Thermomechanical Topology Optimization of Shape-Memory Alloy Structures Using a Transient Bilevel Adjoint Method

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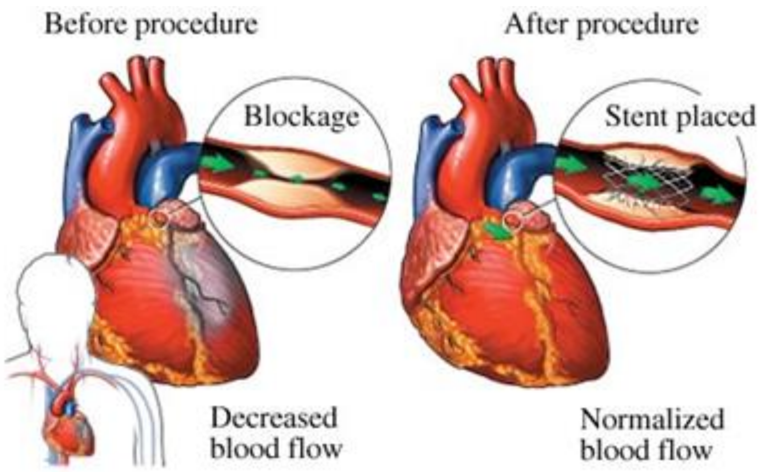
Shape-Memory Alloys: An Overview

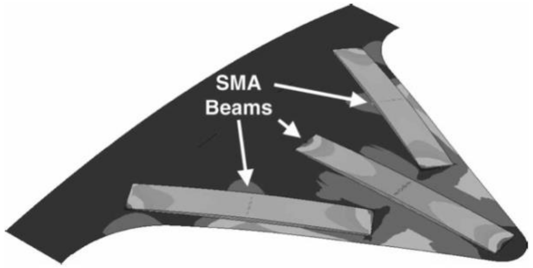
- Shape change in response to thermal loads
- Continuous "fluid" motion with infinite degrees of freedom
- Bio-inspired design
- Programmable motion



ANSYS 2015


The BioRobotics Institute/Scuola Superiore Sant'Anna 2019

NASA 2018



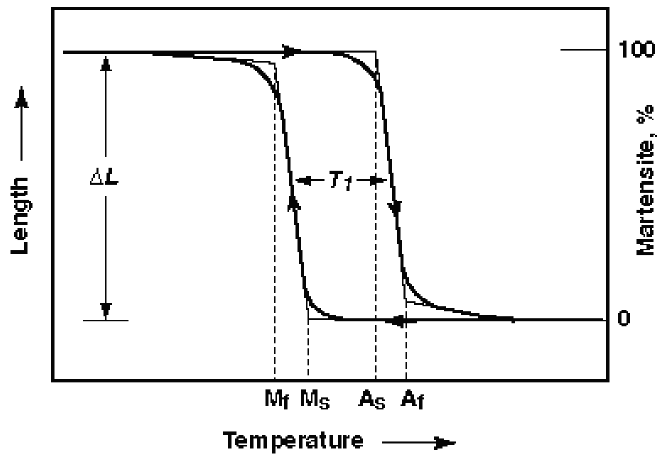
Boeing 2008

❖ Key contributions: - Novel topology optimization formulation
- Bi-level adjoint sensitivity formulation

Phase Transformation

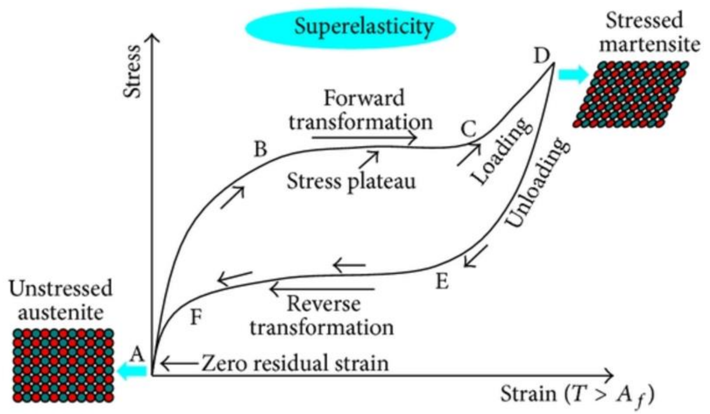
- Inelastic stress and strain response due to phase transformation
- Triggered by latent heat exchange
- Transformation between Martensite and Austenite phases

Two-way shape-memory effect



Lagoudas 2008

Superelasticity



Seo 2015

Lagoudas, DC. *Shape Memory Alloys: Modeling and Engineering Applications*. Berlin, Germany: Springer; 2008.
Seo, Junwon, Young Chan Kim, and Jong Wan Hu. "Pilot study for investigating the cyclic behavior of slit damper systems with recentering shape memory alloy (SMA) bending bars used for seismic restrainers." Applied Sciences 5.3 (2015): 187-208.

