Topology Optimization of Dynamic Acoustic-Mechanical Structures using the Ersatz Material Model

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Outline

- Research Motivation
- Topology Optimization Algorithm
- Numerical Examples
- Concluding Remarks

Research Motivation

- In real applications, there are many acoustic-mechanical structures and devices which have designated functionalities, such as sound insulation, vibration isolation, etc.

- Topology optimization of structures considering acoustic-mechanical coupling has been widely investigated.

G.H. Yoon, J.S. Jensen, O. Sigmund. Topology optimization of acoustic-structure interaction problems using a mixed finite element formulation. *International Journal for Numerical Methods in Engineering*. 70(9) (2007) 1049-1075.

L. Shu, Y.M. Wang, Z.D. Ma. Level set based topology optimization of vibrating structures for coupled acoustic-structural dynamics. *Computers and Structures*. 132 (2014) 34–42.

R. Picelli, W.M. Vicente, R. Pavanello, Y.M. Xie. Evolutionary topology optimization for natural frequency maximization problems considering acoustic–structure interaction. *Finite Elements in Analysis and Design*. 106(15) (2015) 56-64.

B.D. Cetin, S.B. Dilgen, N. Aage, J.S. Jensen. Topology optimization of acoustic mechanical interaction problems: a comparative review. *Structural and Multidisciplinary Optimization*. 60(2) (2019) 779-801.

Research Motivation

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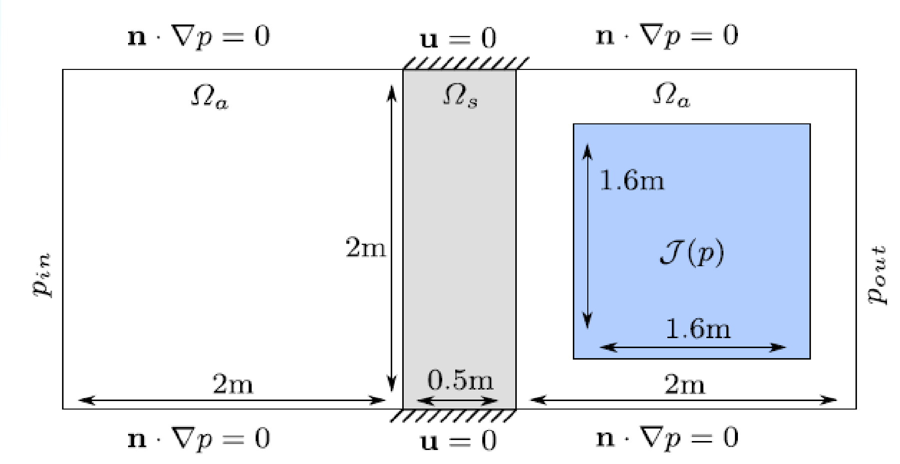
❑ Benchmark problem



$$\text{min: } J(p) = \int_{\Omega_{\mathrm{obj}}} |p| \, \mathrm{d}\Omega$$

$$V_f \leq V^*$$

- the mixed $\mathbf{u}/p$ formulation
- the segregated formulation

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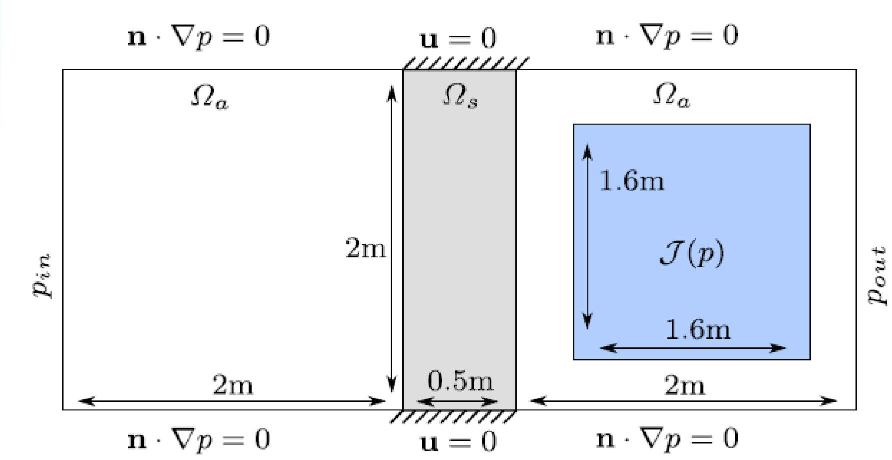
Research Motivation

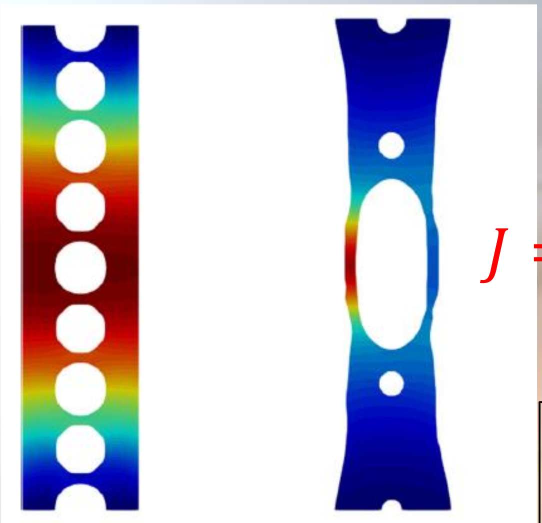
- Benchmark problem



$$\text{min: } J(p) = \int_{\Omega_{\text{obj}}} |p| \, \mathrm{d}\Omega$$

$$V_f \leq V^*$$

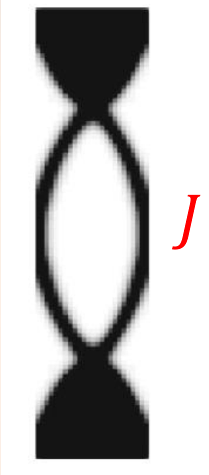
Boundary-based level-set method:



$J = 68.7\text{N}$

the mixed $\mathbf{u}/p$ formulation

Element-based density method (RAMP):



$J = 48.9\text{N}$

However, the thresholded density-based design using the segregated formulation gives a 28.6% reduction in objective value ($J = 34.9$ N).

