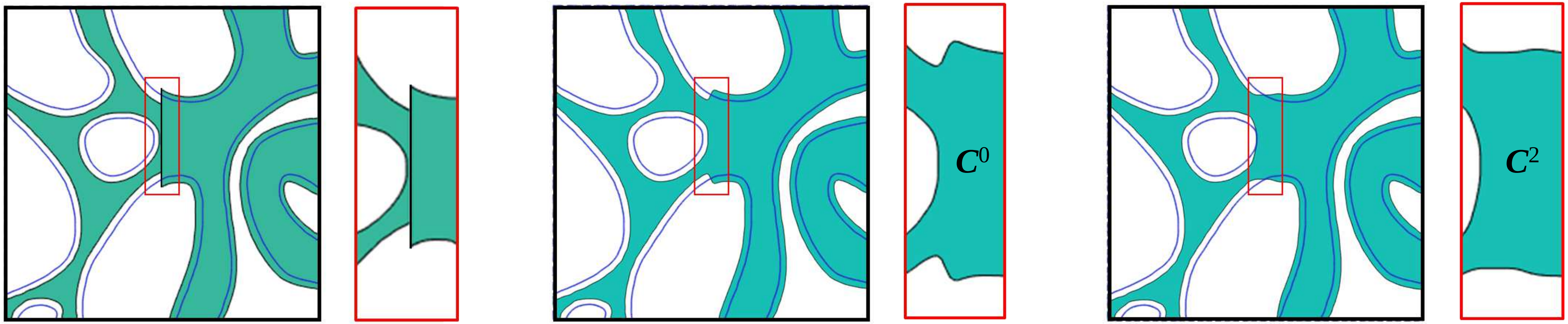
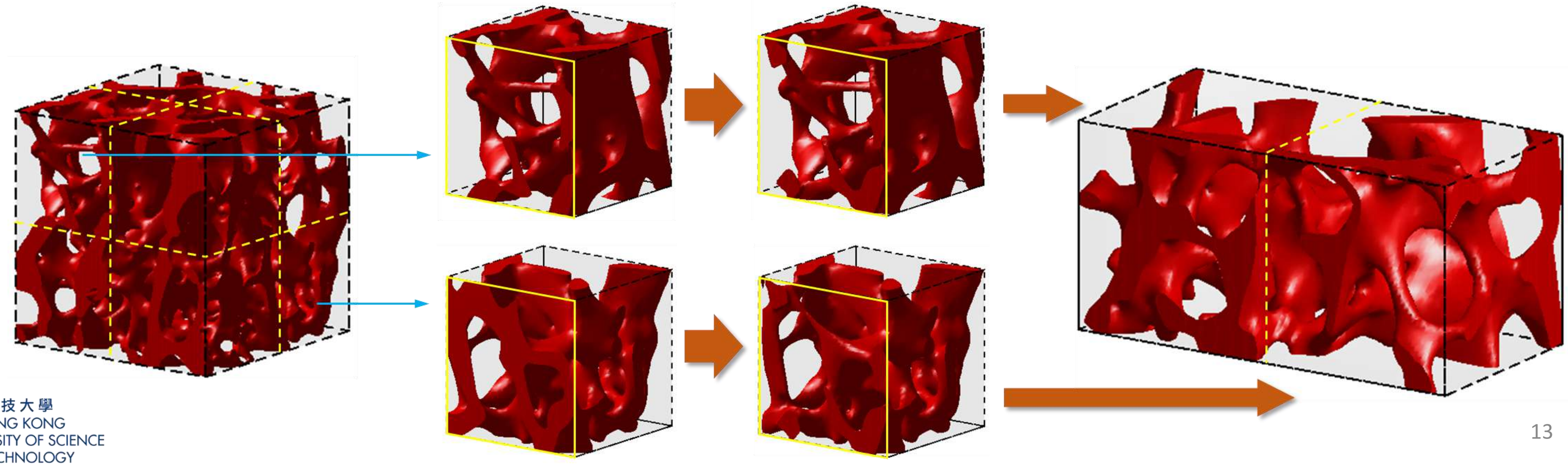
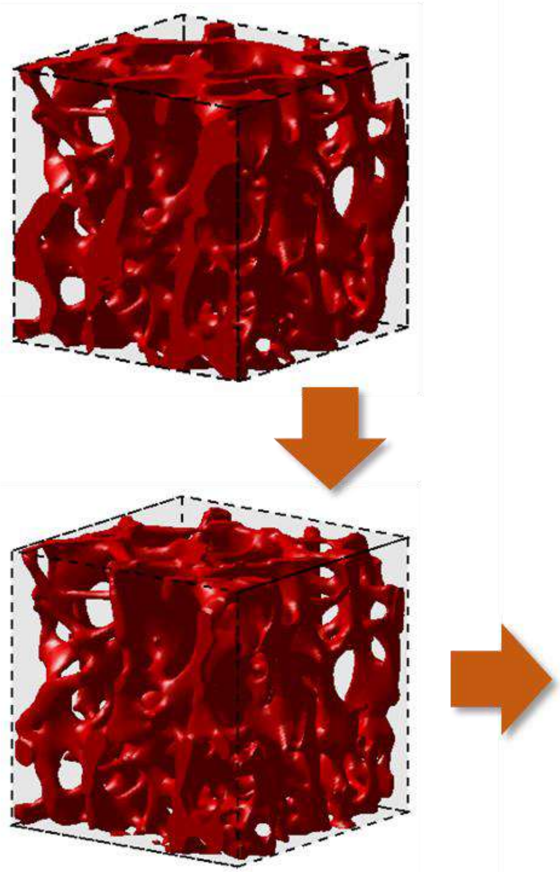
Cell Connectivity

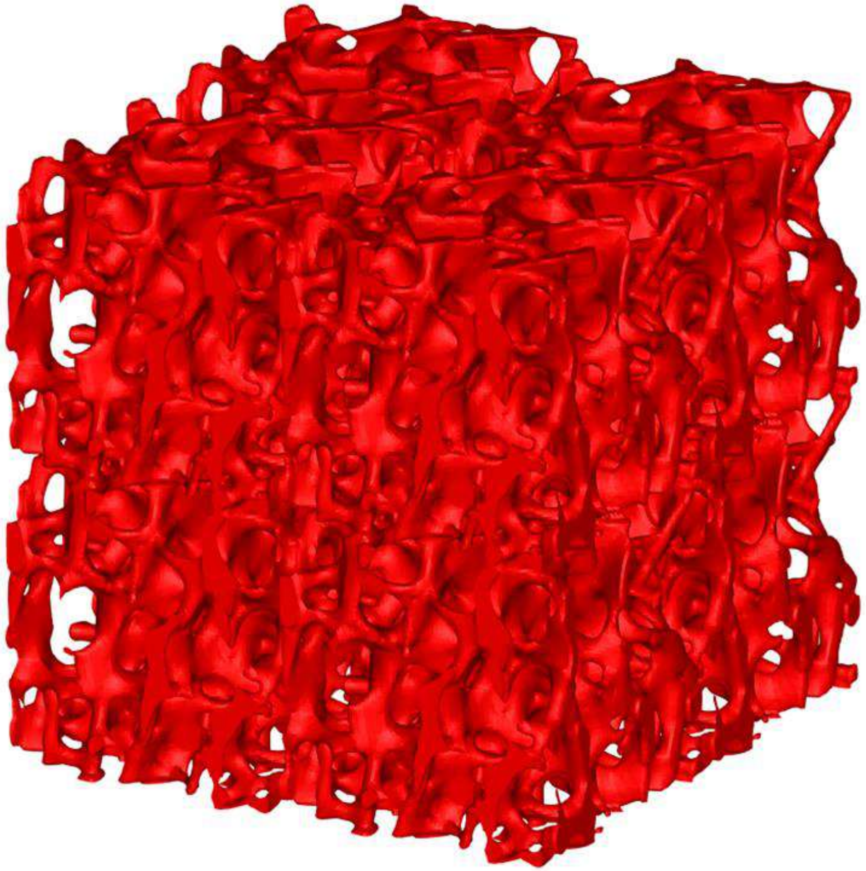


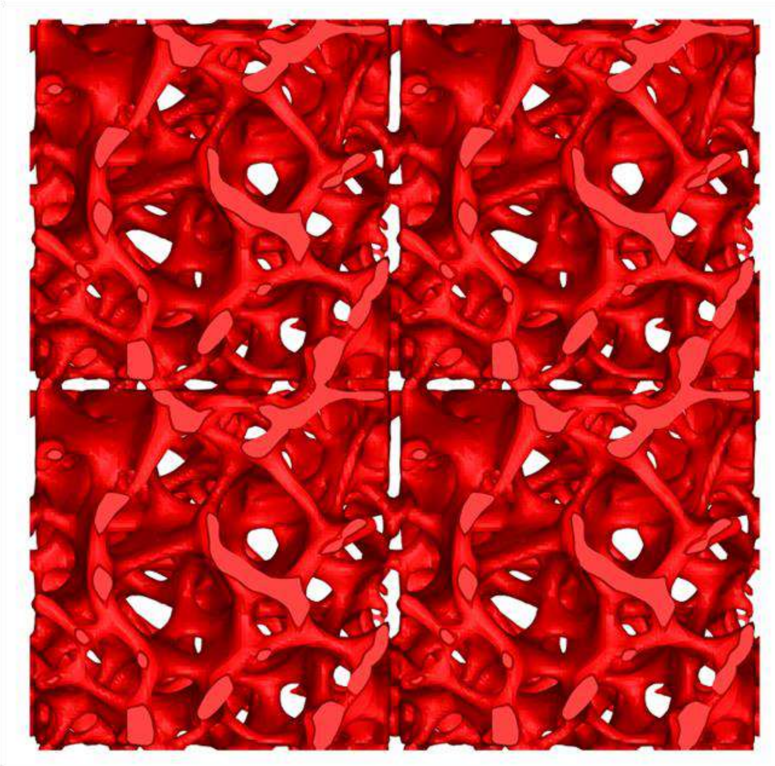


Cell Periodicity

A periodic cell is easily represented when the geometric continuity conditions are applied to **the opposite faces of the cell** in the direction of periodic repetition. Fro example in 1D: $c_0 \;= c_n$