Constitutive Model

- Additive decomposition of small strains

$$\varepsilon = \varepsilon^{e}(\sigma) + \varepsilon^{t}(\sigma, T, \xi) + \varepsilon^{th}(T)$$

$$\boldsymbol{S} = \boldsymbol{S}^A + \xi(\boldsymbol{S}^M - \boldsymbol{S}^A) = \boldsymbol{S}^A + \xi\Delta\boldsymbol{S}$$

$$\alpha = \alpha^A = \alpha^M$$

$$c = c^A = c^M$$

$$\dot{\boldsymbol{\varepsilon}}^t = \boldsymbol{\Lambda}\dot{\xi}$$

$$\boldsymbol{\Lambda} = \begin{cases} \frac{3}{2} H \frac{\boldsymbol{\sigma}_s}{\sigma_s^{eff}} & \dot{\xi} > 0 \\ H \frac{\boldsymbol{\varepsilon}_{t-r}}{\varepsilon_{t-r}^{eff}} & \dot{\xi} < 0 \end{cases}$$

Transformation tensor

$$\mathrm{d}\boldsymbol{\sigma} = \boldsymbol{\mathfrak{L}} : \mathrm{d}\boldsymbol{\varepsilon}$$

$$\boldsymbol{\mathfrak{L}} = \begin{cases} \boldsymbol{S}^{-1} - \frac{\boldsymbol{S}^{-1}:\partial_{\boldsymbol{\sigma}}\Phi \otimes \boldsymbol{S}^{-1}:\partial_{\boldsymbol{\sigma}}\Phi}{\partial_{\boldsymbol{\sigma}}\Phi:\boldsymbol{S}^{-1}:\partial_{\boldsymbol{\sigma}}\Phi - \partial_{\xi}\Phi} & \dot{\xi} > 0 \\ \boldsymbol{S}^{-1} - \frac{\boldsymbol{S}^{-1}:\partial_{\boldsymbol{\sigma}}\Phi \otimes \boldsymbol{S}^{-1}:\partial_{\boldsymbol{\sigma}}\Phi}{\partial_{\boldsymbol{\sigma}}\Phi:\boldsymbol{S}^{-1}:\partial_{\boldsymbol{\sigma}}\Phi + \partial_{\xi}\Phi} & \dot{\xi} < 0 \end{cases}$$

Finite Element Implementation

Global level :

$$\boldsymbol{R}_{n+1} = \bigwedge_{\mathrm{el}}\Big(\sum_{\mathrm{G^d}}(w\boldsymbol{B}^{\mathrm{T}}_{\mathrm{G^d}}\boldsymbol{\sigma}_{\mathrm{G^d},\mathrm{n+1}}) - \sum_{\mathrm{G^f}}(\varpi\boldsymbol{N}_{\mathrm{G^f}}\boldsymbol{p}_{\mathrm{G^f},\mathrm{n+1}})\Big) = 0$$