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Ersatz Material Model

Built on the previous researches, this paper investigates element-based topology optimization of acoustic-mechanical structures using the Ersatz material model. The bulk modulus, shear modulus and density of each element are linearly interpolated as

$$\begin{cases} K(x_i) = K_s x_i + K_a(1 - x_i) \\ G(x_i) = G_s x_i \\ \rho(x_i) = \rho_s x_i + \rho_a(1 - x_i) \end{cases}$$

$x_i = 0$ means the element is composed of the acoustic medium (e.g. air) and $x_i = 1$ for the solid medium.

The potential advantages of using the linear Ersatz material model include

- Achieving the consistency of objective value in the mixed **u**/*p* formulation and the segregated formulation
- Overcoming artificial vibration modes during optimization process

Topology optimization problem

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$$\text{Min.}: \; J(\boldsymbol{x}, \mathbf{U}(\boldsymbol{x}))$$

$$\text{S.t.}: V_f \leq V_f^*$$

$$\begin{cases} x_i = 1 & \text{when } x_i \in \Omega_{\text{solid}} \\ x_i = 0 & \text{when } x_i \in \Omega_{\text{acoustic}} \\ 0 < x_i < 1 & \text{when } x_i \text{ at the acoustic} - \text{solid boundary} \end{cases}$$

→ 0/1 constraints of design variables

To solve the optimization problem using the relaxed design variables, $0 \leq x_i \leq 1$, the objective function is modified by introducing Lagrange multipliers for all constraints as

$$\text{Min.}: \; f = J(\boldsymbol{x}, \mathbf{U}(\boldsymbol{x})) + \boldsymbol{\psi}(V_f - V_f^*) + \sum \boldsymbol{\psi}_j (g_j - g_j^*)$$

the upper and lower bounds of design variables

the floating projection constraint, which simulates the original 0/1 constraints together with the upper and lower bounds of design variables.

Numerical implementation

$$x_{1,i} = (1-\Lambda)\left(1-\frac{\mathrm{d}\,J^k/\mathrm{d}\,x_i}{\partial\,V_f/\partial\,x_i}\right)x_i^{k-1}$$

→ optimality criterion

$$x_{2,i} = \begin{cases} 0 & if\ x_{1,i} \le 0 \\ x_{1,i} & \text{otherwise} \\ 1 & if\ x_{1,i} \ge 1 \end{cases}$$

→ the upper and lower bounds

$$x_{3,i} = \frac{\sum w(r_{ij})x_{2,j}}{\sum w(r_{ij})}$$

→ filter

$$x_i^k = \frac{\tanh(\beta\cdot th^k) + \tanh[\beta\cdot(x_{3,i} - th^k)]}{\tanh(\beta\cdot th^k) + \tanh[\beta\cdot(1 - th^k)]}$$

→ the floating projection constraint
(pushing x_i from inside toward 0 or 1)

$V_f \le V_f^*$?

No → Update Λ

Yes

β starts from a small positive value, e.g. 10^{-6} and then increases with $\Delta\beta$ (e.g., 1) once the solution is convergent.

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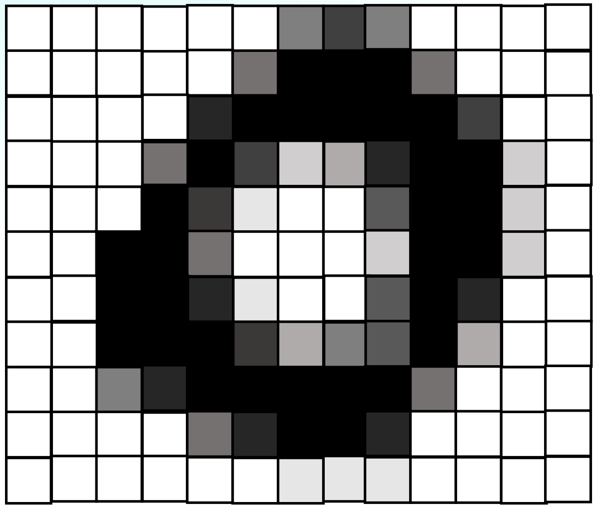
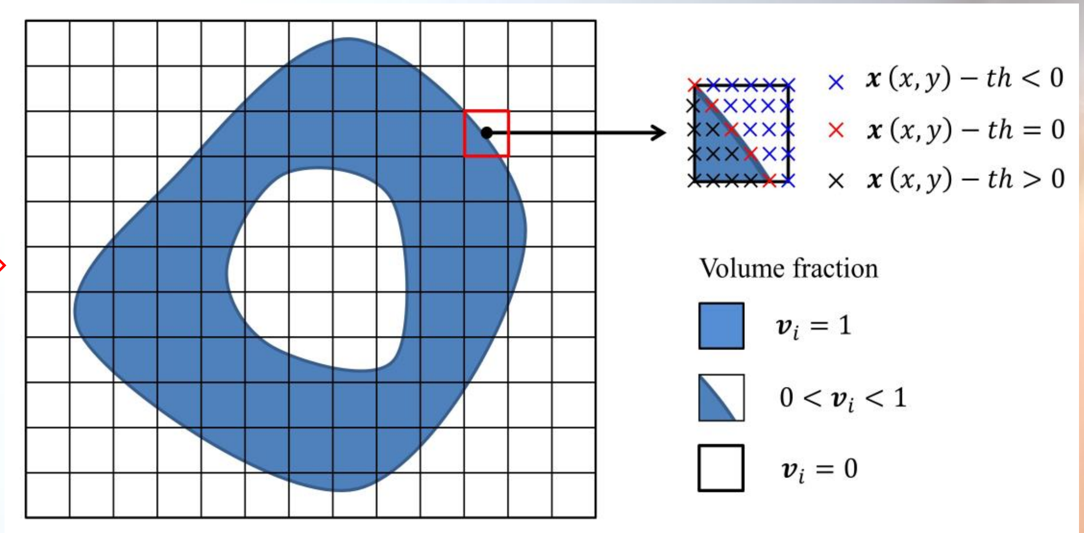
Representation of optimized topology for CAD-ready model