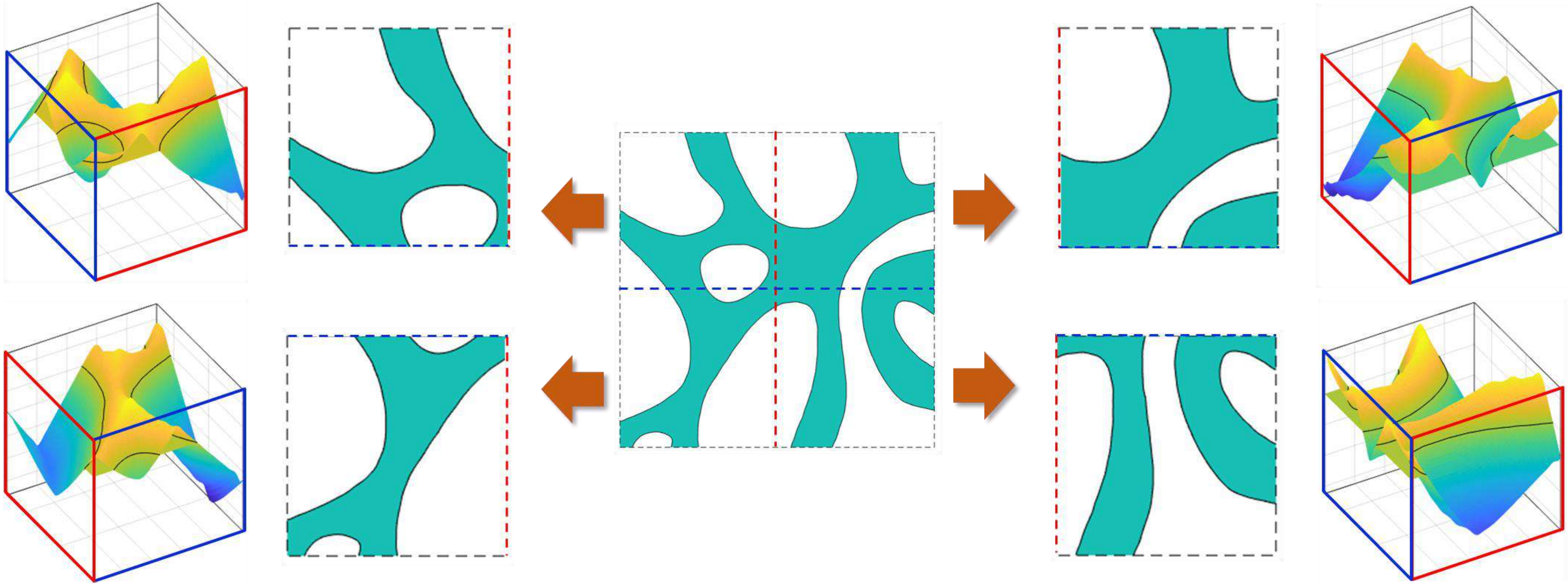






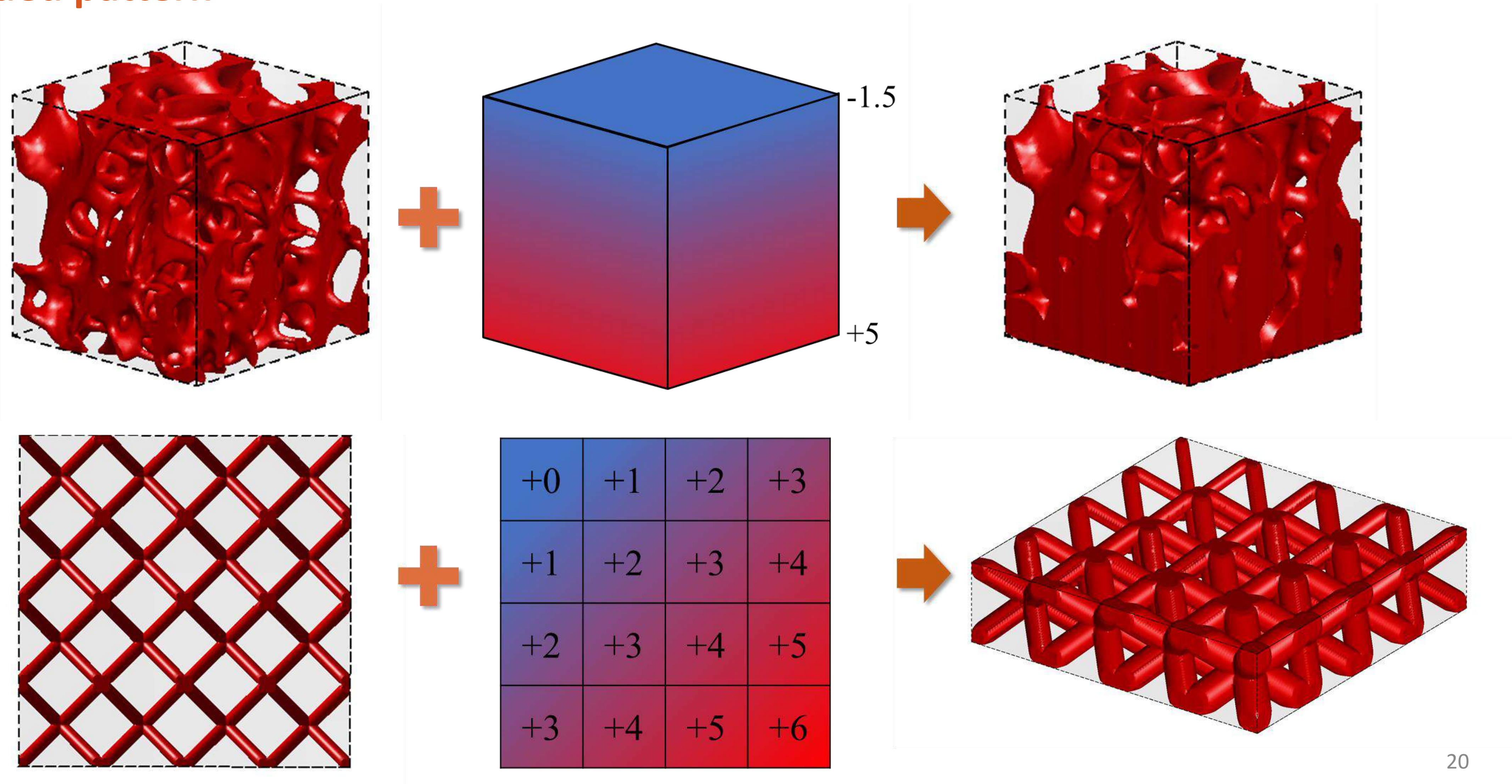
Cell Division

- **2D case**



Cell Manipulation

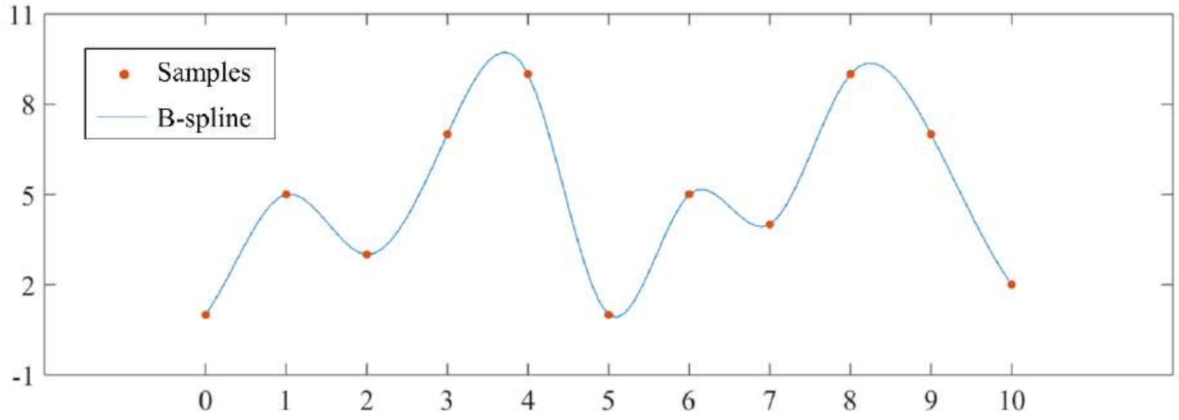
- **Graded pattern**



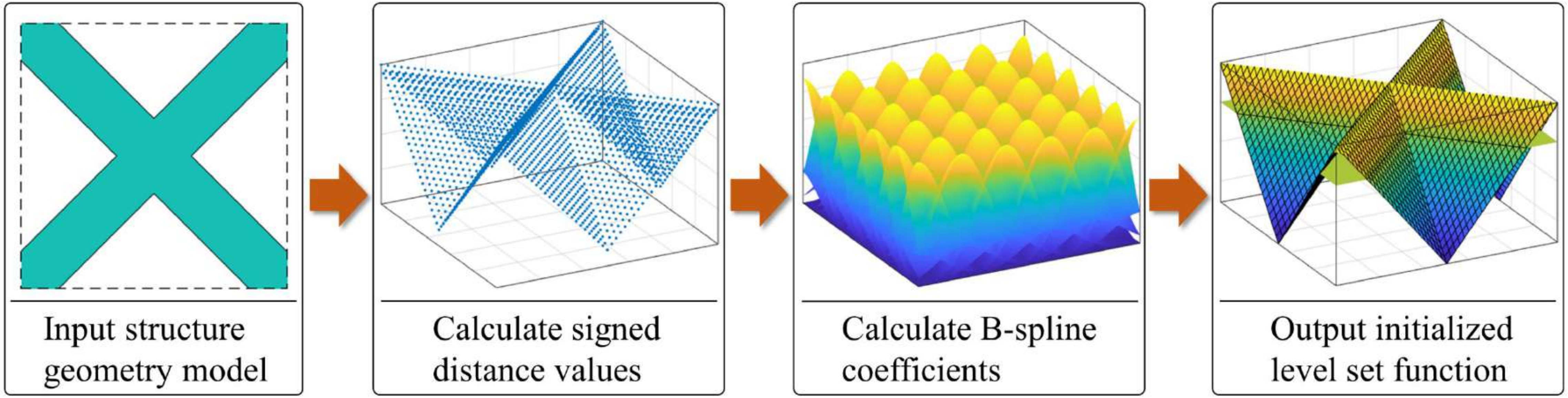
Fast B-spline Interpolation

- **Fast B-spline interpolation**

  Given a serious of samples, the coefficients of a cubic B-spline that passes through all the samples can be calculated **without solving linear equations**.

