July 21, 2020, TOP Webinar

Multifidelity Design Guided by Topology Optimization*

K. Yaji, S. Yamasaki, K. Fujita

Osaka Univ., Japan

* *Struct Multidisc Optim* (2020) 61:1071–1085

OSAKA UNIVERSITY

Outline

1. Background
- Topology Optimization
- Multi-Fidelity Design Optimization

2. Framework
- Topology Optimization with Low-Fidelity Model
- Basic Framework of Our Study

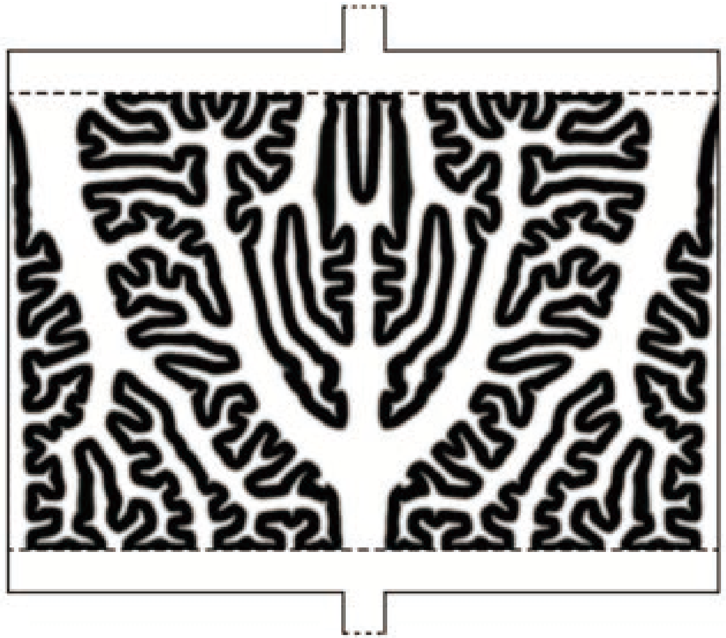
3. Case Studies
- Pressure Drop Minimization
- Heatsink Design

4. Conclusions

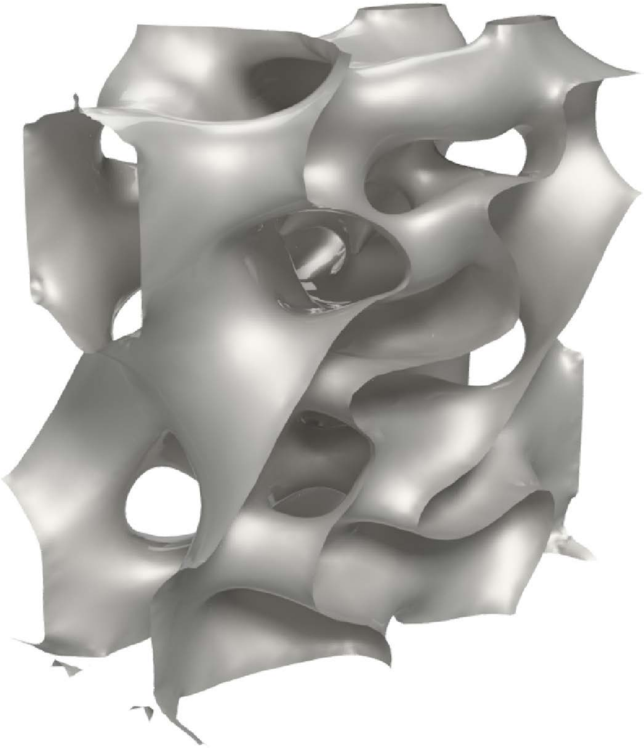
K. Yaji, Osaka Univ

Powerful tool for generating novel designs

Heatsink
[Yaji et al., *SMO*, 2018]



Flow battery
[Yaji et al., *SMO*, 2018]

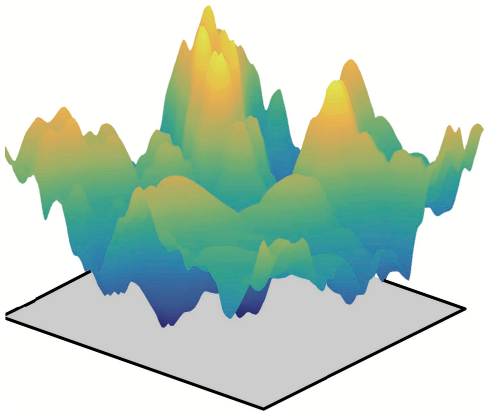


Heat Exchanger
[Kobayashi et al., *arXiv*, 2020]

