Design of Phononic-like Structures and Band-Gap Tuning by Concurrent Two-scale Topology Optimization

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Outline

- Background and motivition
- Concurrent two-scale modeling and method for band-gap design
- Extension of the work connecting additive manufactured lattice infills
- Conclusions

Background and Motivation

Background

Vibration/sound isolation and mitigation : car, aircraft, high speed train, subway, factory machine, et al.

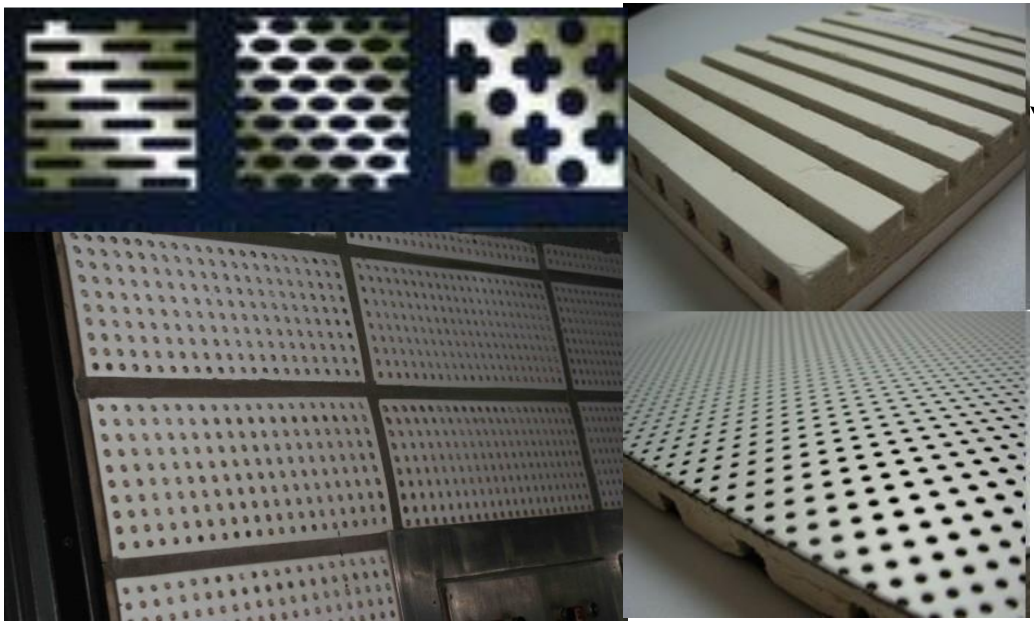
Active control

Passive design:

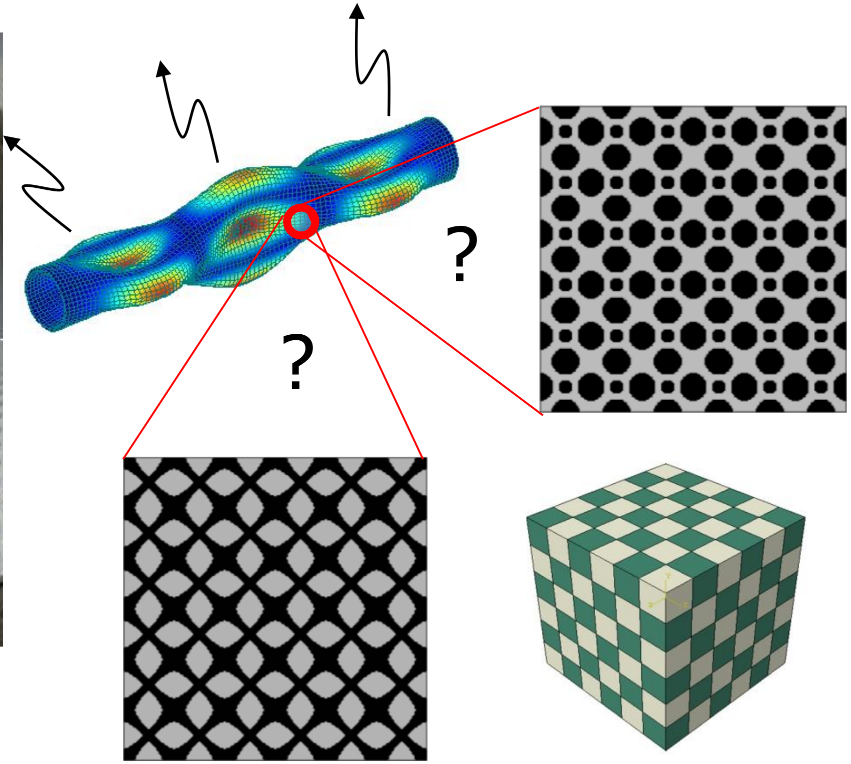
During recent years, the design problem has benefited from the methodology of topology optimization.