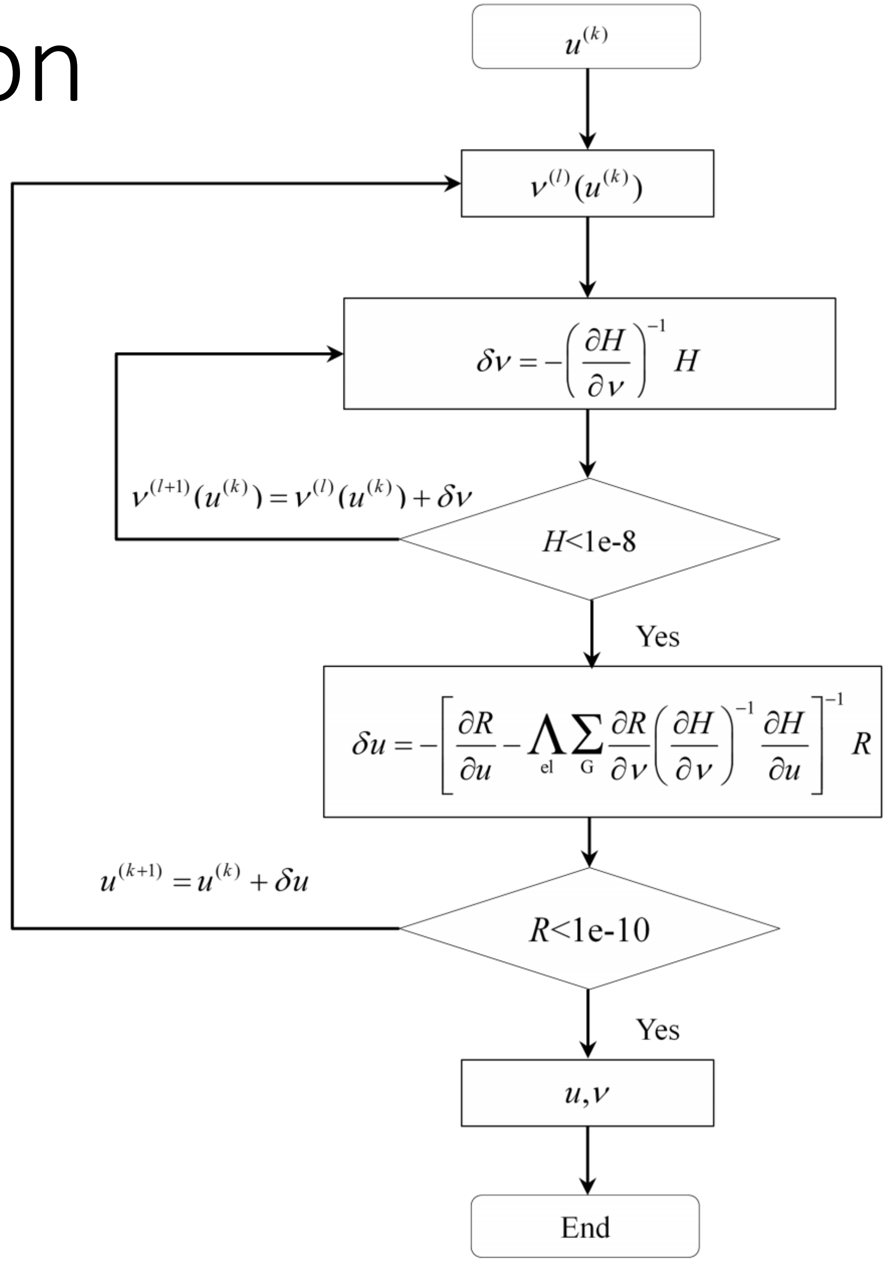
Local level :

$\dot{\xi} \neq 0$:

$$\boldsymbol{H}_{n+1} = \begin{cases} H_{\xi} = \Phi(\boldsymbol{\sigma}_{n+1}, T_{n+1}, \xi_{n+1}) \\ \boldsymbol{H}_{\boldsymbol{\varepsilon}^t} = \boldsymbol{\varepsilon}^t_n + \boldsymbol{\Lambda}(\xi_{n+1} - \xi_n) - \boldsymbol{\varepsilon}^t_{n+1} \\ H_S = S_n + \Delta S(\xi_{n+1} - \xi_n) - S_{n+1} \\ \boldsymbol{H}_{\boldsymbol{\sigma}} = \boldsymbol{\sigma}_n + \boldsymbol{S}^{-1}_{n+1} : [\boldsymbol{B}\boldsymbol{d}_{n+1} - \boldsymbol{\alpha}(T_{n+1} - T_0) - \boldsymbol{\varepsilon}^t_{n+1}] \\ \qquad - \boldsymbol{S}^{-1}_n : [\boldsymbol{B}\boldsymbol{d}_n - \boldsymbol{\alpha}(T_n - T_0) - \boldsymbol{\varepsilon}^t_n] - \boldsymbol{\sigma}_{n+1} \end{cases}$$