Fundamental Issues of Topology Optimization

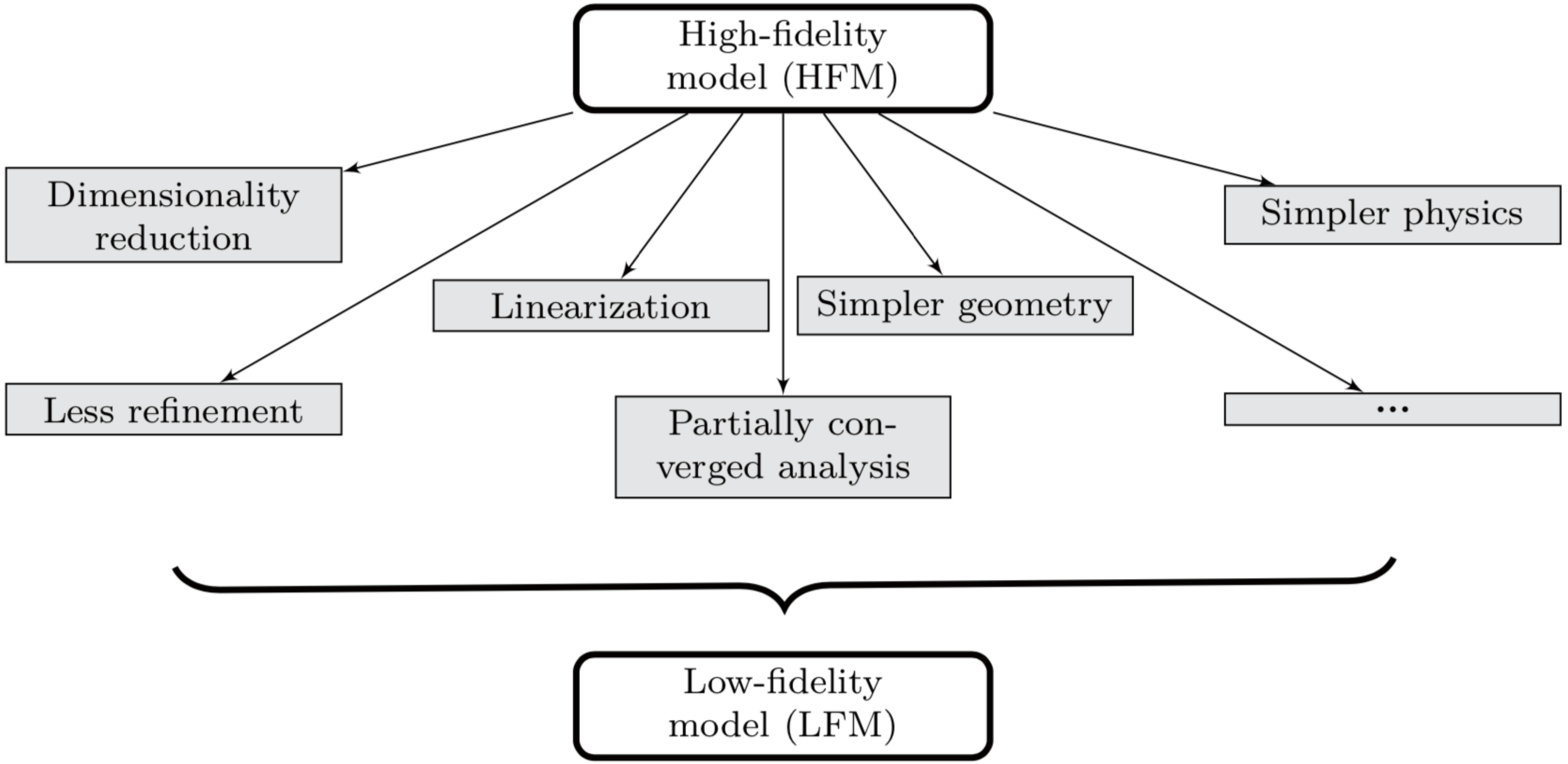
Solving optimization problems becomes hard
when dealing with complex physics.

Topology optimization
cannot avoid multimodality.



Massive computational cost for getting a local optimum
&
Sometimes patient parameter study…

Original complex optimization problem is efficiently solved by **low-fidelity analysis** with a few **high-fidelity analysis**.

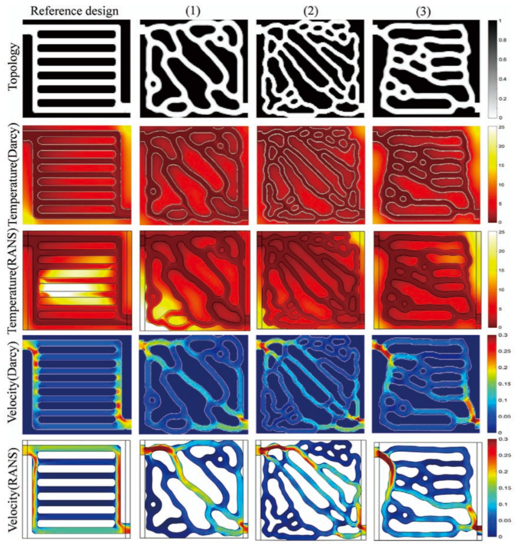


[Fernandez-Godino et al., *AIAA Journal*, 2019]