Background and Motivation

Porous material/structure



Metamaterial design



Design of microscopic level:

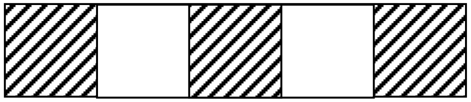
Reduce noise/vibration by microstructural or two-scale topology optimization

Background and Motivation

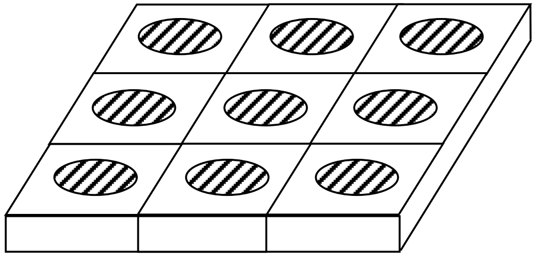
➢ Phononic crystal

Periodically structured functional material;
Energy band gap property for elastic wave.

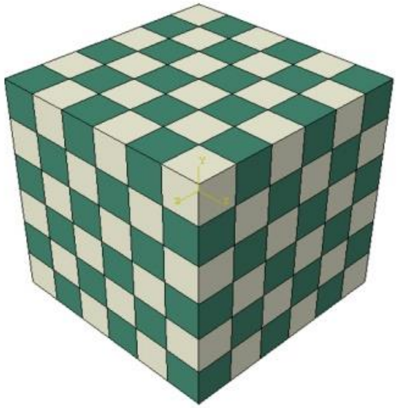
1D

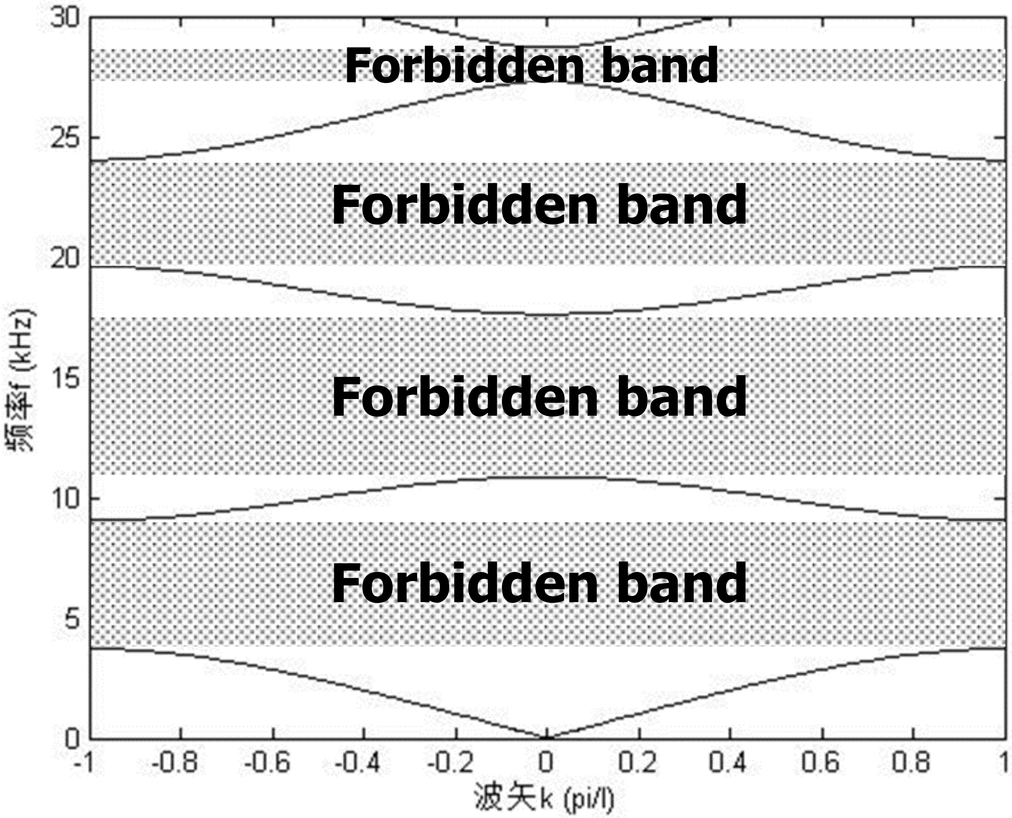


2D



3D





Dispersion plot

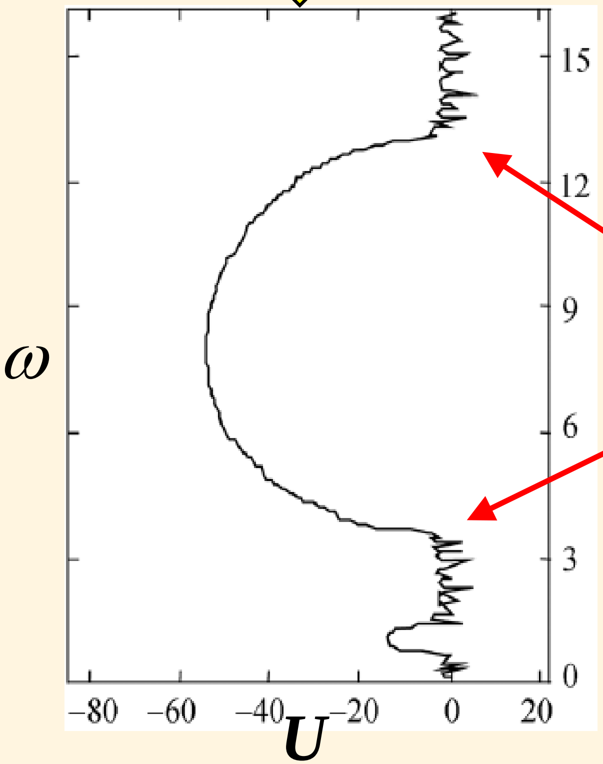
Background and Motivation