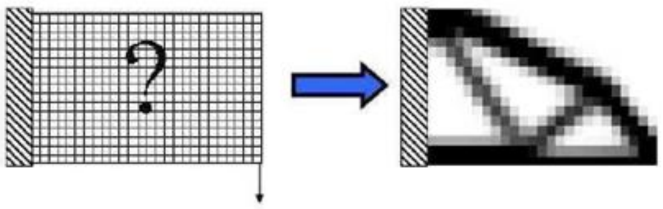
$\dot{\xi} = 0$:

$$\begin{aligned} \boldsymbol{H}_{n+1} = \boldsymbol{\sigma}_n + \boldsymbol{S}^{-1} &: [(\boldsymbol{B}\boldsymbol{d}_{n+1} - \boldsymbol{\alpha}(T_{n+1} - T_0) - \boldsymbol{\varepsilon}^t) \\ &- (\boldsymbol{B}\boldsymbol{d}_n - \boldsymbol{\alpha}(T_n - T_0) - \boldsymbol{\varepsilon}^t)] - \boldsymbol{\sigma}_{n+1} = 0 \end{aligned}$$



Optimization Problem Formulation

Two-phase topology optimization:



Two-way shape-memory effect (geometric advantage):

$$f_{obj} = \boldsymbol{L}^{\mathrm{T}} \boldsymbol{d}$$

Superelasticity (mechanical advantage):

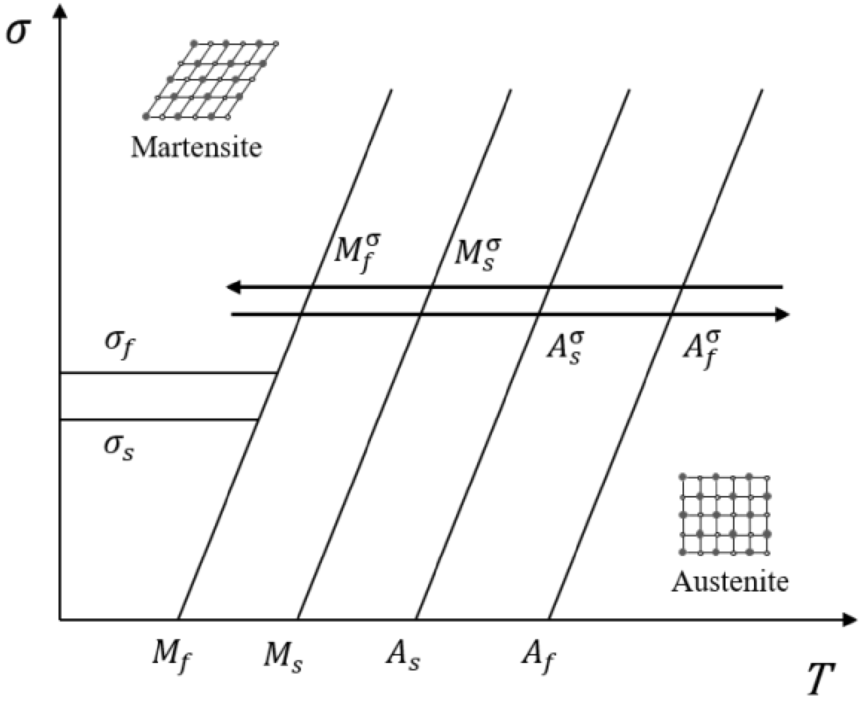
$$f_{obj} = \frac{\boldsymbol{F}_{out}}{|\boldsymbol{F}_{in}|} = \frac{\boldsymbol{L}_{out}^{\mathrm{T}} \boldsymbol{F}}{|\boldsymbol{F}_{in}|}$$