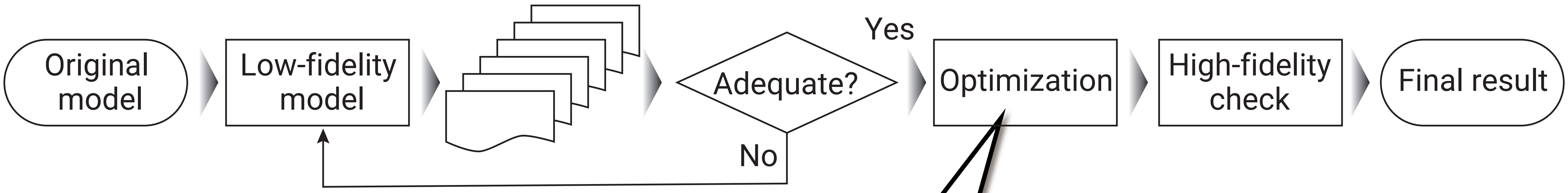
Original analysis model is replaced with **low-fidelity model**.

Related works

- ✓ Zhao et al., *Int J Heat Mass Transf* (2018): Forced convection (**HF: turbulence**, **LF: Darcy**)
- ✓ Haertel et al., *Int J Heat Mass Transf* (2018): Heatsink (**HF: 3D**, **LF: 2D**)
- ✓ Asmussen et al., *Struct Multidisc Optim* (2019): Natural convection (**HF: Navier-Stokes**, **LF: Darcy**)

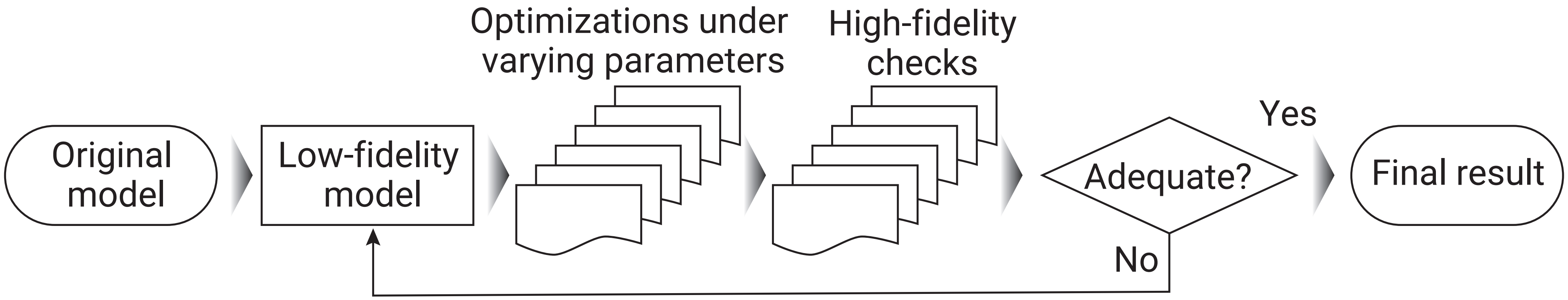


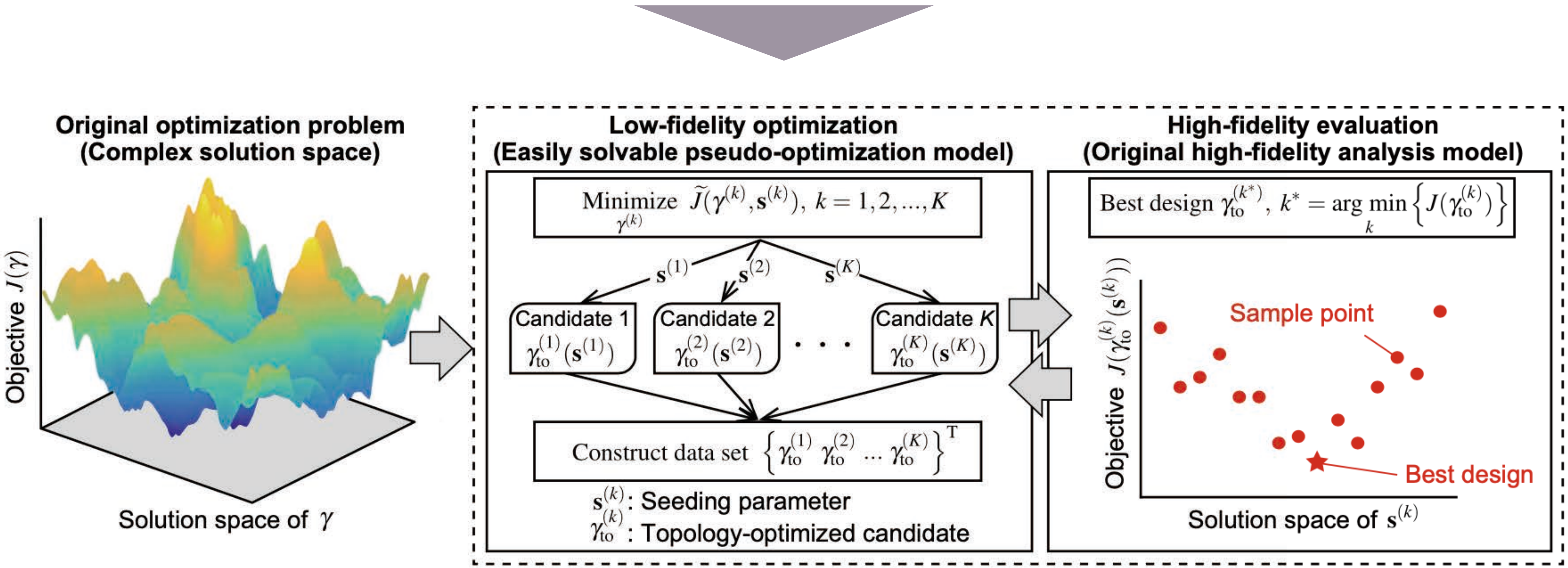
[Zhao et al., *IJHMT*, 2018]