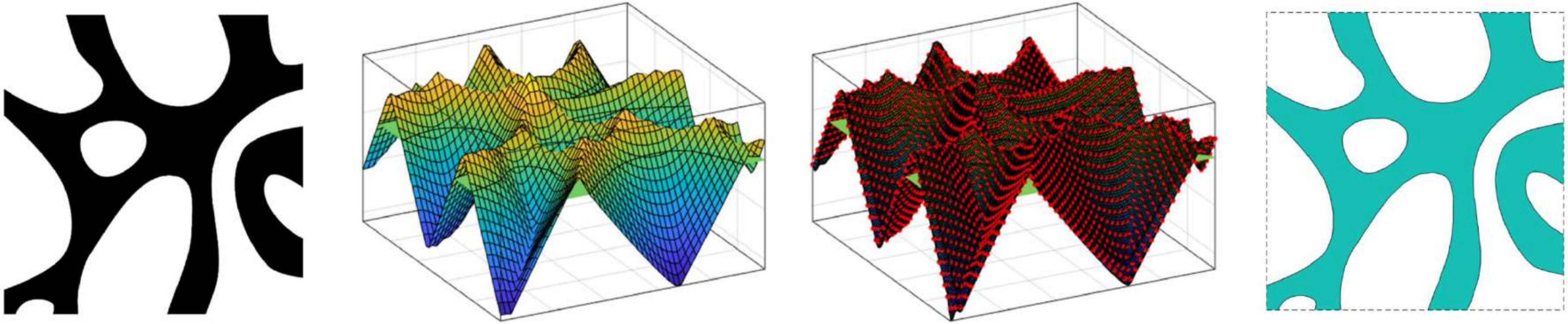


- **Initialization and reconstruction**

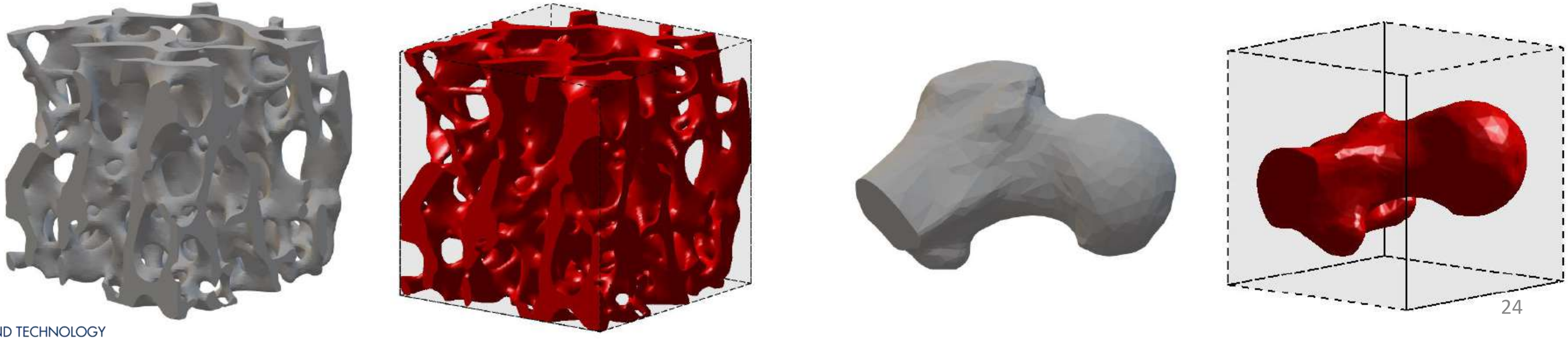


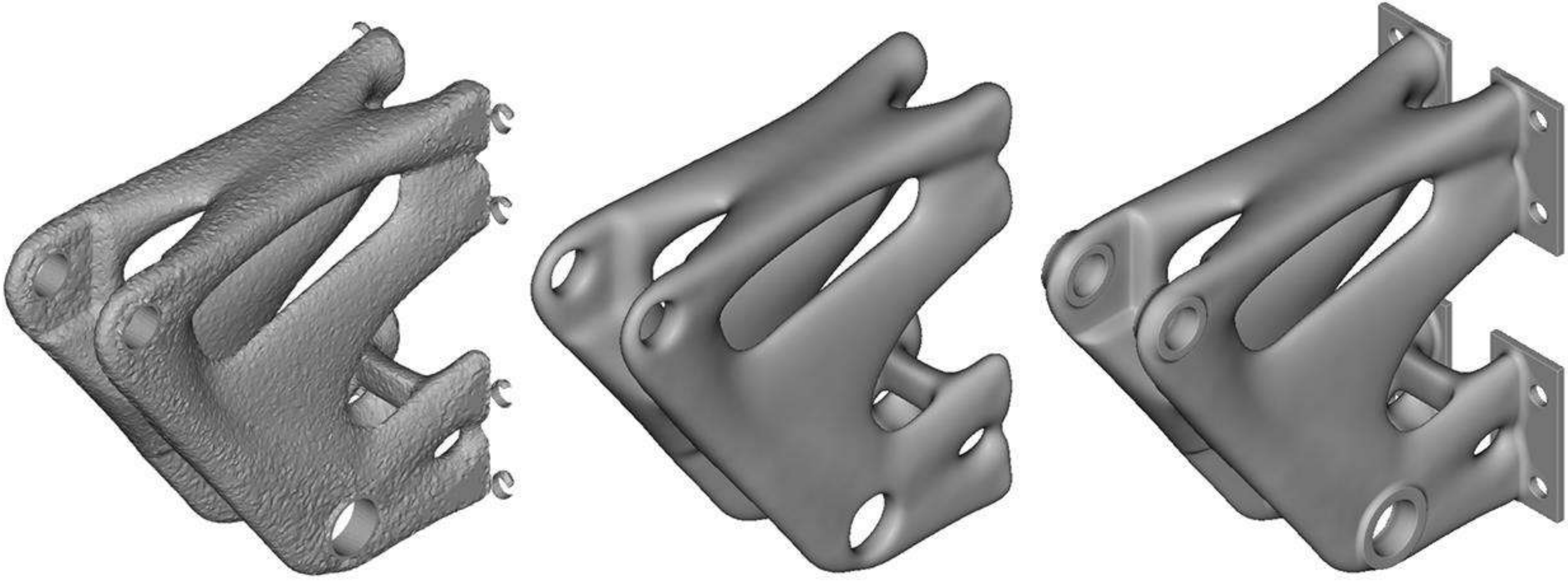
Fast B-spline Interpolation

- **2D case**



- **3D case**





Automated smoothing process (middle) is immediately usable for additional modeling operations (right). *Source: nTopology*

Sensitivity Analysis

Optimization problem

$$\underset{\phi}{\text{Minimize:}} \quad J(\boldsymbol{u},\phi) = \int_D \varepsilon_{ij}(\boldsymbol{u}) C_{ijkl} \varepsilon_{kl}(\boldsymbol{u}) H(\phi) d\Omega$$

$$\text{Suject to} \quad \begin{cases} a(\boldsymbol{u},\boldsymbol{v},\phi) = l(\boldsymbol{v},\phi), \boldsymbol{v} \in \boldsymbol{U} \\ G(\phi) = \int_D H(\phi) d\Omega - V_{max} \le 0 \\ \boldsymbol{u} = \bar{\boldsymbol{u}}, \quad \text{on } \partial\Omega_u \\ \sigma_{ij}^n = \bar{\sigma}_{ij}^n, \quad \text{on } \partial\Omega_\sigma \end{cases}$$

Shape derivative

$$\dot{L} = \int_{\partial\Omega} g(\boldsymbol{x}) V^n d\boldsymbol{x}, \quad g(\boldsymbol{x}) = -\varepsilon_{ij}\big(\boldsymbol{u}(\boldsymbol{x},t)\big) C_{ijkl} \varepsilon_{kl}\big(\boldsymbol{u}(\boldsymbol{x},t)\big) + \lambda$$

Cellular level set model in B-splines

$$V_s^n(\boldsymbol{x},t) = \frac{1}{|\nabla\phi_s(\boldsymbol{x},t)|} \frac{\partial\phi_s(\boldsymbol{x},t)}{\partial t}$$
$$= \frac{1}{|\nabla\phi_s(\boldsymbol{x},t)|} \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=0}^{l} \dot{c}_{i,j,k}^s(t)\, B_{i,p}^s(x) B_{j,q}^s(y) B_{k,r}^s(z)$$

$$\dot{L} = \int_{\partial\Omega} \frac{g(\boldsymbol{x})}{|\nabla\phi_s(\boldsymbol{x},t)|} \left[\sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=0}^{l} \dot{c}_{i,j,k}^s(t)\, B_{i,p}^s(x) B_{j,q}^s(y) B_{k,r}^s(z) \right] d\boldsymbol{x}$$

$$\dot{L} = \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=0}^{l} \dot{c}_{i,j,k}^s(t) \int_D g(\boldsymbol{x}) \delta(\phi) B_{i,p}^s(x) B_{j,q}^s(y) B_{k,r}^s(z)\, d\boldsymbol{x}$$

Coefficient derivative

$$\dot{c}_{i,j,k}^s(t) = \int_{D_s} W(u,w,\phi) B_{i,p}^s(x) B_{j,q}^s(y) B_{k,r}^s(z) d\boldsymbol{x}, \text{ for } s = 1,\cdots,M$$

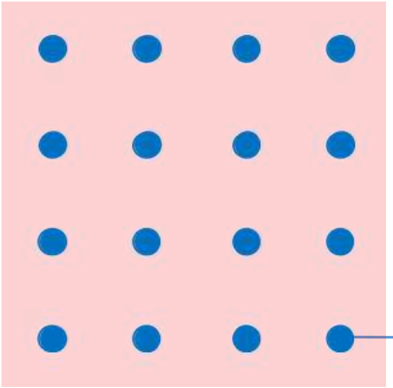
Update scheme

$$c_{i,j,k}^s(t+1) = c_{i,j,k}^s(t) + \Delta t\, \dot{c}_{i,j,k}^s(t)$$

or MMA.

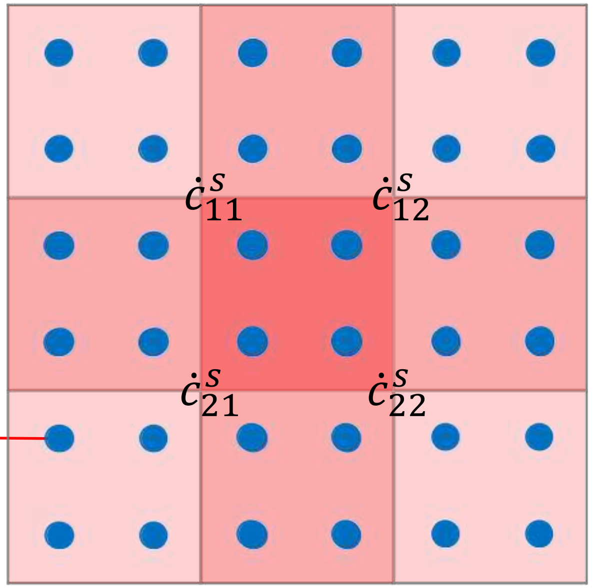
- **Discrete Separable Convolutions for Gradient Calculation**

$$\dot{c}^s_{i,j,k} = \sum_{\boldsymbol{x} \in D^s_{i,j,k}} W_h(\boldsymbol{x})\, \beta^3_h\left(\frac{x - x_i}{h_x}\right) \beta^3_h\left(\frac{y - y_j}{h_y}\right) \beta^3_h\left(\frac{z - z_k}{h_z}\right)$$



Discretized kernel function $\beta^3_h \beta^3_h$

Discretized feature function W_h



- **Re-Normalization**

$$|\phi_s(\boldsymbol{x})| < \|c^s_{i,j,k}\|_\infty$$

$$\tilde{c}^s_{i,j,k}(t+1) := \frac{c^s_{i,j,k}(t+1)}{\|c^s_{i,j,k}(t+1)\|_\infty}$$