Material interpolation scheme:

$$E^{\mathrm{eff}} = r^p E^*$$

$$\mathrm{d}T^{\mathrm{eff}} = r^p \mathrm{d}T^*$$

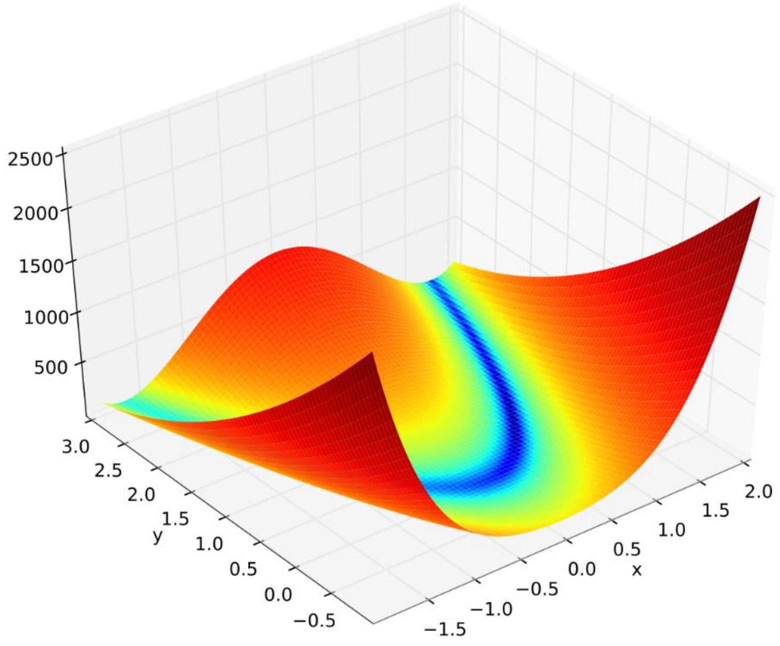
$$V^{\mathrm{eff}} = r V^*$$



Adjoint Sensitivity Analysis

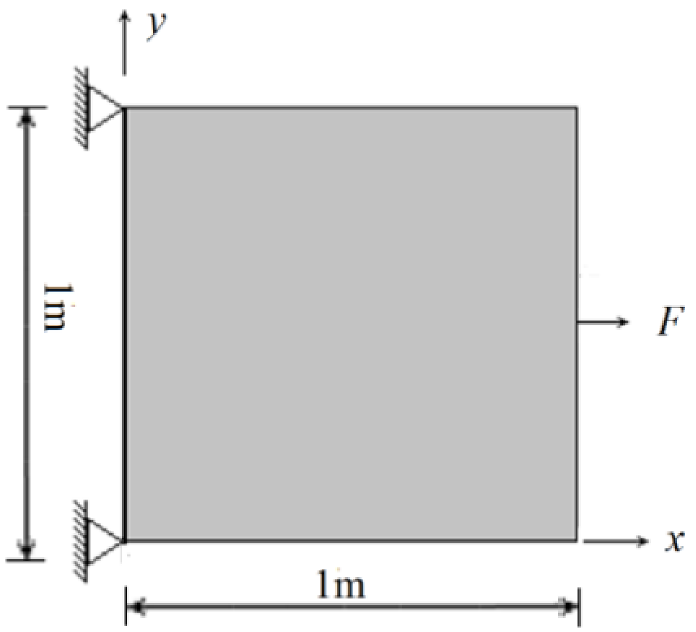
$$n = N_t : \begin{cases} \boldsymbol{\lambda}_{N_t} = \left[\dfrac{\partial \boldsymbol{R}_{N_t}}{\partial \boldsymbol{u}_{N_t}} - \bigwedge_{\mathrm{el}} \sum_{\mathrm{G}} \dfrac{\partial \boldsymbol{R}_{N_t}}{\partial \boldsymbol{\nu}_{\mathrm{G},N_t}} \left(\dfrac{\partial \boldsymbol{H}_{\mathrm{G},N_t}}{\partial \boldsymbol{\nu}_{\mathrm{G},N_t}} \right)^{-1} \dfrac{\partial \boldsymbol{H}_{\mathrm{G},N_t}}{\partial \boldsymbol{u}_{N_t}} \right]^{-\mathrm{T}} \\ \qquad \cdot \left[\bigwedge_{\mathrm{el}} \sum_{\mathrm{G}} \dfrac{\partial f_{int}}{\partial \boldsymbol{\nu}_{\mathrm{G},N_t}} \left(\dfrac{\partial \boldsymbol{H}_{\mathrm{G},N_t}}{\partial \boldsymbol{\nu}_{\mathrm{G},N_t}} \right)^{-1} \dfrac{\partial \boldsymbol{H}_{\mathrm{G},N_t}}{\partial \boldsymbol{u}_{N_t}} - \dfrac{\partial f_{int}}{\partial \boldsymbol{u}_{N_t}} \right]^{\mathrm{T}} \\ \boldsymbol{\gamma}_{\mathrm{G},N_t} = - \left(\dfrac{\partial \boldsymbol{H}_{\mathrm{G},N_t}}{\partial \boldsymbol{\nu}_{\mathrm{G},N_t}} \right)^{-\mathrm{T}} \left(\dfrac{\partial f_{int}}{\partial \boldsymbol{\nu}_{\mathrm{G},N_t}} + \boldsymbol{\lambda}_{N_t}^{\mathrm{T}} \dfrac{\partial \boldsymbol{R}_{N_t}}{\partial \boldsymbol{\nu}_{\mathrm{G},N_t}} \right)^{\mathrm{T}}, \ \forall\, \mathrm{G} \end{cases}$$