Convergent solution for a given β



Graphic processing



$$\boldsymbol{x}^k = \{x_1^k, x_2^k, \dots x_i^k, \dots x_N^k\}^T$$

$$\boldsymbol{v}^k = \{v_1^k, v_2^k, \dots, v_i^k, \dots v_N^k\}$$

$$\tau = \frac{1}{N}\sum_{i=1}^{N}\left(x_i^k - v_i^k\right)^2 \leq 0.001$$

and/or

$$\tau = \left|\frac{J(\boldsymbol{x}, \mathbf{U}(\boldsymbol{x})) - J(\boldsymbol{v}, \mathbf{U}(\boldsymbol{v}))}{J(\boldsymbol{x}, \mathbf{U}(\boldsymbol{x}))}\right| \leq 0.01$$

FEA on $\boldsymbol{v}^k$

$\beta = \beta + \Delta\beta$

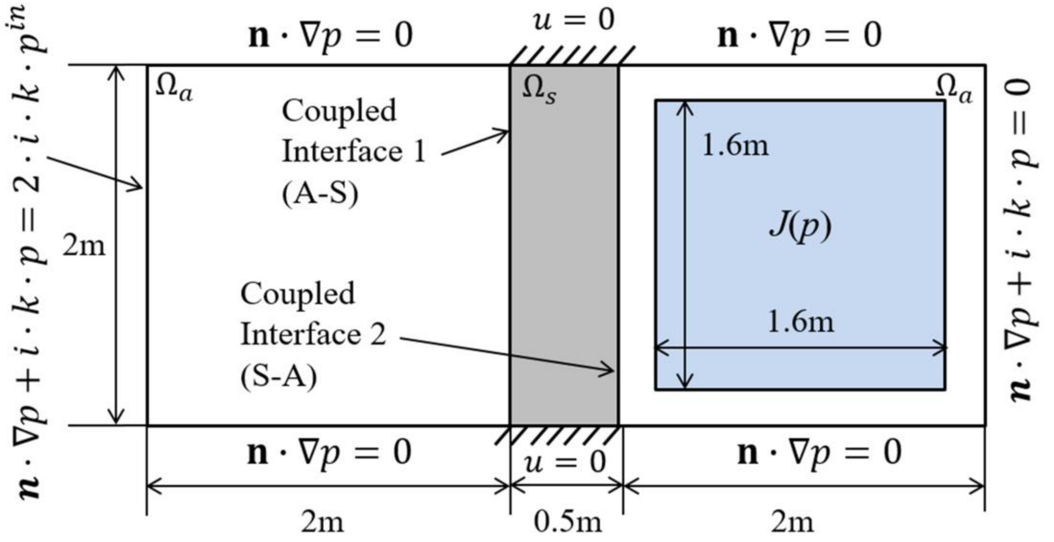
Stop optimization

Sound insulation