- **Bound on the Slope**

$$A_3 \le (p+1)(q+1)(r+1)\sqrt{K_p^2 + K_q^2 + K_r^2}$$

$$K_3 = 2/3$$

- **HPC Computing**

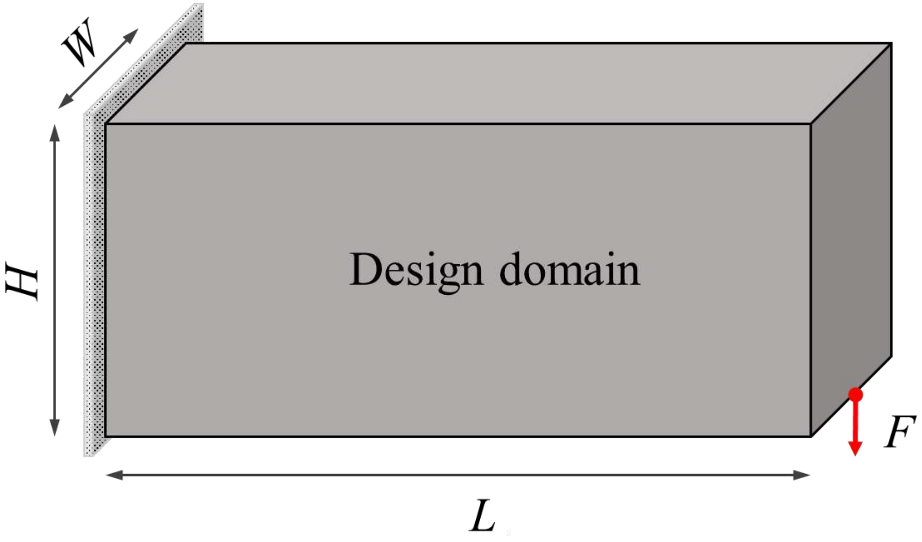
Discrete Separable Convolution

On Separable-Support Cells

$$\text{for } s = 1, \cdots, M$$

Examples

- **Case 1**



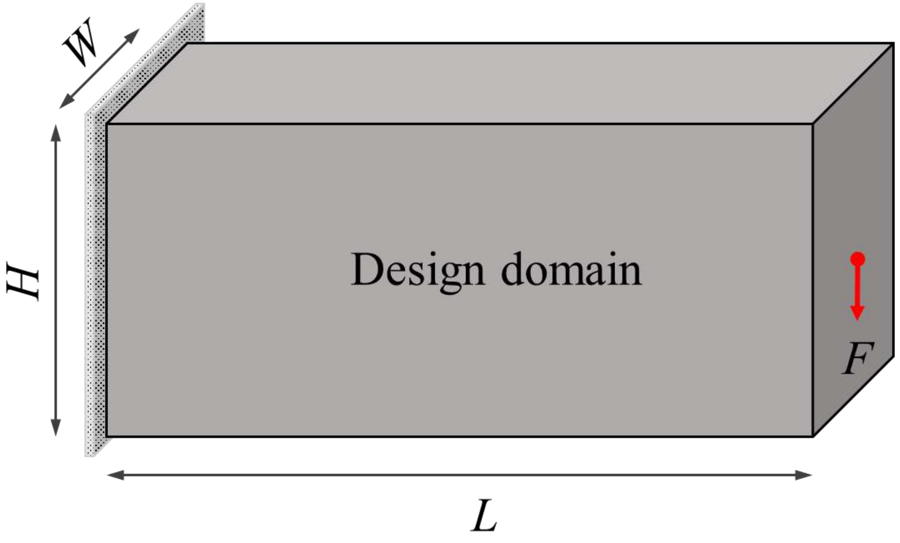
Domain 4×2×2

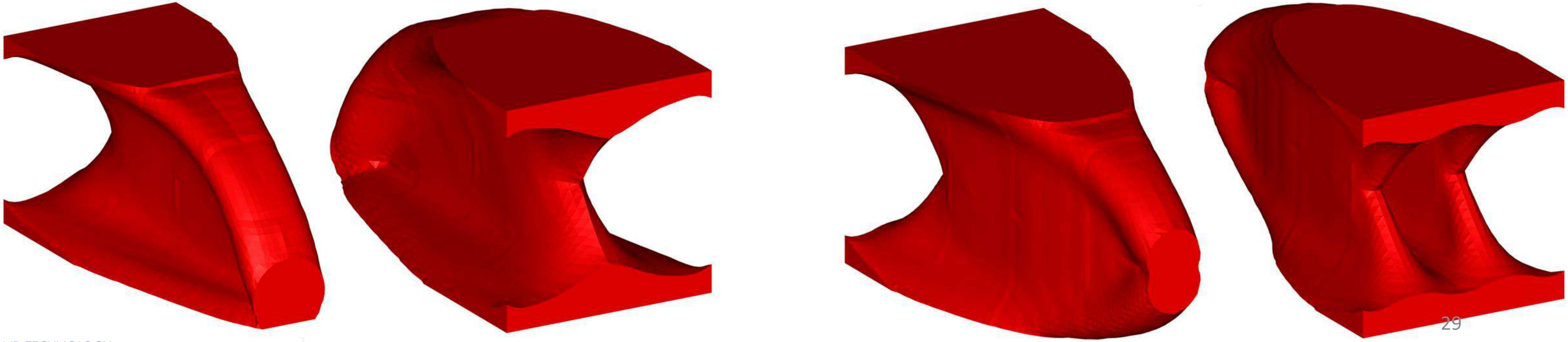
Cell 1×1×1

Grid 0.0625

Knot Span 0.0625