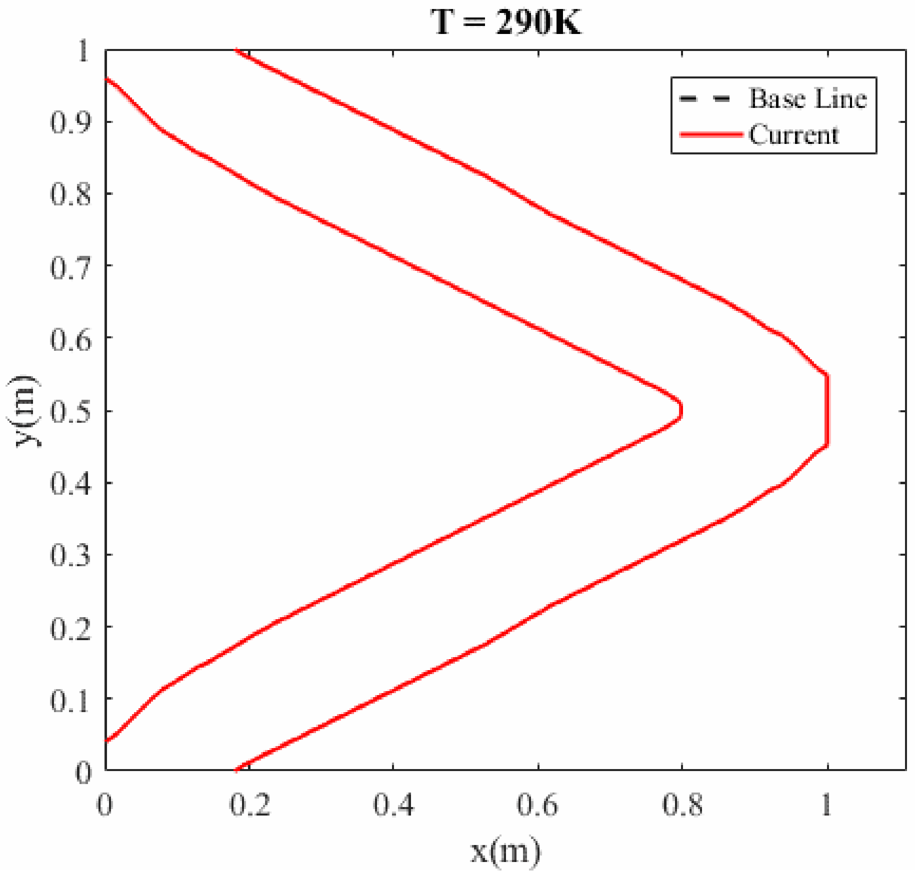
$$n < N_t : \begin{cases} \boldsymbol{\lambda}_{n} = \left[\bigwedge_{\mathrm{el}} \sum_{\mathrm{G}} \dfrac{\partial \boldsymbol{R}_{n}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} \left(\dfrac{\partial \boldsymbol{H}_{\mathrm{G},n}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} \right)^{-1} \dfrac{\partial \boldsymbol{H}_{\mathrm{G},n}}{\partial \boldsymbol{u}_{n}} - \dfrac{\partial \boldsymbol{R}_{n}}{\partial \boldsymbol{u}_{n}} \right]^{-\mathrm{T}} \\ \qquad \cdot \left\{ \boldsymbol{\lambda}_{n+1}^{\mathrm{T}} \left[\dfrac{\partial \boldsymbol{R}_{n+1}}{\partial \boldsymbol{u}_{n}} - \bigwedge_{\mathrm{el}} \sum_{\mathrm{G}} \dfrac{\partial \boldsymbol{R}_{n+1}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} \left(\dfrac{\partial \boldsymbol{H}_{\mathrm{G},n}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} \right)^{-1} \dfrac{\partial \boldsymbol{H}_{\mathrm{G},n}}{\partial \boldsymbol{u}_{n}} \right] \right. \\ \qquad \left. + \bigwedge_{\mathrm{el}} \sum_{\mathrm{G}} \boldsymbol{\gamma}_{\mathrm{G},n+1}^{\mathrm{T}} \left[\dfrac{\partial \boldsymbol{H}_{\mathrm{G},n+1}}{\partial \boldsymbol{u}_{n}} - \dfrac{\partial \boldsymbol{H}_{n+1}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} \left(\dfrac{\partial \boldsymbol{H}_{\mathrm{G},n}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} \right)^{-1} \dfrac{\partial \boldsymbol{H}_{\mathrm{G},n}}{\partial \boldsymbol{u}_{n}} \right] \right\}^{\mathrm{T}} \\ \boldsymbol{\gamma}_{\mathrm{G},n} = - \left(\dfrac{\partial \boldsymbol{H}_{\mathrm{G},n}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} \right)^{-\mathrm{T}} \left(\boldsymbol{\lambda}_{n}^{\mathrm{T}} \dfrac{\partial \boldsymbol{R}_{n}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} + \boldsymbol{\lambda}_{n+1}^{\mathrm{T}} \dfrac{\partial \boldsymbol{R}_{n+1}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} + \boldsymbol{\gamma}_{\mathrm{G},n+1}^{\mathrm{T}} \dfrac{\partial \boldsymbol{H}_{\mathrm{G},n+1}}{\partial \boldsymbol{\nu}_{\mathrm{G},n}} \right)^{\mathrm{T}}, \ \forall\, \mathrm{G} \end{cases}$$



