Topology Optimization of Dynamic Acoustic-Mechanical Structures using the Ersatz Material Model

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Outline

- Research Motivation
- Topology Optimization Algorithm
- Numerical Examples
- Concluding Remarks
Research Motivation

- In real applications, there are many acoustic-mechanical structures and devices which have designated functionalities, such as sound insulation, vibration isolation, etc.

- Topology optimization of structures considering acoustic-mechanical coupling has been widely investigated.


Research Motivation

- Benchmark problem

\[ \min J(p) = \int_{\Omega_{\text{obj}}} |p| \, d\Omega \]

\[ V_f \leq V^* \]

- the mixed \( u/p \) formulation
- the segregated formulation

Research Motivation

Benchmark problem

\[
\min: J(p) = \int_{\Omega_{\text{obj}}} |p| \, d\Omega \\
V_f \leq V^*
\]

However, the thresholded density-based design using the segregated formulation gives a 28.6% reduction in objective value \((J = 34.9 \, \text{N})\).


Boundary-based level-set method:

\[ J = 68.7\,\text{N} \]

Element-based density method (RAMP):

\[ J = 48.9\,\text{N} \]

the mixed \(u/p\) formulation
Ersatz Material Model

Built on the previous researches, this paper investigates element-based topology optimization of acoustic-mechanical structures using the Ersatz material model. The bulk modulus, shear modulus and density of each element are linearly interpolated as

\[
\begin{aligned}
K(x_i) &= K_s x_i + K_a (1 - x_i) \\
G(x_i) &= G_s x_i \\
\rho(x_i) &= \rho_s x_i + \rho_a (1 - x_i)
\end{aligned}
\]

\(x_i = 0\) means the element is composed of the acoustic medium (e.g. air) and \(x_i = 1\) for the solid medium.

The potential advantages of using the linear Ersatz material model include

- Achieving the consistency of objective value in the mixed \(u/p\) formulation and the segregated formulation
- Overcoming artificial vibration modes during optimization process
Topology optimization problem

Min.: $J(x, U(x))$

S.t.: $V_f \leq V_f^*$

\[
\begin{cases}
  x_i = 1 & \text{when } x_i \in \Omega_{\text{solid}} \\
  x_i = 0 & \text{when } x_i \in \Omega_{\text{acoustic}} \\
  0 < x_i < 1 & \text{when } x_i \text{ at the acoustic} - \text{solid boundary}
\end{cases}
\]

To solve the optimization problem using the relaxed design variables, $0 \leq x_i \leq 1$, the objective function is modified by introducing Lagrange multipliers for all constraints as

\[
\text{Min.: } f = J(x, U(x)) + \Psi(V_f - V_f^*) + \sum \psi_j (g_j - g_j^*)
\]

the upper and lower bounds of design variables

the floating projection constraint, which simulates the original 0/1 constraints together with the upper and lower bounds of design variables.
Numerical implementation

\[ x_{1,i} = (1 - \Lambda)(1 - \frac{d J^k}{d x_i}(\frac{\partial V_f}{\partial x_i})x_i^{k-1}) \]

\[ x_{2,i} = \begin{cases} 
0 & \text{if } x_{1,i} \leq 0 \\
x_{1,i} & \text{otherwise} \\
1 & \text{if } x_{1,i} \geq 1 
\end{cases} \]

\[ x_{3,i} = \frac{\sum w(r_{ij})x_{2,j}}{\sum w(r_{ij})} \]

\[ x_i^k = \frac{\tanh(\beta \cdot th^k) + \tanh(\beta \cdot (x_{3,i} - th^k))}{\tanh(\beta \cdot th^k) + \tanh(\beta \cdot (1 - th^k))} x_i^k \]

Update \( \Lambda \)

No

\[ V_f \leq V_f^* ? \]

Yes

→ optimality criterion

→ the upper and lower bounds

→ filter

→ the floating projection constraint (pushing \( x_i \) from inside toward 0 or 1)