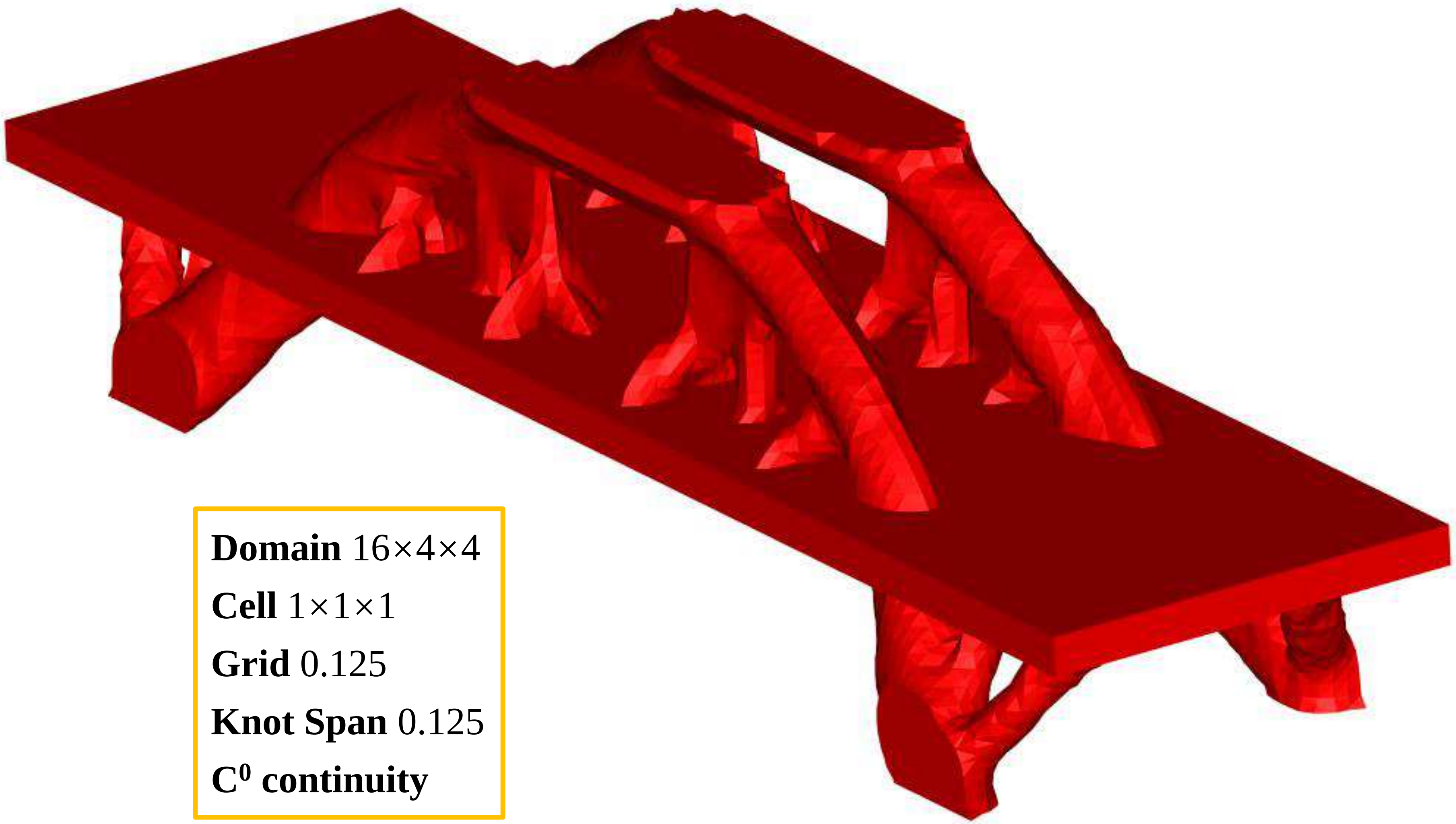
C^0 continuity





Examples

- **Case 2**



Domain 16×4×4
Cell 1×1×1
Grid 0.125
Knot Span 0.125
C^0 continuity

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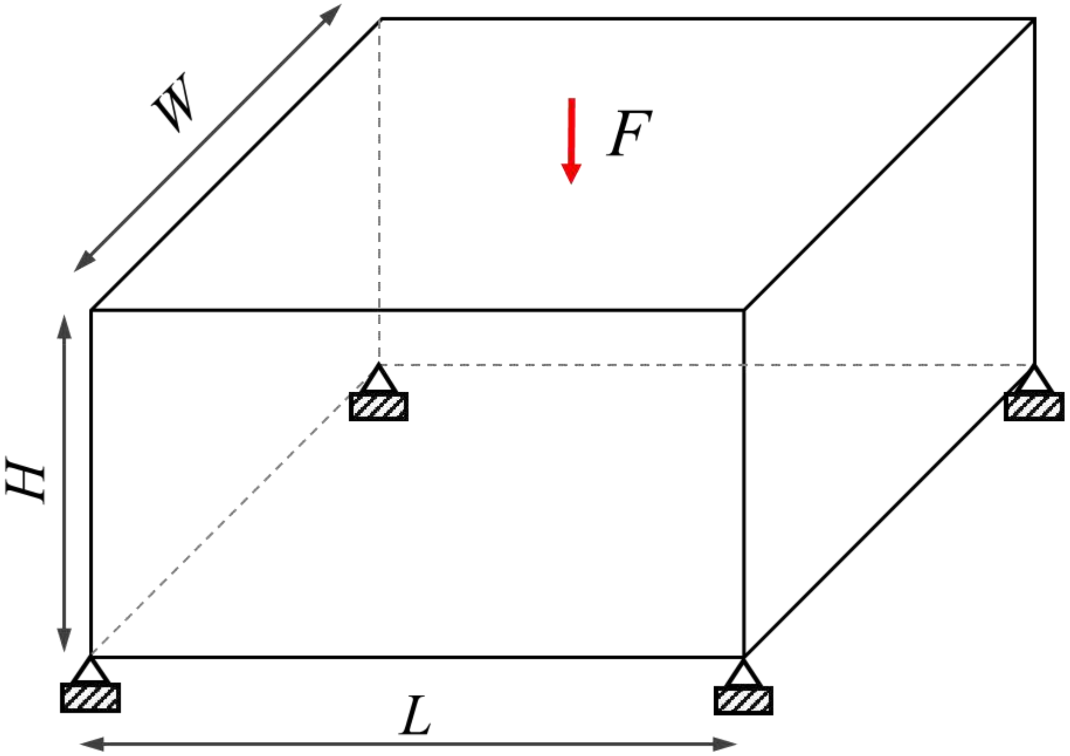
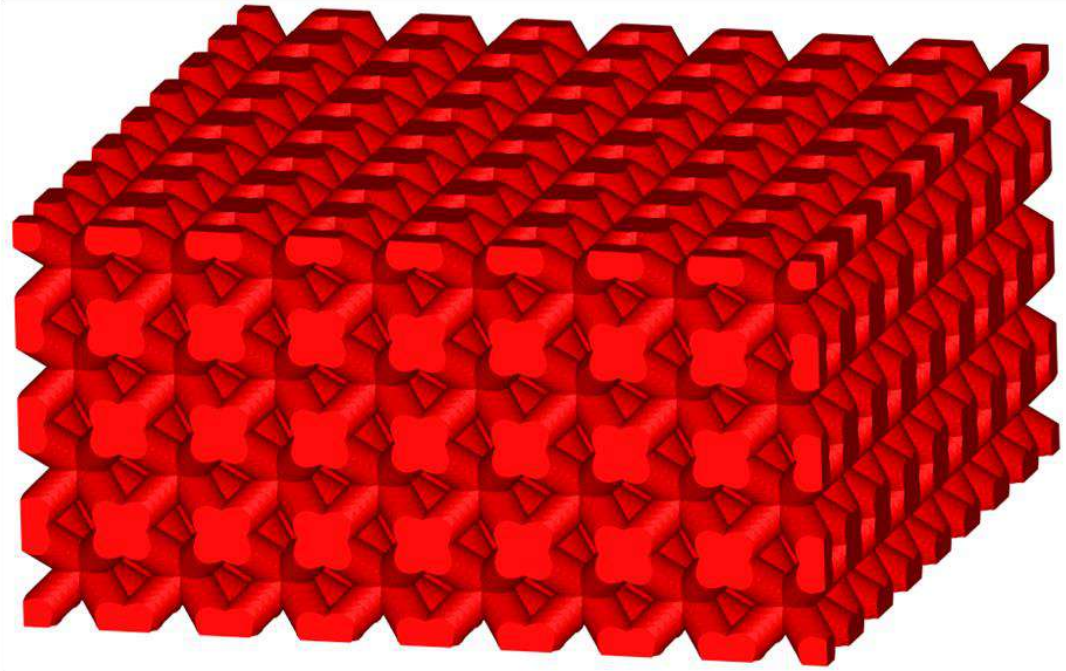
Examples

- **Case 3**



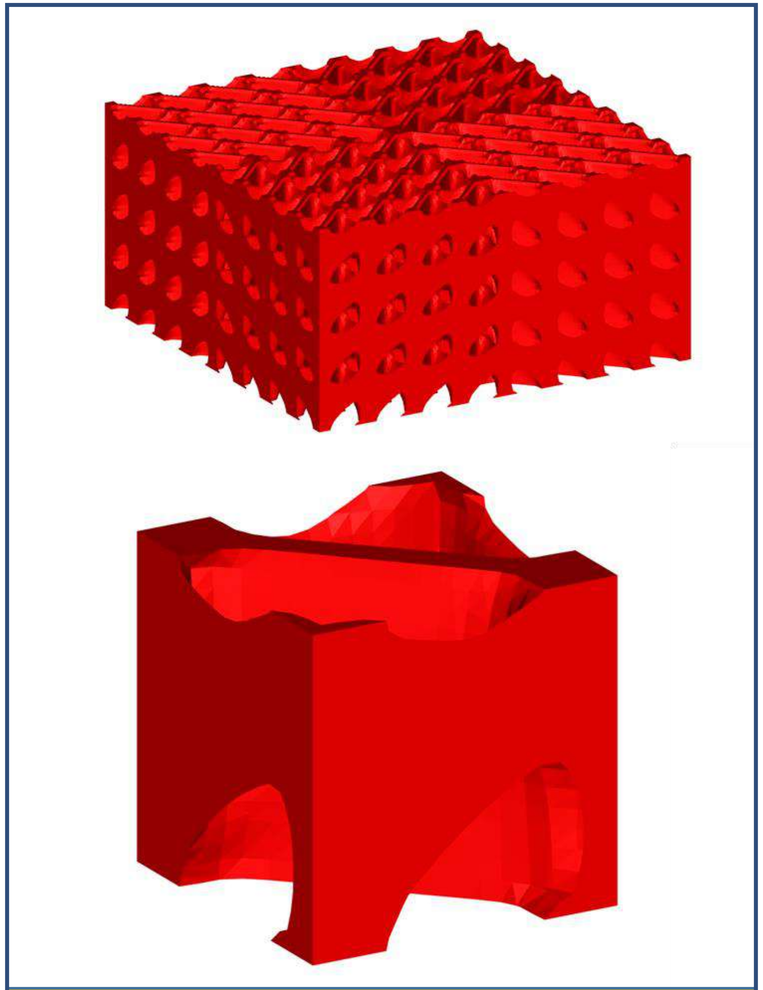


Domain 8×8×4 **Cell** 1×1×1 **Grid** 0.0625 **Knot Span** 0.0625 **C^0 continuity**