Two ways of band gap tuning

Homogenization based solution

Frequency response

$$f(\omega, U) = 0$$



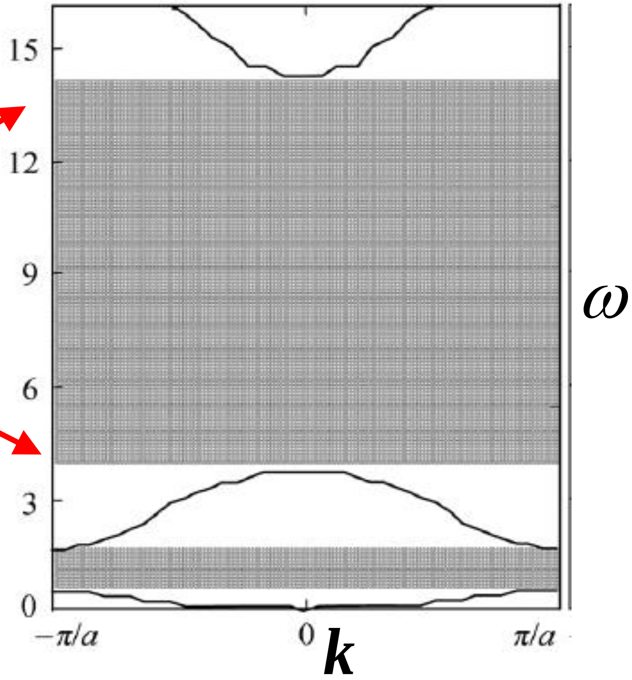
Frequency response curve

Band gap

Bloch wave solution

Eigenvalue analysis

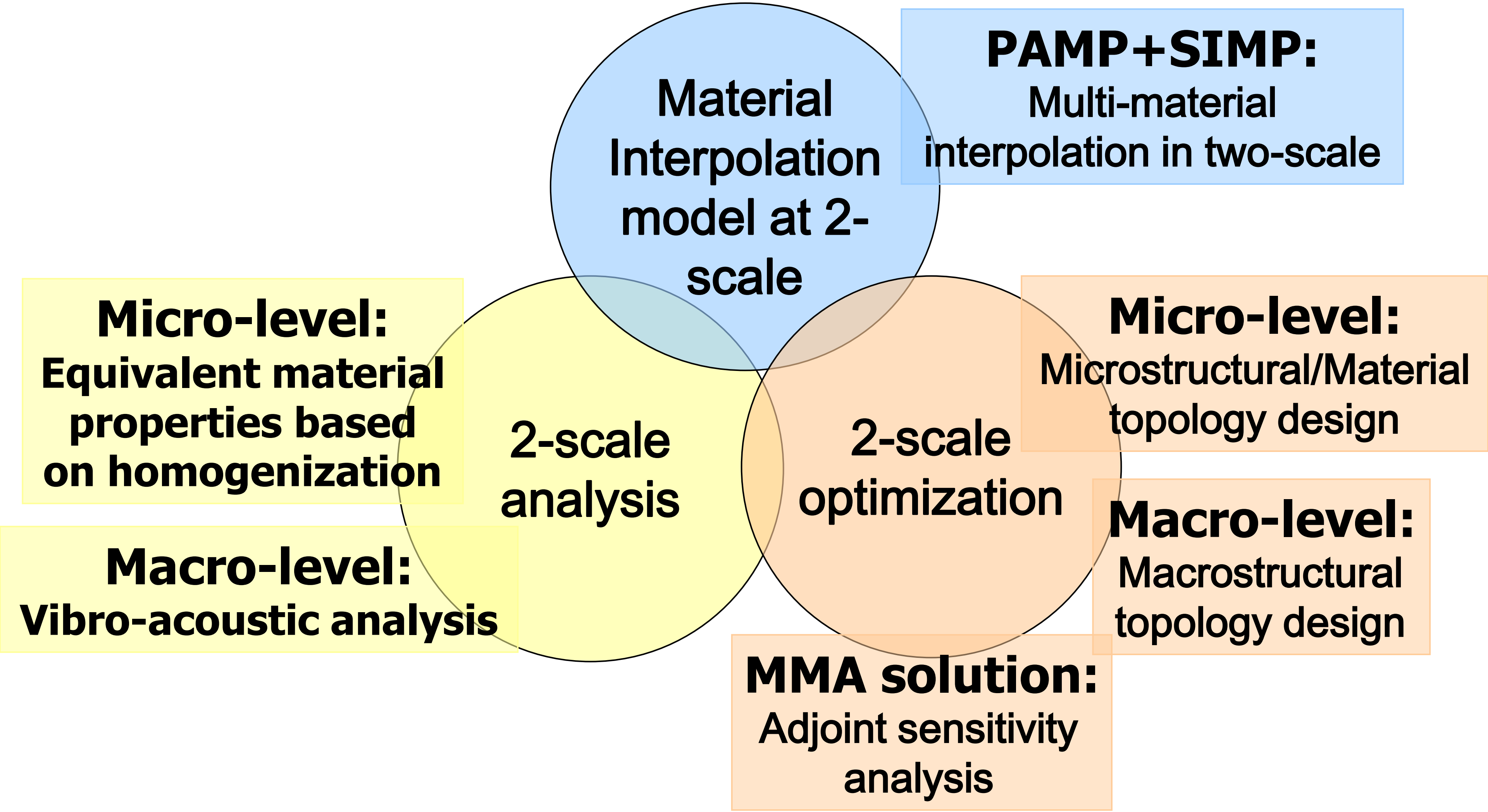
$$\tilde{f}(\omega, \boldsymbol{k}) = 0$$