Appropriate parameter in low-fidelity model is changed during optimization.

Basic Framework of Our Study





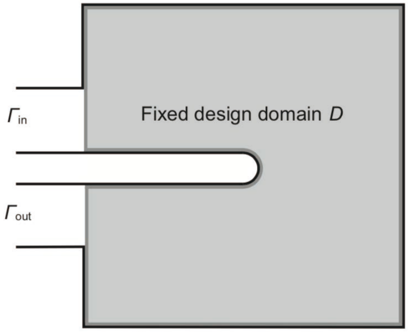
Case 1: Pressure Drop Minimization

Original problem

$$\begin{aligned} \underset{\gamma}{\text{minimize}} \quad & J_{pd} = \int_\Gamma (-\mathbf{n}\cdot\mathbf{u})(p+\rho|\mathbf{u}|^2/2)\,\mathrm{d}\Gamma \\ \text{subject to} \quad & G = \int_D \gamma\,\mathrm{d}\Omega - V_{max} \leqslant 0 \\ & 0 \leqslant \gamma(\mathbf{x}) \leqslant 1,\ \forall \mathbf{x}\in D \end{aligned}$$

- ✓ Velocity **u** and pressure *p* are given by solving turbulence model (k-ε RANS).
- ✓ Objective function: pressure drop
- ✓ Reynolds number: Re = 10,000

Parameter	Symbol	Value	Unit
Inlet width	L_{in}	1.0	m
Inlet speed	U_{in}	0.01	m/s
Kinematic viscosity	ν	1.0×10^{-6}	m^2/s
Fluid density	ρ_f	1.0×10^3	kg/m^3

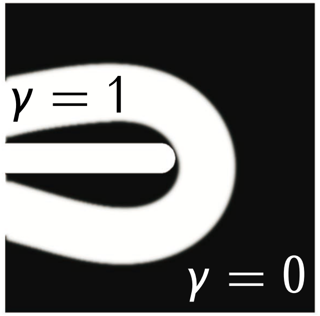


Low-fidelity problem

$$\begin{aligned} \underset{\gamma^{(k)}}{\text{minimize}} \quad & \tilde{J}_{pd}(\gamma^{(k)}, d_1^{(k)}) = \int_\Gamma (-\mathbf{n}\cdot\tilde{\mathbf{u}})(\tilde{p}+|\tilde{\mathbf{u}}|^2/2)\,\mathrm{d}\Gamma \\ \text{subject to} \quad & G(\gamma^{(k)}) = \int_D \gamma^{(k)}\,\mathrm{d}\Omega - V_{max} \leqslant 0 \\ & 0 \leqslant \gamma^{(k)}(\mathbf{x}) \leqslant 1,\ \forall \mathbf{x}\in D \\ \text{for given} \quad & s_1^{(k)},\ k = 1, \dots, K \end{aligned}$$

- ✓ Velocity **u** and pressure *p* are given by solving Navier-Stokes equations (Low-Re Laminar flow).
- ✓ $Re(s_1) = \underline{Re} + (\overline{Re} - \underline{Re})s_1 \quad 0 \leqslant s_1 \leqslant 1$
- ✓ Density approach [Borrvall & Petersson, *IJNMF*, 2003]