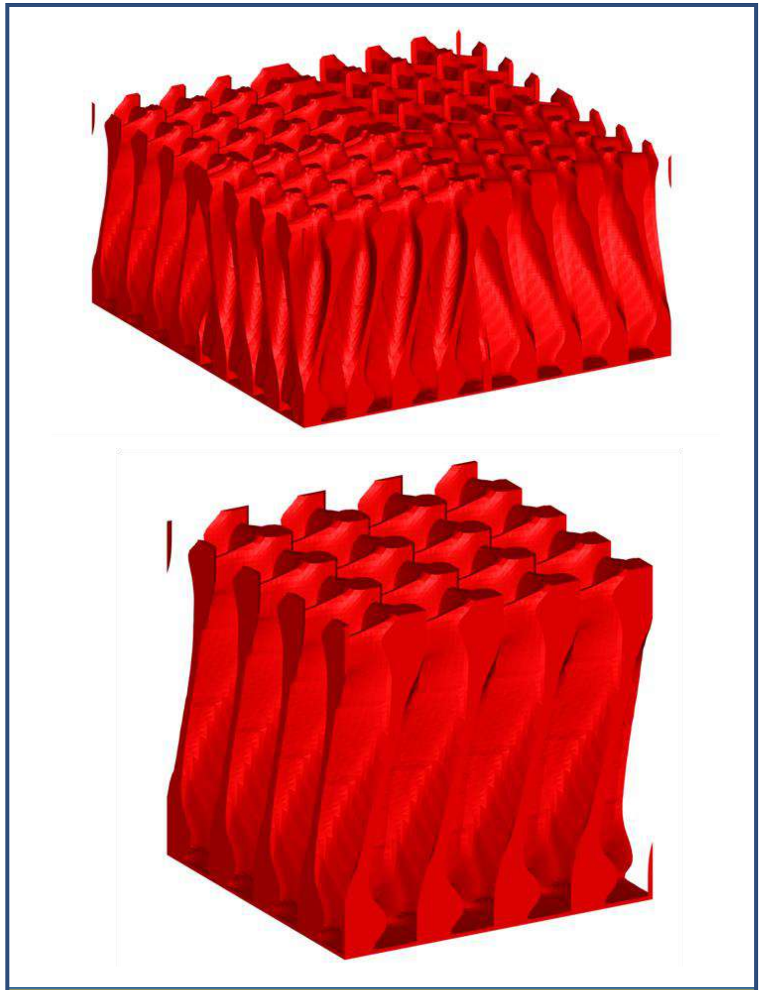
Examples

- **Case 3**

One scale solid design



Periodic cellular structure

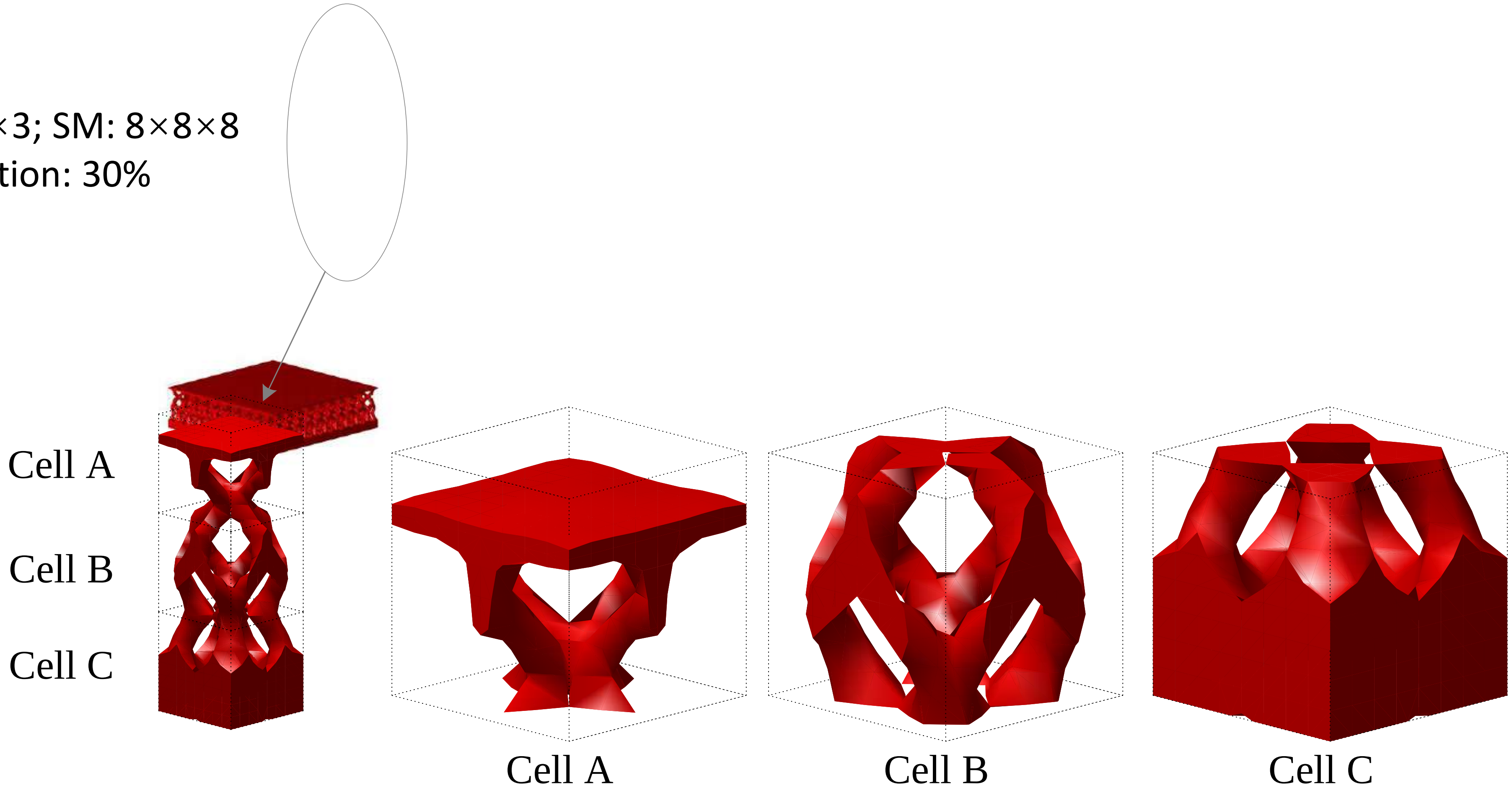


Layered cellular structure

3D Layered Plate (three layers)

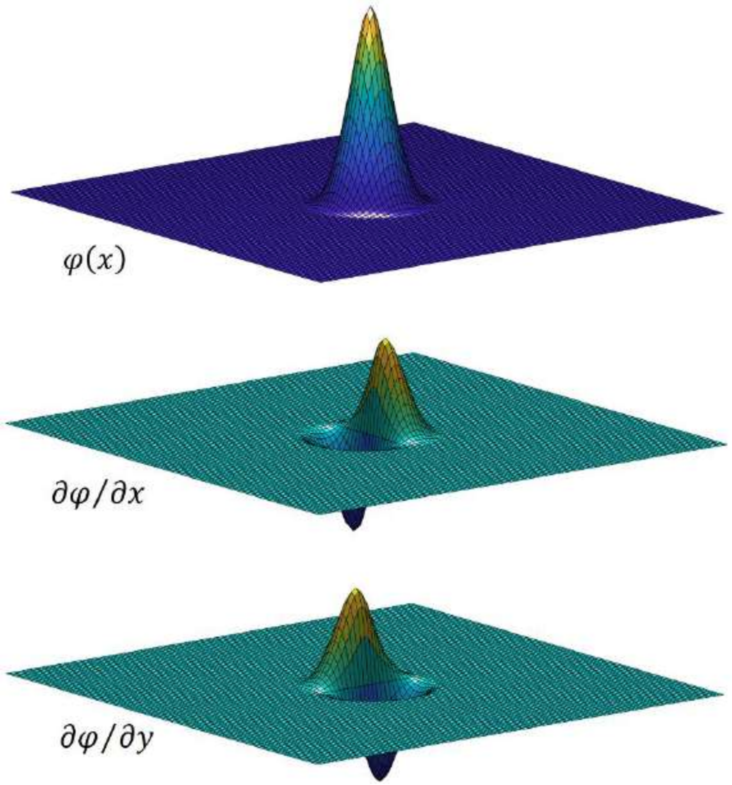
Solved with EMsFEM

CM: 10×10×3; SM: 8×8×8
Volume fraction: 30%



Cellular Level Set in RBFs

Parameterization with *compactly supported radial basis functions (CS-RBFs)*:



$$\phi(x,t) = \sum_{i=1}^{n} \varphi_i(x)\alpha_i(t) + p(x,t)$$

$$\mathbf{H}\boldsymbol{\alpha}(t) = \boldsymbol{\phi}(t)$$

$$\boldsymbol{\alpha}(t_{k+1}) = \boldsymbol{\alpha}(t_k) + \Delta t \mathbf{H}^{-1}\boldsymbol{\omega}\big(t_k, \boldsymbol{\alpha}(t_k)\big)$$

$$\boldsymbol{\alpha}(t_0) = \mathbf{H}^{-1}\boldsymbol{\phi}(t_0)$$

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Fully parallel level set method

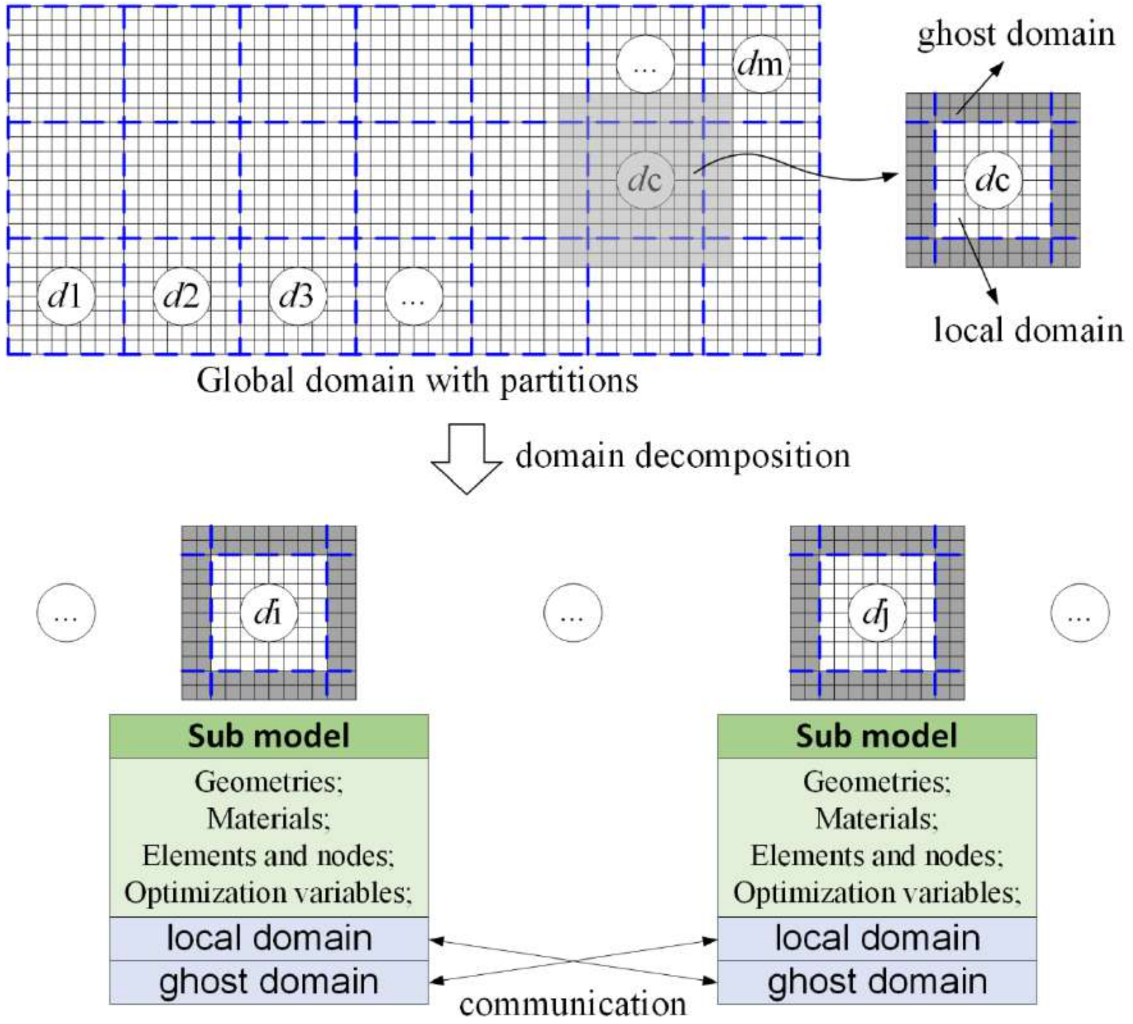

Illustration of domain decomposition

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Fully parallel level set method

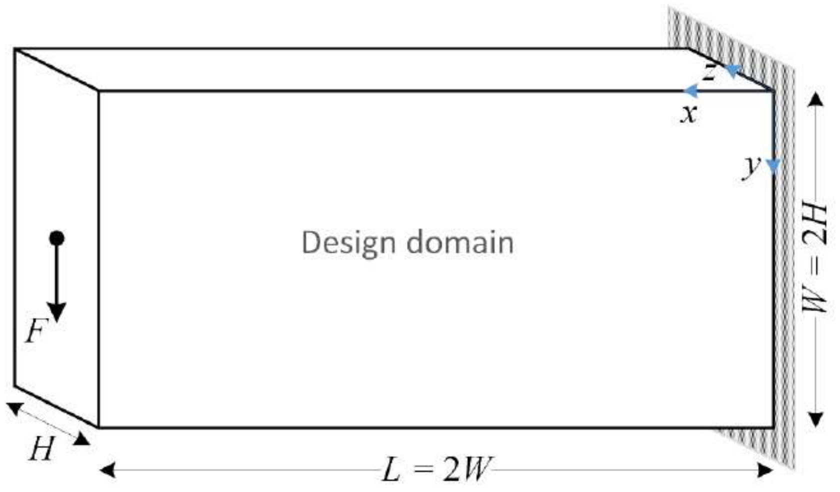
thickness $H = 24$ and the load $F = 1$

Mesh 1: 96×48×24 (110,592)

Mesh 2: 192×96×48 (884,736)