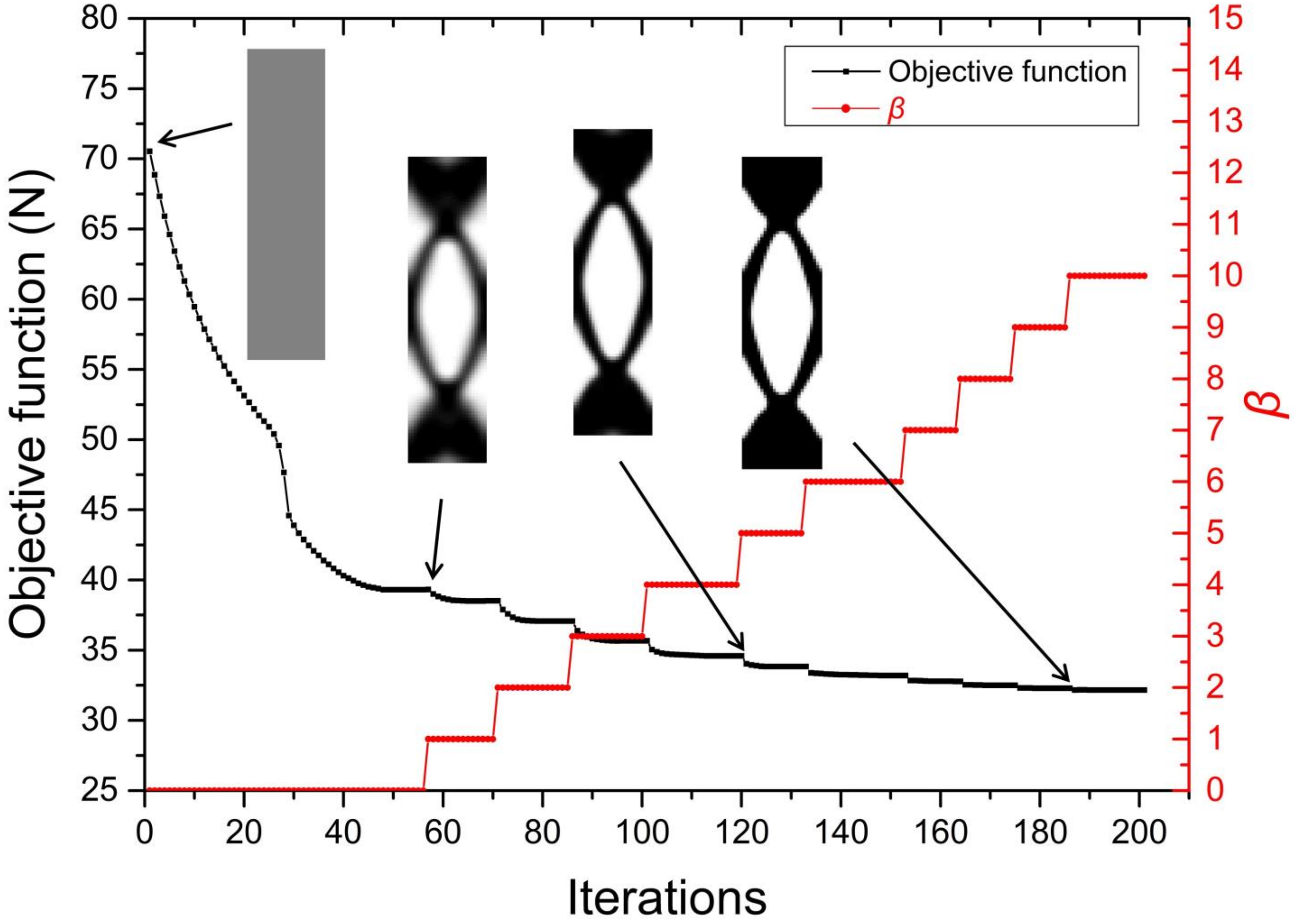
- Revisiting the benchmark problem

$$\text{min: } J(p) = \int_{\Omega_{\text{obj}}} |p| \, \mathrm{d}\Omega$$

$$\text{s.t.}: V_f \leq V^*$$

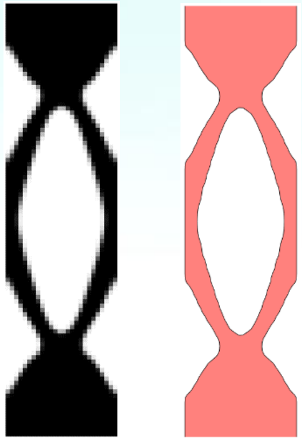


$f = 1/\pi$ Hz



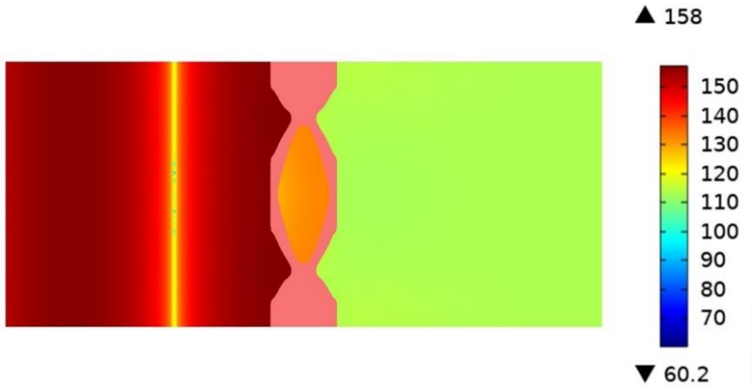
Sound insulation

❑ Revisiting the benchmark problem



J = 32.16 N
(**u**/*p*)

Current design



J = 32.43 N
(segregated)

Only 0.84% relative difference

J = 48.9N
(**u**/*p*)

J = 34.9 N
(segregated)

Density-based design*

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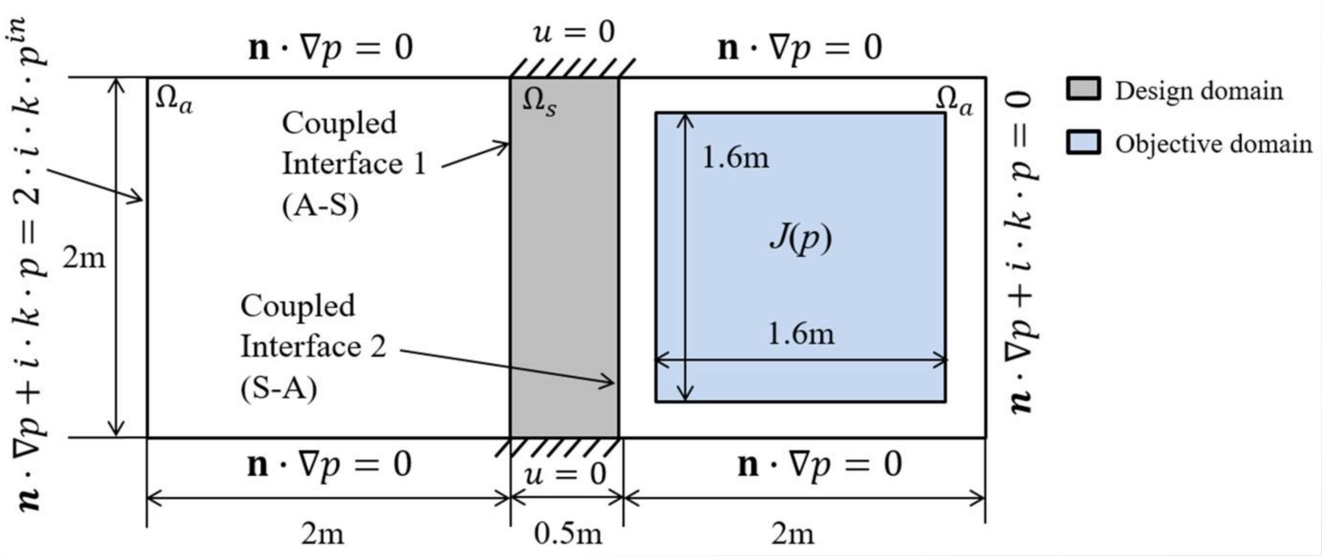
Sound insulation