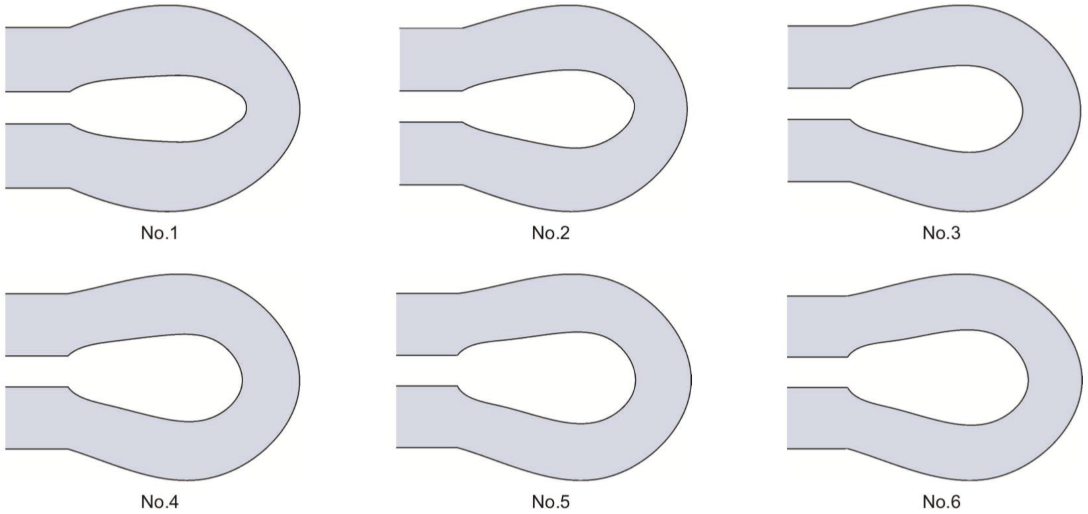
Parameter	Symbol	Value
Tuning parameter	q	0.01
Inverse permeability	α	1.0×10^4
Inlet speed	$\tilde{U}_{in}$	1.0
Reynolds number	$(\underline{Re}_{pd}, \overline{Re}_{pd})$	(30, 120)