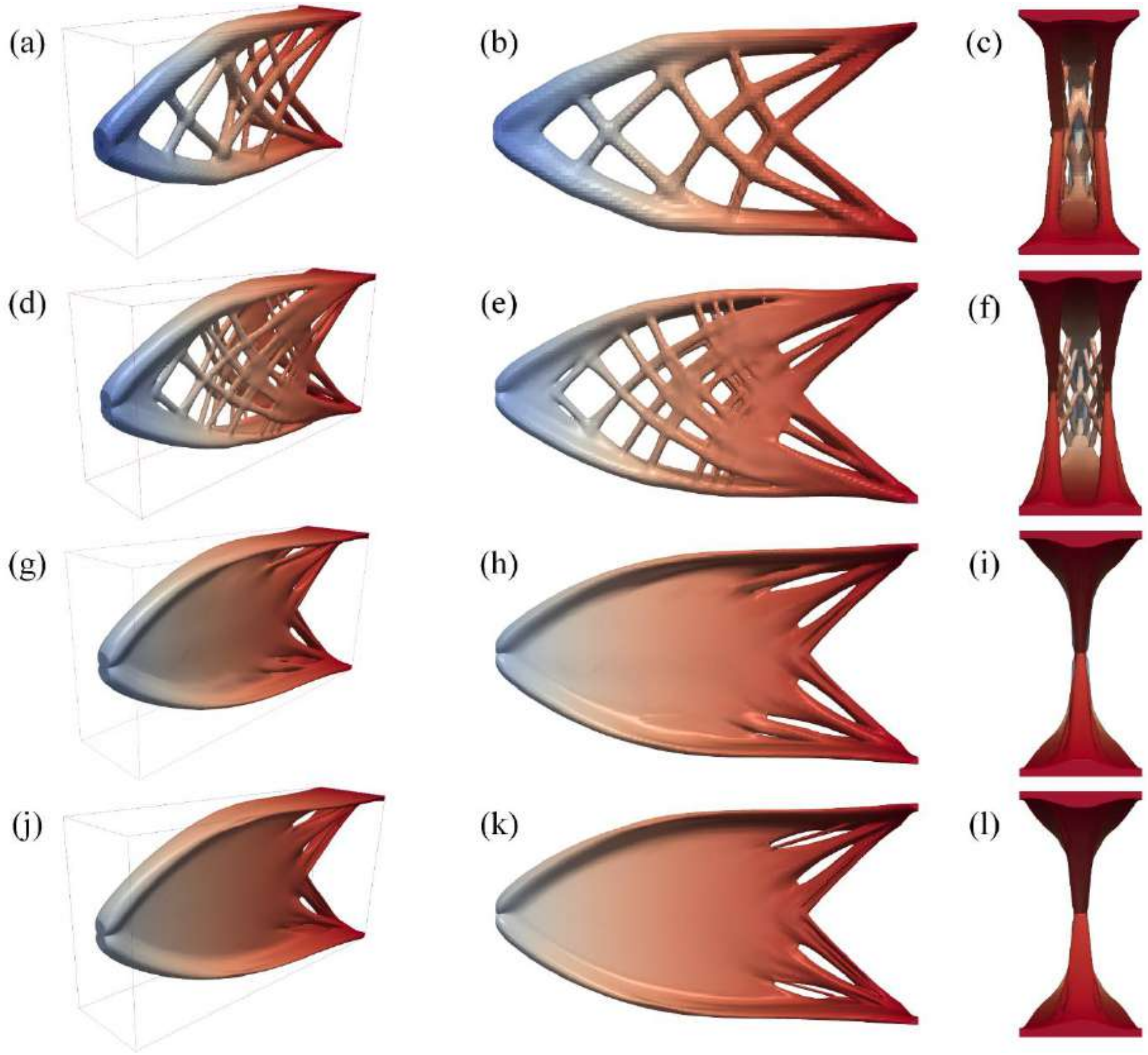
Mesh 3: 288×144×72 (2,985,984)

Mesh 4: 384×192×96 (7,077,888)



The maximum usable volume fraction is set as 0.12

Fully parallel level set method



Mesh 1: 96×48×24 (110,592)

About 45 minutes on the PC

Mesh 2: 192×96×48 (884,736)
Almost 6.5 hours on the WS

Mesh 3: 288×144×72 (2,985,984)

HPC1 with 120 cores and about 2.67 hours

Mesh 4: 384×192×96 (7,077,888)

HPC2 with 160 cores and about 6 hours

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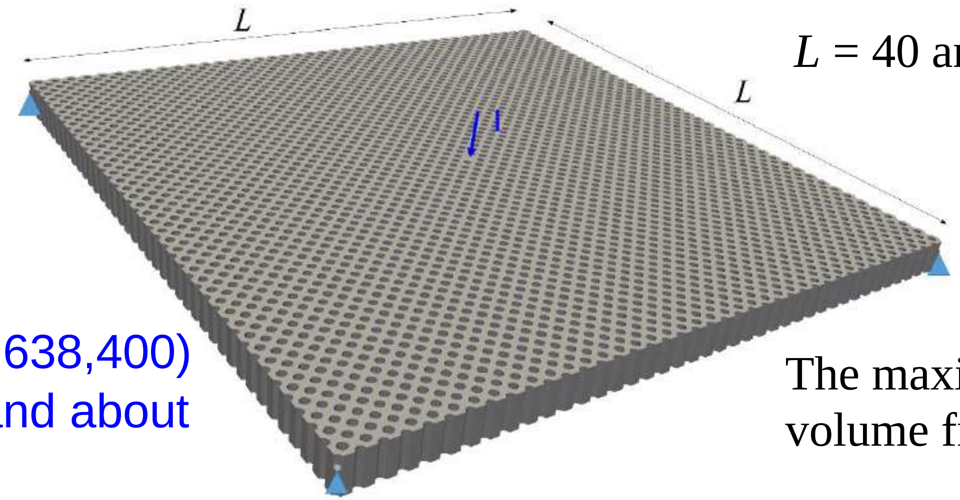
Fully parallel level set method

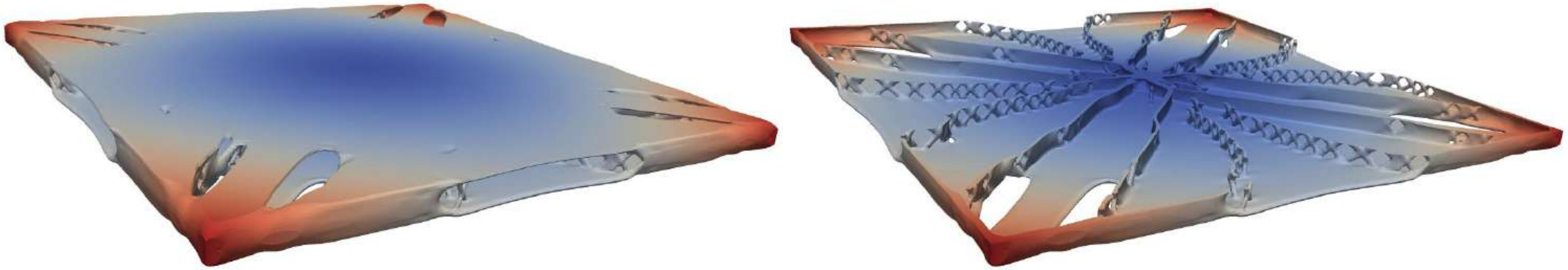


$L = 40$ and the thickness $T = 2$

Mesh: 320×320×16 (1,638,400)
HPC1 with 120 cores and about 1.5 hours

The maximum usable material volume fraction is set as 0.3.

Three-dimensional plate model and its boundary conditions



(a) (b)

Optimized result of the plate, colored by the out-plane displacement field:
(a) the whole structure; (b) the internal structure of the plate

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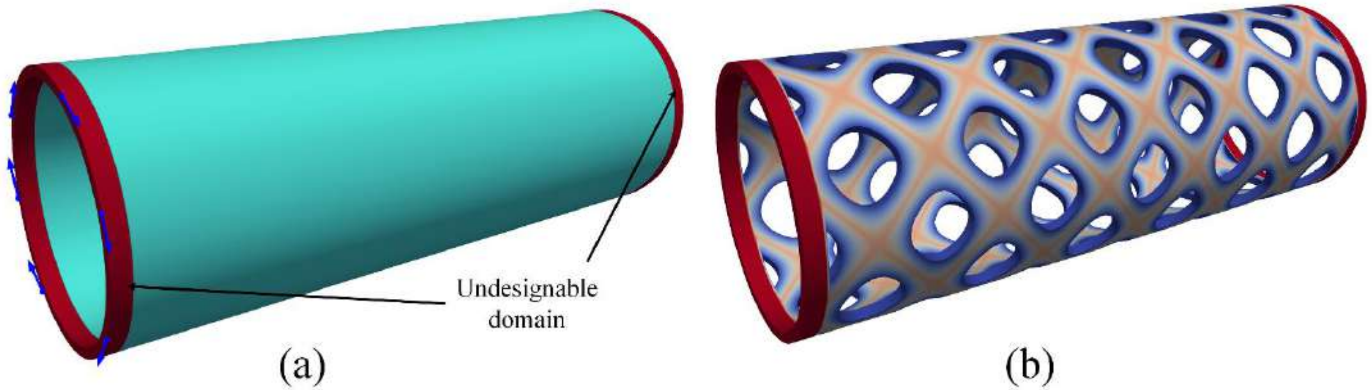
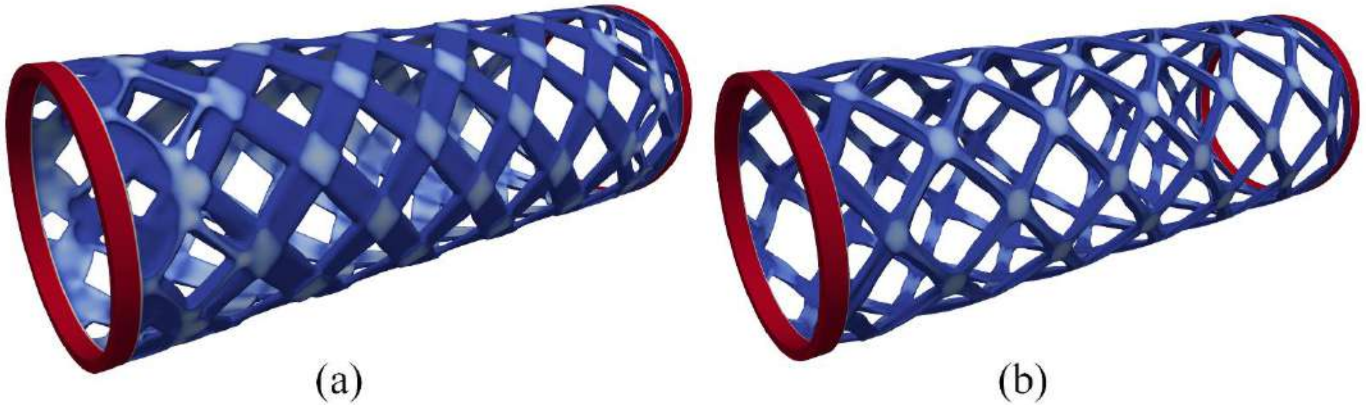
Fully parallel level set method