- Lightweight design

$$\min.: V_f$$

$$\text{s.t.}: J(\boldsymbol{x}, \mathbf{U}(\boldsymbol{x})) \le J^* = 40\text{N}$$



$$f = 1/\pi \text{ Hz}$$

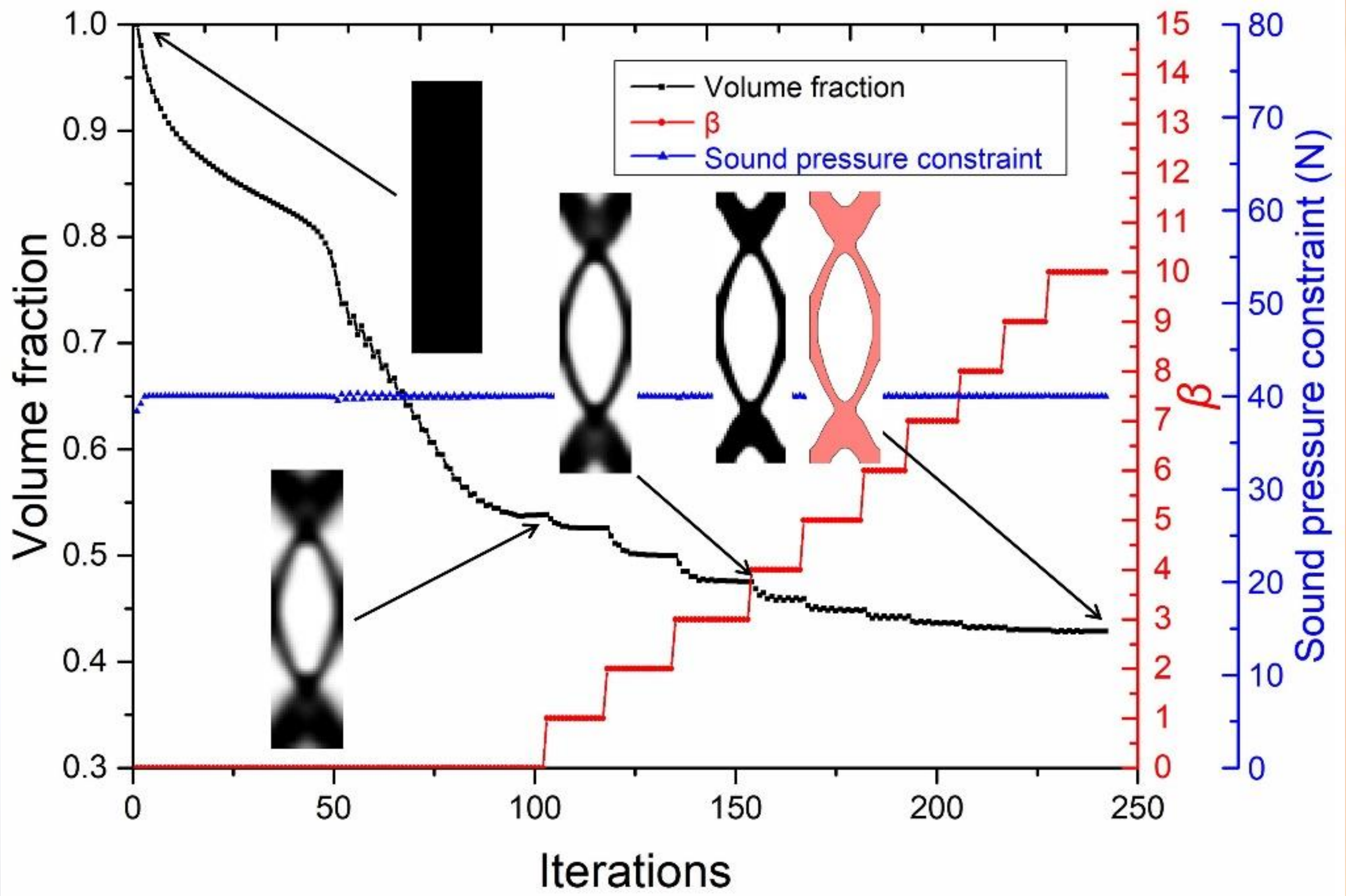
Sound insulation

❑ Lightweight design

$\min.: V_f$

$\text{s.t.}: J(\boldsymbol{x}, \mathbf{U}(\boldsymbol{x})) \leq J^* = 40\text{N}$