\( \beta \) starts from a small positive value, e.g. \( 10^{-6} \) and then increases with \( \Delta \beta \) (e.g., 1) once the solution is convergent.
Representation of optimized topology for CAD-ready model

Convergent solution for a given $\beta$

$\mathbf{x}^k = \{x_1^k, x_2^k, \ldots, x_i^k, \ldots, x_N^k\}^T$

$\mathbf{v}^k = \{v_1^k, v_2^k, \ldots, v_i^k, \ldots, v_N^k\}$

$$\tau = \frac{1}{N} \sum_{i=1}^{N} (x_i^k - v_i^k)^2 \leq 0.001$$

and/or

$$\tau = \left| \frac{J(x, U(x)) - J(v, U(v))}{J(x, U(x))} \right| \leq 0.01$$

$\beta = \beta + \Delta \beta$

Stop optimization

Graphic processing

Volume fraction

- $v_i = 1$
- $0 < v_i < 1$
- $v_i = 0$

FEA on $\mathbf{v}^k$
Sound insulation

- Revisiting the benchmark problem

\[
\min J(p) = \int_{\Omega_{\text{obj}}} |p| \, d\Omega
\]

s.t.: \(V_f \leq V^*\)

\[ f = \frac{1}{\pi} \text{Hz} \]
Sound insulation

- Revisiting the benchmark problem

\[ J = 32.16 \text{ N} \]  
\[ \left( \frac{u}{p} \right) \]

Current design

\[ J = 32.43 \text{ N} \]  
\[ \left( \text{segregated} \right) \]

Only 0.84% relative difference

\[ J = 48.9 \text{ N} \]  
\[ \left( \frac{u}{p} \right) \]

Density-based design*

\[ J = 34.9 \text{ N} \]  
\[ \left( \text{segregated} \right) \]

Sound insulation

- Lightweight design

\[
\text{min. } V_f \\
\text{s. t.: } J(x, U(x)) \leq J^* = 40N
\]
Sound insulation

- Lightweight design
  - min. \( V_f \)
  - s. t. \( J(\mathbf{x}, \mathbf{U}(\mathbf{x})) \leq J^* = 40 \text{N} \)

\[ J = 39.71 \text{N} \]
(segregated)

\[ V_{f_{\text{min}}} = 0.428 \]
Vibration reduction

\[
\min J(\mathbf{u}) = \int_{\Omega_{obj}} |\mathbf{n} \cdot \mathbf{u}| \, d\Omega \quad \text{(vibration amplitude)}
\]
Vibration reduction

\[ J = 4.47 \text{ m}^2 \]  
(\( \frac{u}{p} \))

\[ J = 4.51 \text{ m}^2 \]  
(segregated)

\( f = 20 \text{ Hz} \)
Maximizing sound transmission loss

\[
\min: -J(p) = -20\log_{10}\left(\frac{|P_{\text{in}}|}{|P_{\text{out}}|}\right)
\]

\[J = 67.27 \, \text{dB} \ (u/p)\]

\[J = 68.76 \, \text{dB} \ \text{(segregated)}\]
Concluding remarks

- This paper proposes a new topology algorithm for designing acoustic-mechanical structures using the linear ersatz material model.

- The advantages of the proposed algorithm include
  - the consistency between the mixed $u/p$ formulation and the segregate formulation
  - no artificial vibration modes during optimization
  - integration of the post-processing of element-based optimized design in optimization
Topology optimization of dynamic acoustic–mechanical structures using the ersatz material model

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Abstract

Topology optimization of dynamic acoustic–mechanical structures is challenging due to the interaction between the acoustic and structural domains and artificial localized vibration modes of structures. This paper presents a floating projection topology optimization (FPTO) method based on the mixed displacement/pressure (u/p) finite element formulation and the ersatz material model. The former is able to release the need for tracking the interface boundaries explicitly between the structural and acoustic

Thank you