Design domain

Initial design

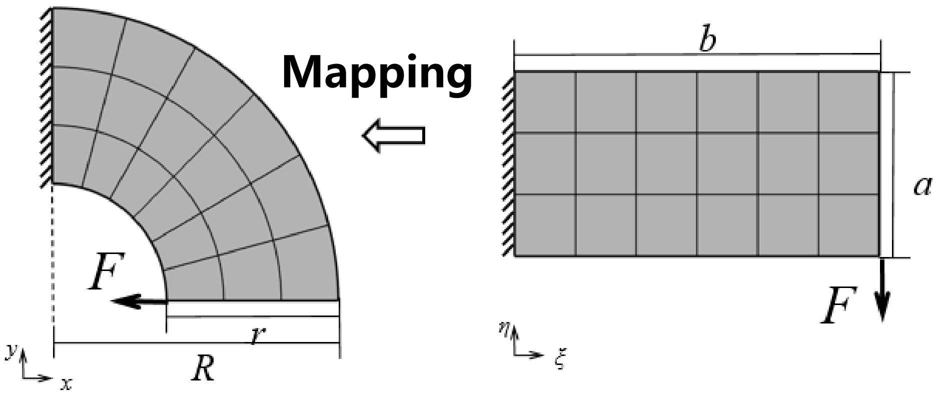


Volume fraction: vf<=0.5
Mesh: 288×288×4 (331,776)
about 4.3 hours on the WS

Volume fraction: vf<=0.3
Mesh: 576×576×8 (2,654,208)
HPC1 with 120 cores and about
2.5 hours

Curved cantilever beam

Problem definition



Definition of the optimization problem & the mapping strategy

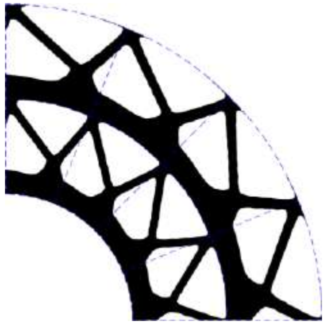
Optimization problem

$$\min \quad \sigma_{pn}$$
$$s.t. \quad V_{frac} \leq 0.4$$

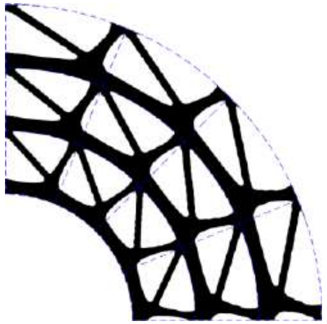
Parameters

- Elements: 288×144
- *h*: 0.02
- C0 continuity
- Cell-wise periodic structure

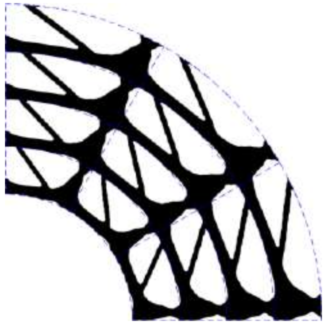
Optimized results with different cell partition schemes



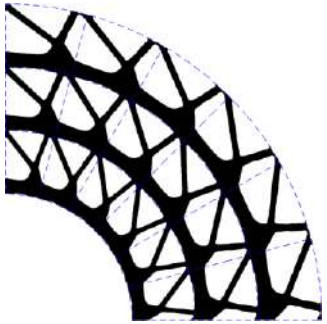
4×2 cells $\sigma_{pn} = 5.18$



4×3 cells $\sigma_{pn} = 5.78$



3×4 cells $\sigma_{pn} = 5.78$



6×3 cells $\sigma_{pn} = 5.85$

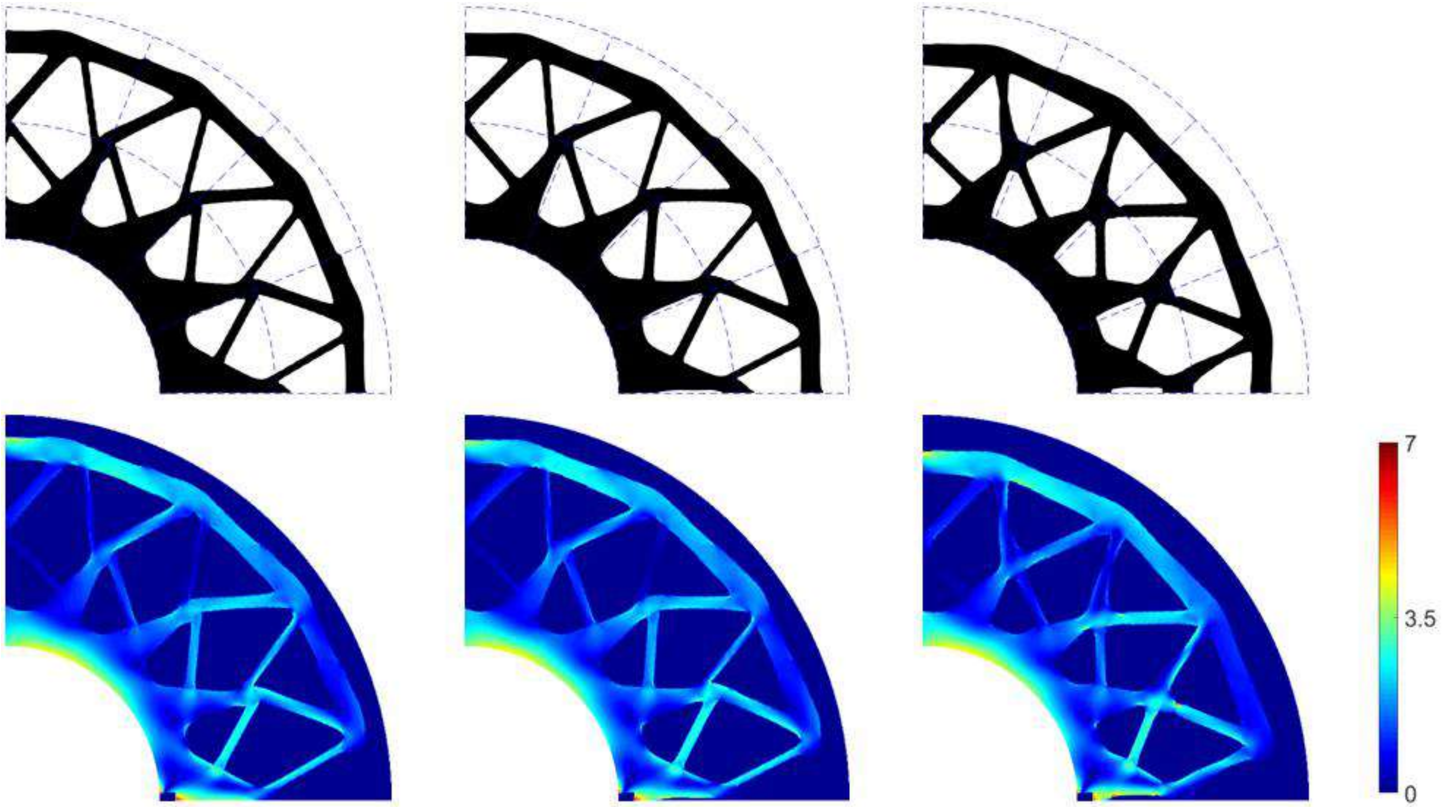
➢ **The stress value becomes higher as cell number increases.**

Continuity order

Optimized results with different continuity orders

Parameters

- Elements: 288×144
- Cells: 4×2
- *h*: 0.02
- Periodicity : radial



C0 $\sigma_{pn} = 5.18$ **C1** $\sigma_{pn} = 5.35$ **C2** $\sigma_{pn} = 5.47$

- Three designs share a similar topology
- A higher order of continuity condition results in a **smoother geometric connection**
- **The additional geometric requirement** of continuities restricts the design freedom at the cell connection regions

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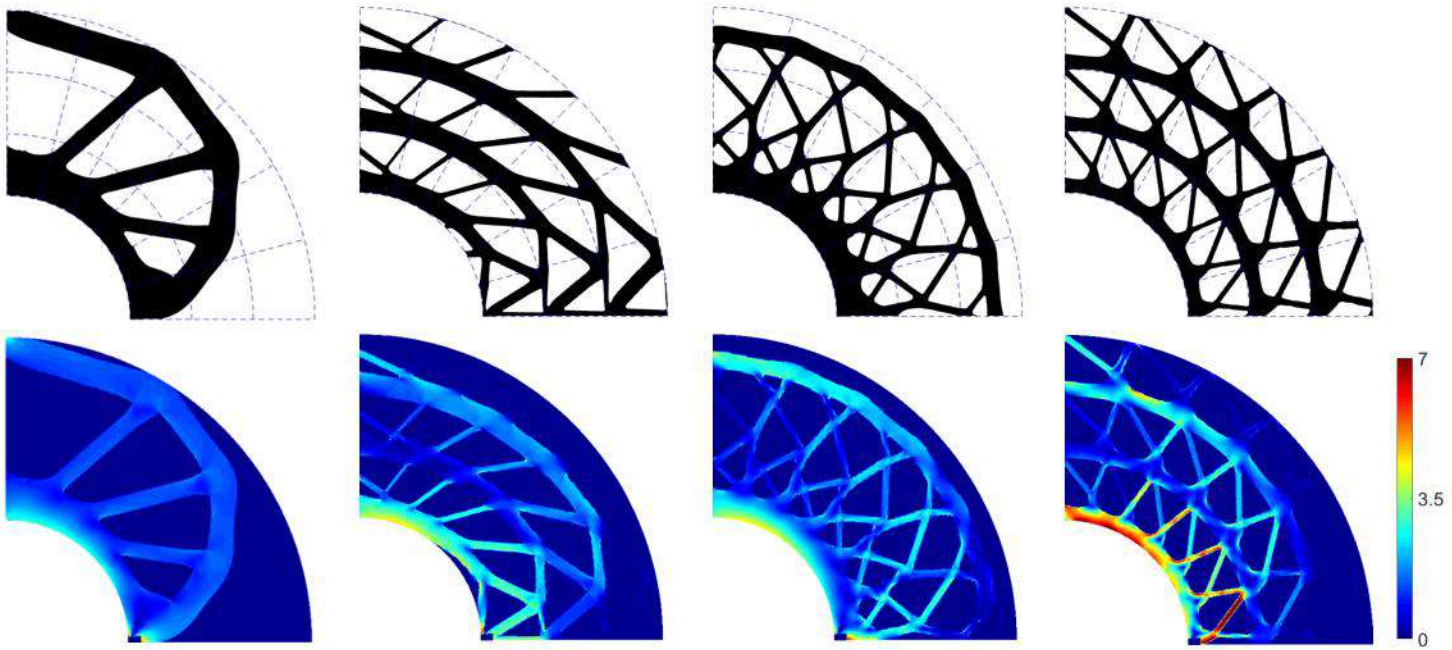
Cell periodicity

Optimized results with different continuity orders

Parameters

- Elements: 288×144
- Cells: 6×3
- *h*: 0.02
- Continuity : C0



Non-periodicity	Circumferential periodic structure	Radial periodic structure	Cell-wise periodic structure
$\sigma_{pn} = 4.24$	$\sigma_{pn} = 4.73$	$\sigma_{pn} = 4.54$	$\sigma_{pn} = 5.85$

- Since periodicity can be viewed as a geometric constraint, it is difficult to obtain a fully-stress design for the periodic cellular structure.
- It is in principle a very difficult task to propose a general strategy for cellular periodicity, especially for a structure with complex stress distribution

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Thank You!

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