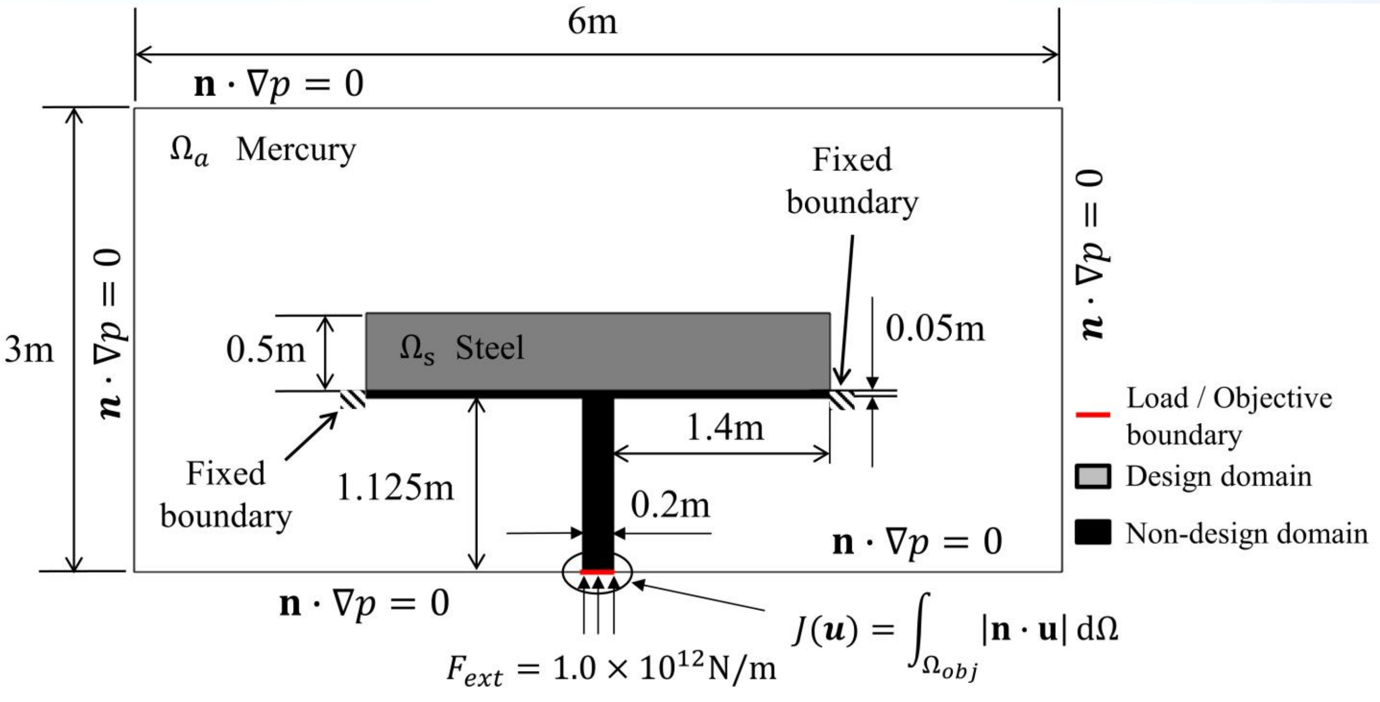
$J = 39.71\text{N}$
(segregated)