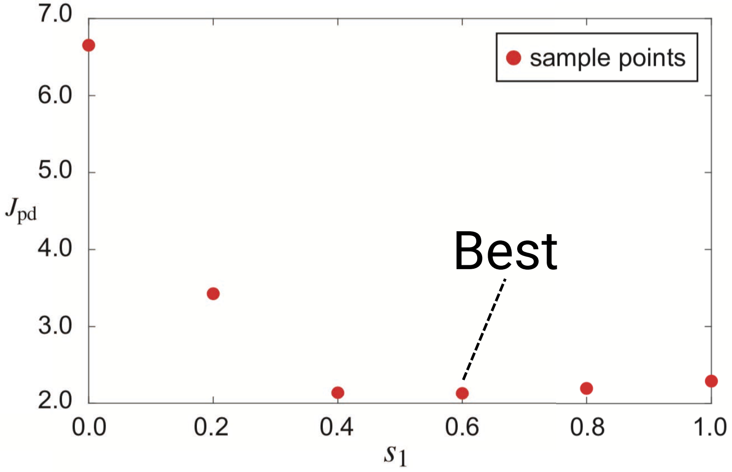


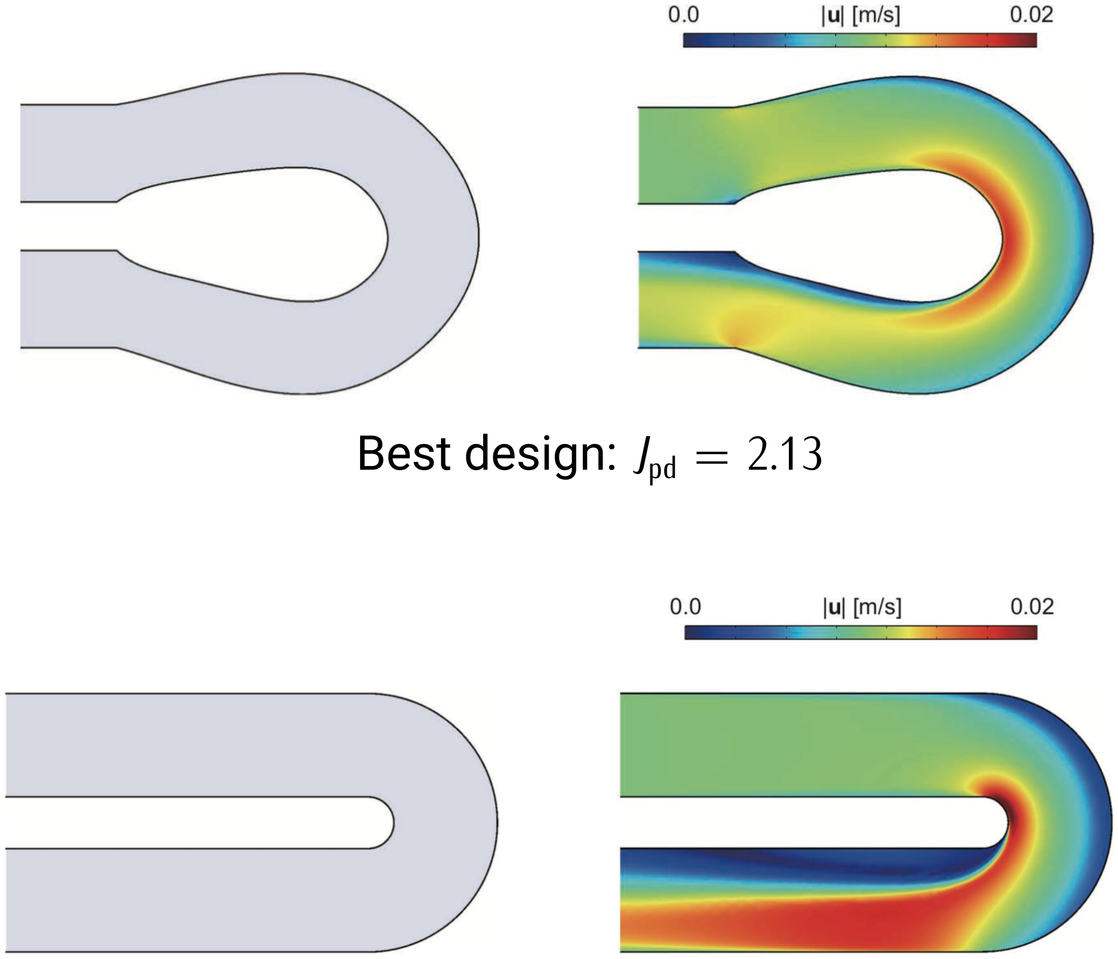
K. Yaji, Osaka Univ

High-fidelity evaluation

$$\text{find } \gamma_{\text{to}}^{(k^*)}, k^* = \arg\min_{k} \left\{ J_{\text{pd}}(\gamma_{\text{to}}^{(k)}) \right\}$$







Best design: $J_{\text{pd}} = 2.13$

Reference design: $J_{\text{pd}} = 6.24$

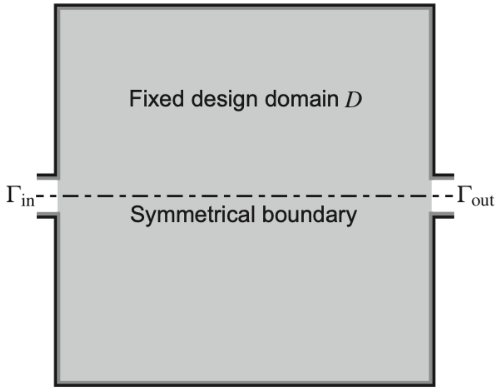
Case 2: Heatsink Design

Original problem

$$\underset{\gamma}{\text{maximize}} \quad J_{\text{hs}} = C/P$$
$$\text{subject to} \quad 0 \leqslant \gamma(\mathbf{x}) \leqslant 1, \ \forall \mathbf{x} \in D$$

- ✓ Turbulent heat transfer (k-ε RANS model)
- ✓ Objective function: thermal conductance / pumping power
- ✓ Reynolds number: Re = 5,000

Parameter	Symbol	Value	Unit
Inlet width	L_{in}	0.02	m
Inlet speed	U_{in}	0.2	m/s
Kinematic viscosity	ν	1.0×10^{-6}	m^2/s
Fluid density	ρ_{f}	1.0×10^3	kg/m^3
Fluid specific heat	c_{f}	4.2×10^3	J/(kg K)
Prandtl number	Pr	7.0	-
Turbulent Prandtl number	Pr_{t}	0.9	-
Inlet temperature	T_{in}	293	K
Solid density	ρ_{s}	2.7×10^3	kg/m^3
Solid specific heat	c_{s}	910	J/(kg K)
Solid thermal conductivity	k_{s}	237	W/(m K)
Prescribed heat source	Q_{hs}	1.0×10^6	W/m^3



Low-fidelity problem

$$\underset{\gamma^{(k)}}{\text{maximize}} \quad \tilde{J}_{\text{hs}}(\gamma^{(k)}, \mathbf{s}^{(k)}) = \int_D \tilde{Q}(\gamma^{(k)})\mathrm{d}\Omega$$
$$\text{subject to} \quad 0 \leqslant \gamma^{(k)}(\mathbf{x}) \leqslant 1, \ \forall \mathbf{x} \in D$$
$$\text{for given} \quad \mathbf{s}^{(k)} = \{s_1^{(k)} \ s_2^{(k)}\}^{\mathsf{T}}, \ k = 1, \ldots, K$$

- ✓ Laminar heat transfer
- ✓ Objective function: amount of heat exchange
- ✓ $Re(s_1) = \underline{Re} + (\overline{Re} - \underline{Re})s_1, 0 \leqslant s_1 \leqslant 1$

  $\beta(s_2) = \underline{\beta} + (\overline{\beta} - \underline{\beta})s_2, 0 \leqslant s_2 \leqslant 1$
- ✓ Density approach [Matsumori et al., *SMO*, 2013]