Example Problem 1: TWSME

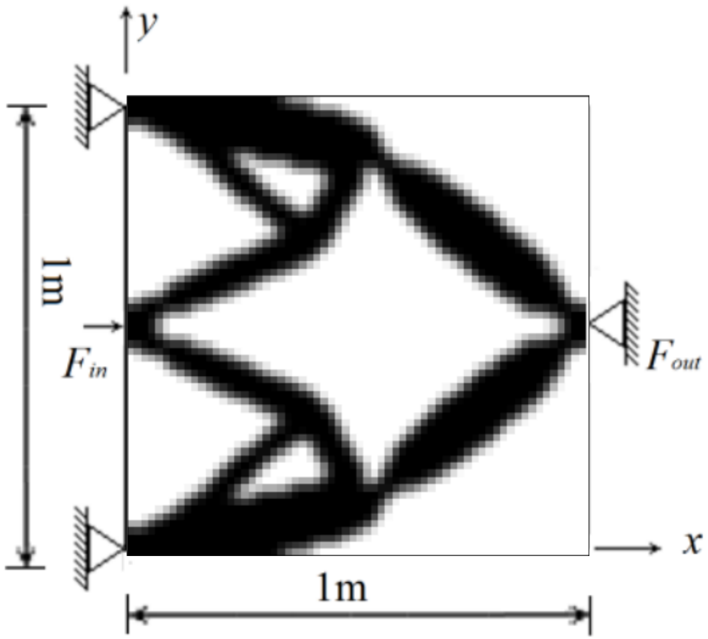
$$\min \quad f_{obj} = -\boldsymbol{L}^{\mathrm{T}}\boldsymbol{d}$$
$$s.t \quad \boldsymbol{L}^{\mathrm{T}}\tilde{\boldsymbol{d}} \leq 3 \times 10^{-4}\mathrm{m}$$
$$V \leq 0.3$$