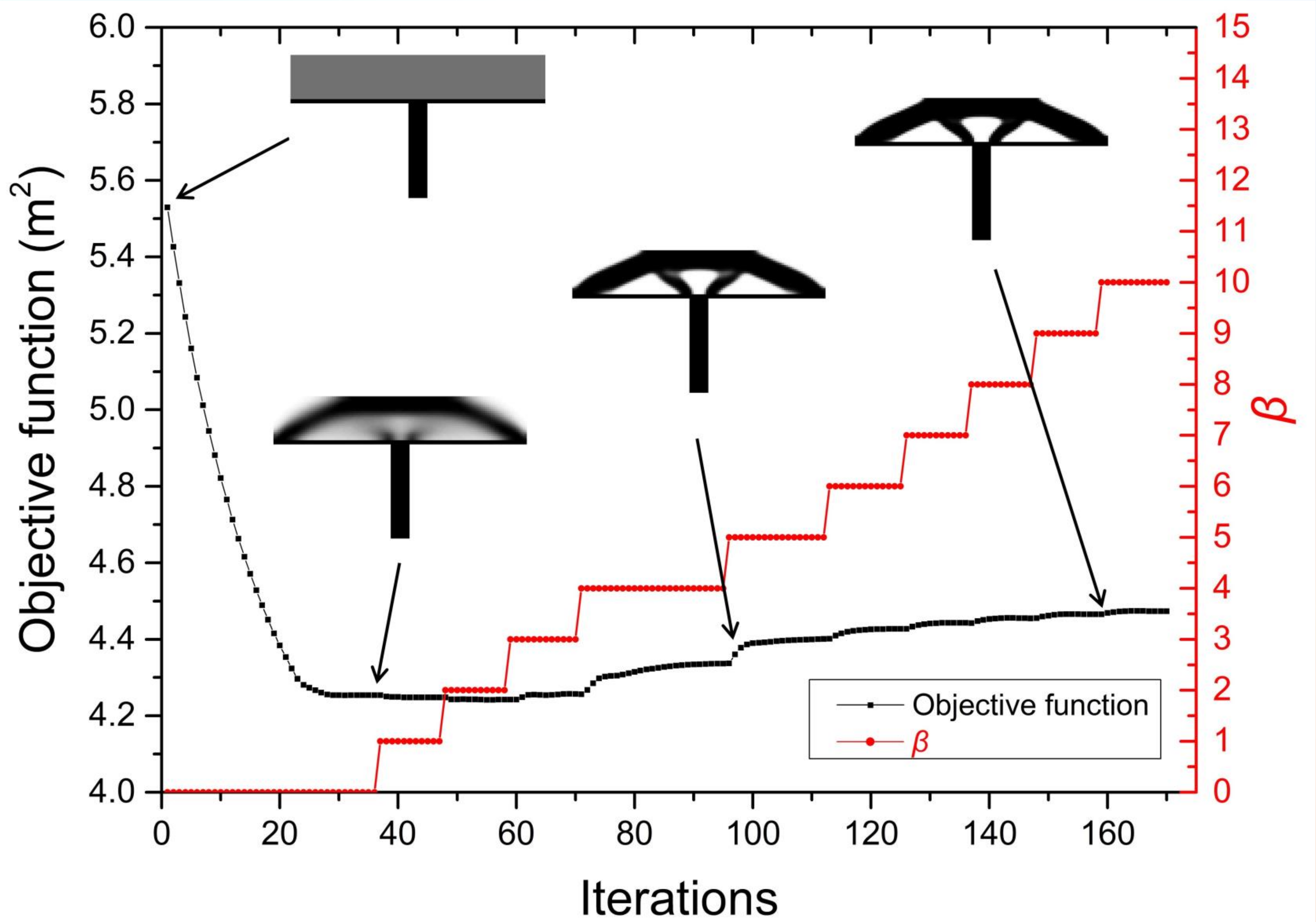
$V_{f\min} = 0.428$

Vibration reduction

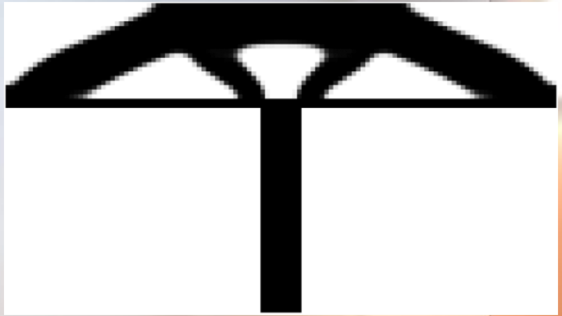
$$\text{min: } J(\boldsymbol{u}) = \int_{\Omega_{obj}} |\mathbf{n} \cdot \mathbf{u}| \, \mathrm{d}\Omega \qquad \text{(vibration amplitude)}$$



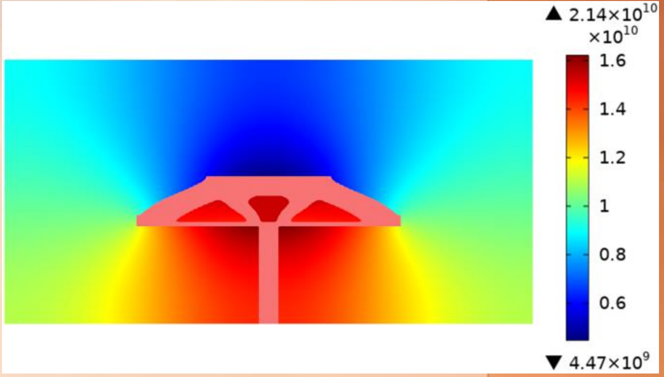
Vibration reduction



f =20 Hz



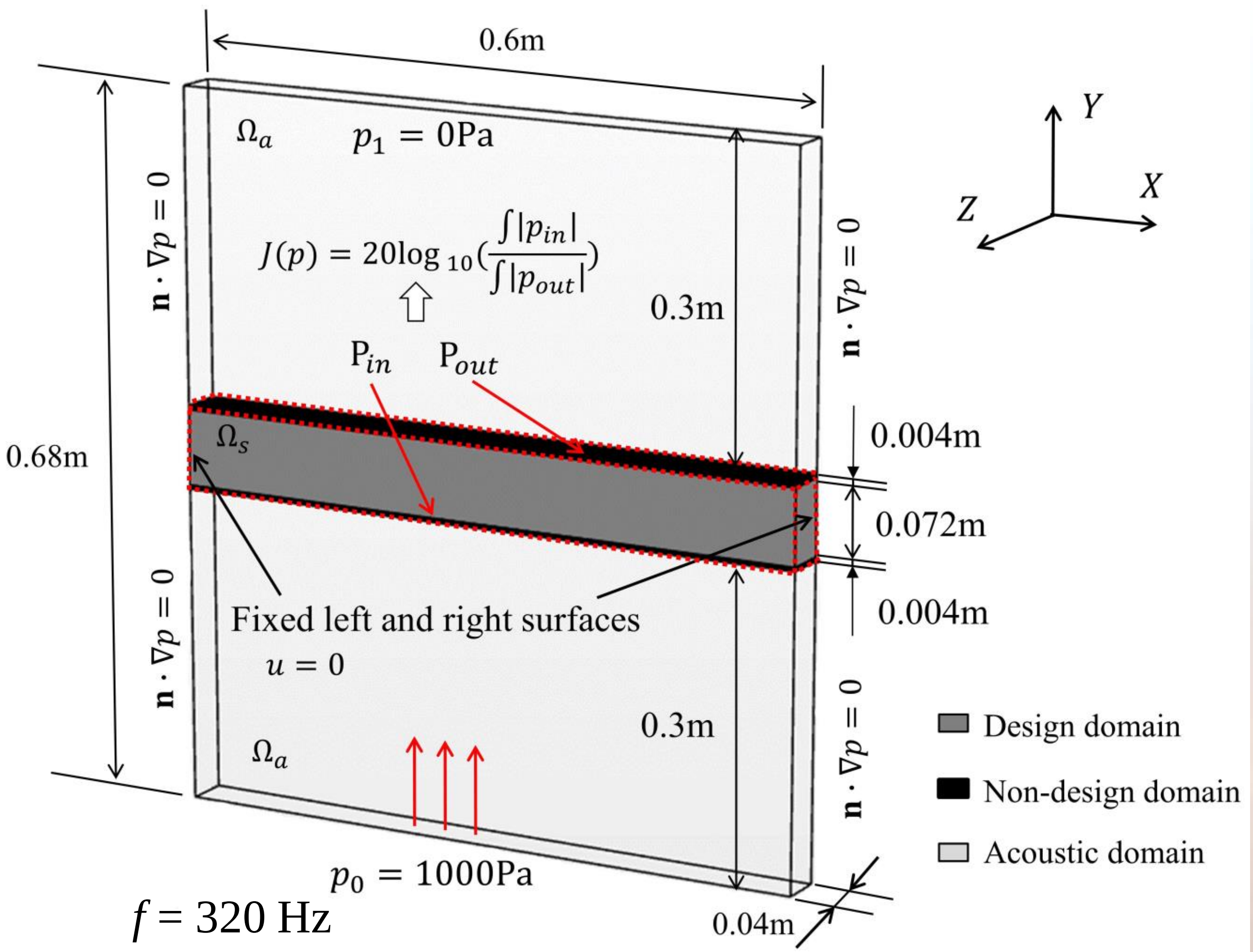
J = 4.47 m^2
(**u**/p)