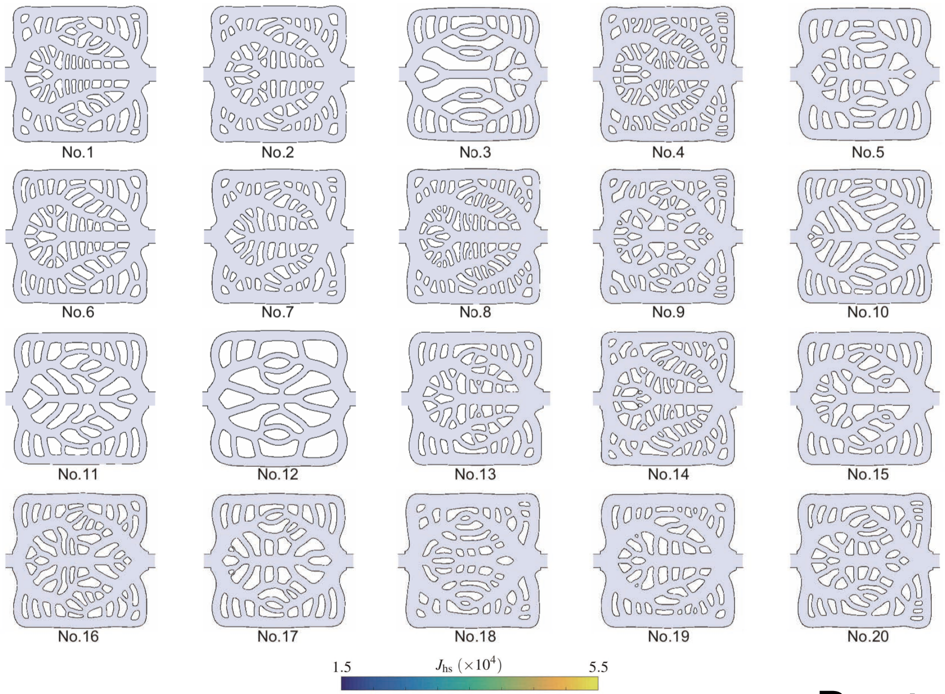
Parameter	Symbol	Value
Prandtl number	Pr	7.0
Tuning parameter	q	0.01
Inverse permeability	α	1.0×10^4
Pressure loss	$\Delta\tilde{p}$	1.0
Inlet temperature	$\tilde{T}_{\text{in}}$	0.0
Reynolds number	$(\underline{Re}_{\text{hs}}, \overline{Re}_{\text{hs}})$	(500, 1200)
Heat transfer coefficient	$(\underline{\beta}, \overline{\beta})$	(100, 1500)