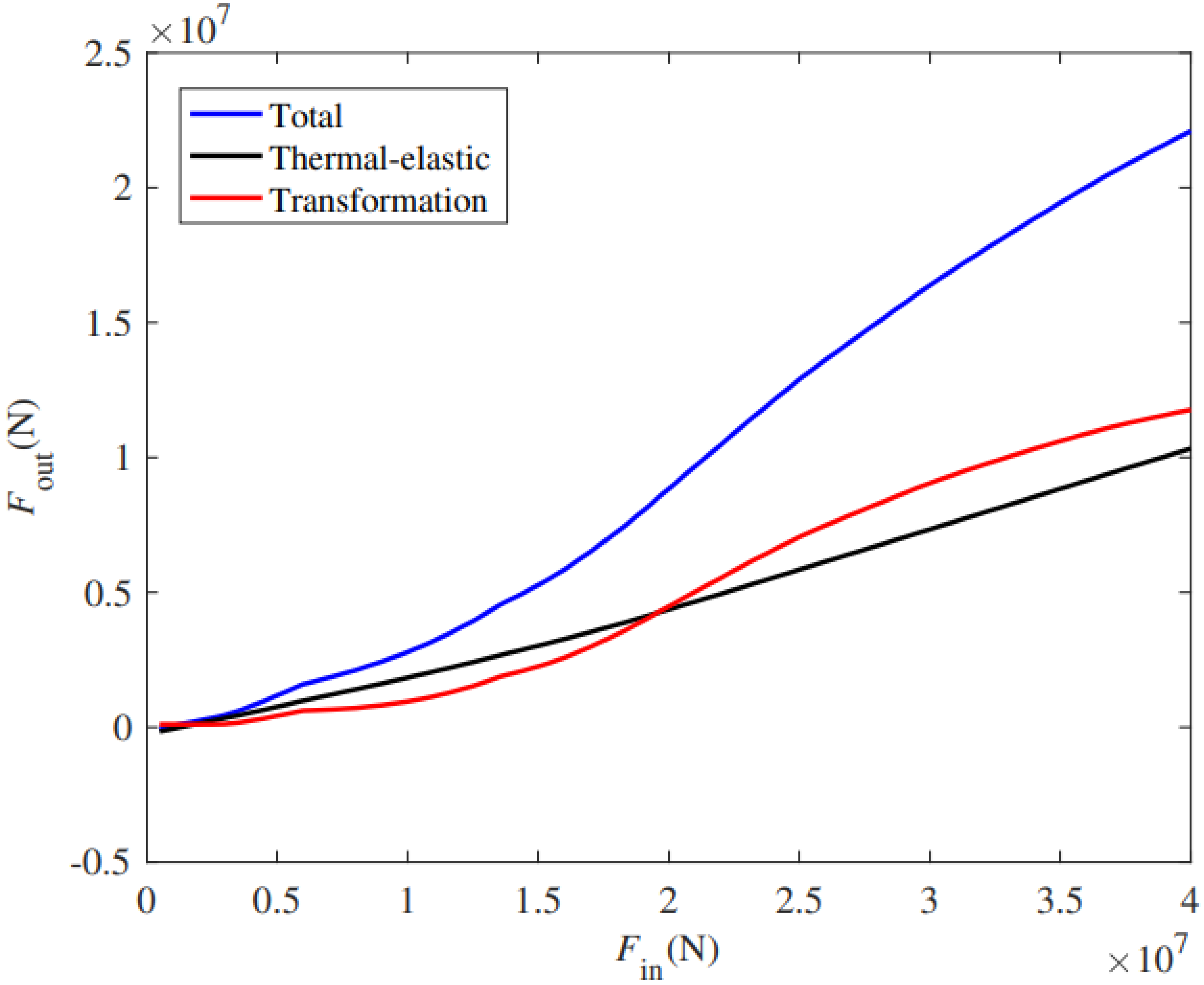




Example Problem 2: Superelasticity

$$\begin{aligned} \min \quad & f_{obj} = -\boldsymbol{F}_{out}/\boldsymbol{F}_{in}, \\ s.t \quad & d_{in} < 2 \times 10^{-3}\text{m} \\ & V \leq 0.3 \end{aligned}$$





Summary & Future Work

- Topology optimization/material interpolation scheme
- Path-dependent, transient adjoint formulation
- 2D examples for TWSME and superelasticity
- Future work:
  - Coupled thermal conductivity (heat diffusion, joule heating)
  - 3D robotic mechanisms; 4D printing

Acknowledgments

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Thank you!

Z. Kang and K.A. James, “Thermomechanical topology optimization of shape-memory alloy structures using a transient bi-level adjoint method”. *International Journal for Numerical Methods in Engineering*, 121(11):2558-2580, 2020.

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