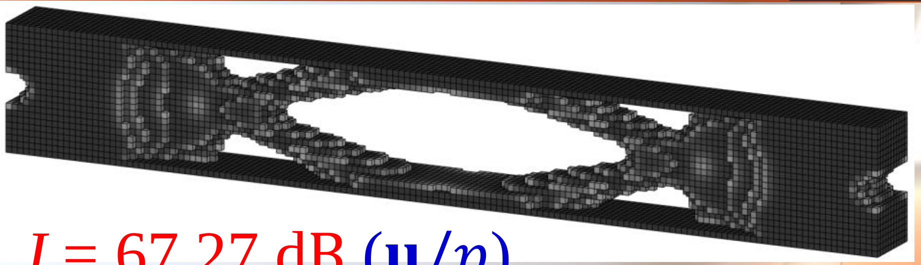
J = 4.51 m^2
(segregated)

Maximizing sound transmission loss

$$\min: \ -J(p) = -20\log_{10}\left(\frac{|P_{in}|}{|P_{out}|}\right)$$

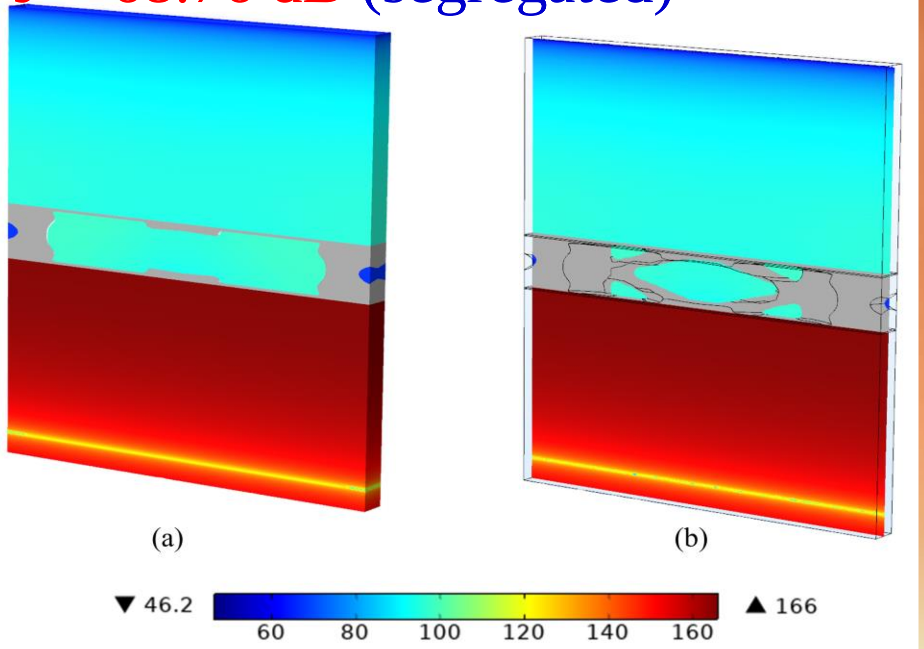


f = 320 Hz



J = 67.27 dB (**u**/p)

J = 68.76 dB (segregated)



Concluding remarks

- This paper proposes a new topology algorithm for designing acoustic-mechanical structures using the linear ersatz material model.

- The advantages of the proposed algorithm include

  - the consistency between the the mixed **u**/*p* formulation and the segregate formulation
  - no artificial vibration modes during optimization
  - integration of the post-processing of element-based optimized design in optimization

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Abstract

Topology optimization of dynamic acoustic–mechanical structures is challenging due to the interaction between the acoustic and structural domains and artificial localized vibration modes of structures. This paper presents a floating projection topology optimization (FPTO) method based on the mixed displacement/pressure (**u**/p) finite element formulation and the ersatz material model. The former is able to release the need for tracking the interface boundaries explicitly between the structural and acoustic

Thank you