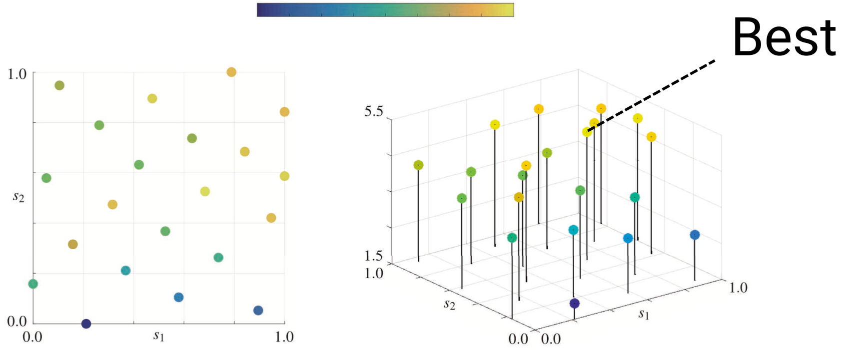
Numerical Results

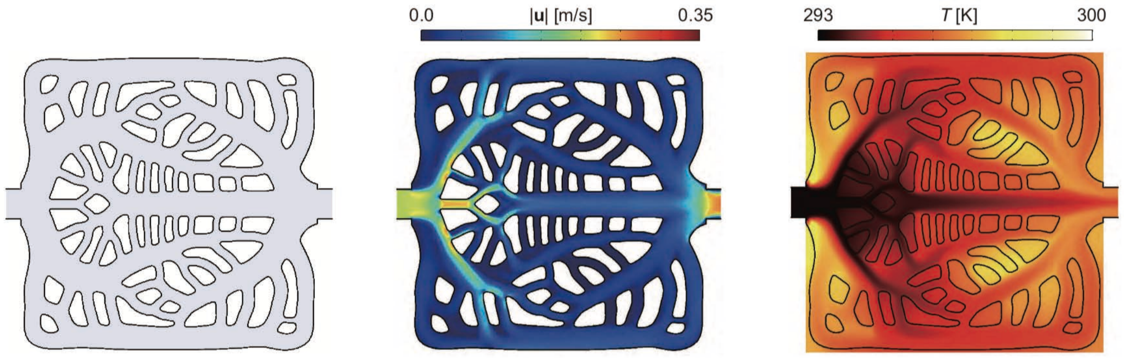
K. Yaji, Osaka Univ

High-fidelity evaluation

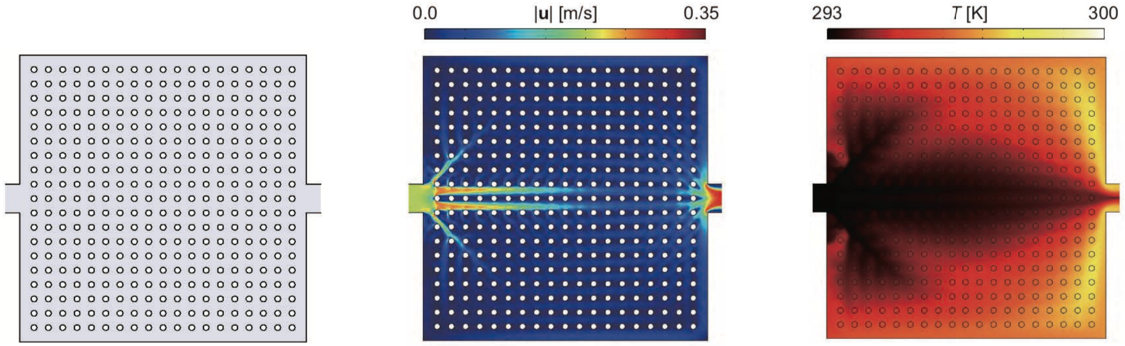
$$\text{find } \gamma_{\text{to}}^{(k^*)},\, k^* = \arg\min_{k} \left\{ J_{\text{hs}}(\gamma_{\text{to}}^{(k)}) \right\}$$







Best design: $J_{\text{hs}} = 5.07$



Reference design: $J_{\text{hs}} = 3.91$

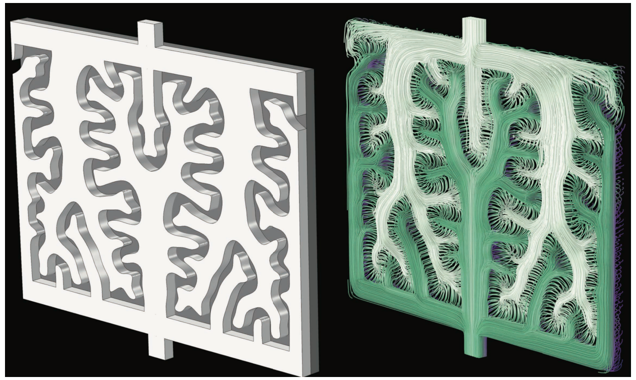
Conclusions

We proposed a framework of multi-fidelity design for topology optimization problems, and applied it to turbulent flow design problems.

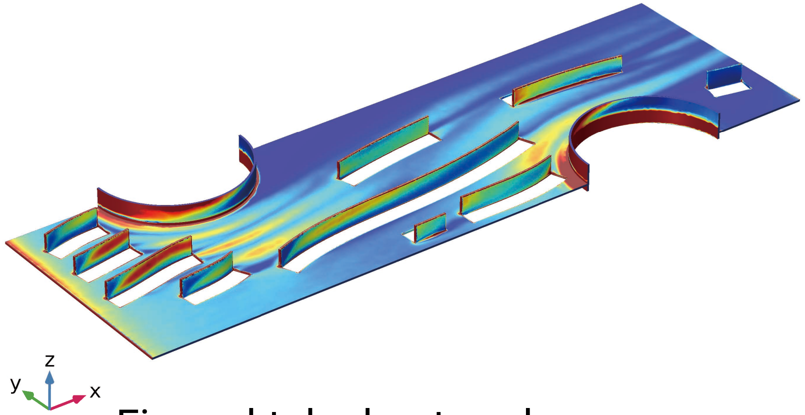
Future works

- ✓ Applications to design optimization problems considering other complex physics—nonlinear structural mechanics, etc.
- ✓ Strategy for determining formulation of low-fidelity optimization problems

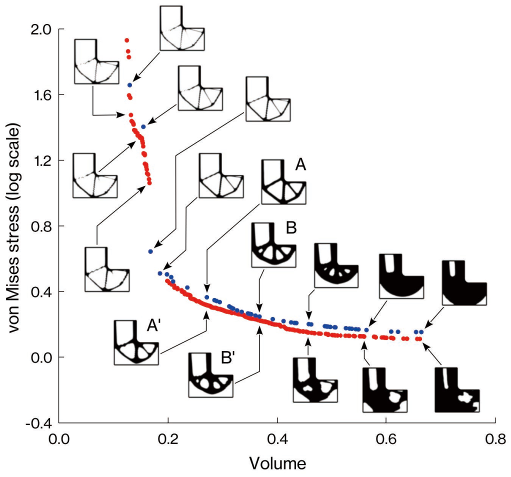
Recent related works



Flow battery
[Yaji et al., *Proc ASME IDETC/CIE*, 2019]



Fin-and-tube heat exchanger
[Kobayashi et al., *Appl Therm Eng*, 2019]



Deep generative model
[Yamasaki et al., *arXiv*, 2020]