Dispersion plot

Modeling and Methods

Multimaterial interpolation model at 2-scale + 2-scale analysis + 2-scale optimization + :



Material Interpolation model at 2-scale

PAMP+SIMP:
Multi-material interpolation in two-scale

2-scale analysis

Micro-level:
Equivalent material properties based on homogenization

Macro-level:
Vibro-acoustic analysis

2-scale optimization

Micro-level:
Microstructural/Material topology design

Macro-level:
Macrostructural topology design

MMA solution:
Adjoint sensitivity analysis

Modeling and Methods

Design objectives

– **Minimization** of the **Transmission Coefficient of the Sound power flow (*STC*)**

Sound power flow (energy flux) $\prod_S$ is defined by

$$\Pi_S = \int_S I_n \, dS = \int_S \frac{1}{2}\mathrm{Re}(p_f v_n^*)\, dS. \qquad (3)$$

S – Structural surface

I_n – Power flow density

p_f – Acoustic sound pressure at the structural surface

v_n^* – Complex conjugate of the normal velocity of the structural surface

$$STC = \frac{\Pi_{SO}}{\Pi_{SI}} = \frac{\int_{SO} \frac{1}{2}\mathrm{Re}(p_f v_n^*)\, dS}{\int_{SI} \frac{1}{2}\mathrm{Re}(p_f v_n^*)\, dS}.$$

Π_{SO} – Sound power at specified output area

Π_{SI} – Sound power at specified input area

Modeling and Methods

Multi-material interpolation at 2 scale (λ、 μ、 ρ)

Microscale: (n+1) base material interpolation

$$ {}_1\boldsymbol{D}^{\mathrm{MI}} = \boldsymbol{D}^{\mathrm{MI}} = \lambda_n^p \left\{ \lambda_{n-1}^p [\lambda_{n-2}^p (\cdots) + (1-\lambda_{n-2}^p)\boldsymbol{D}_{n-1}] + (1-\lambda_{n-1}^p)\boldsymbol{D}_n \right\} + (1-\lambda_n^p)\boldsymbol{D}_{n+1} $$

$$ {}_1\eta^{\mathrm{MI}} = \eta^{\mathrm{MI}} = \lambda_n^q \left\{ \lambda_{n-1}^q [\lambda_{n-2}^q (\cdots) + (1-\lambda_{n-2}^q)\eta_{n-1}] + (1-\lambda_{n-1}^q)\eta_n \right\} + (1-\lambda_n^q)\eta_{n+1} $$

Homogenization ↓

First microstructure design variable: λ

$$ \boldsymbol{D}_1^{\mathrm{H}}(\boldsymbol{\lambda}) = \frac{1}{|Y|}\int_Y {}_1\boldsymbol{D}^{\mathrm{MI}}(\boldsymbol{I}-\boldsymbol{bu})\mathrm{d}Y \qquad \eta_1^{\mathrm{H}}(\boldsymbol{\lambda}) = \frac{1}{|Y|}\int_Y {}_1\boldsymbol{\eta}^{\mathrm{MI}}\mathrm{d}Y $$

Second microstructure design variable: μ

$$ \boldsymbol{D}_2^{\mathrm{H}}(\boldsymbol{\mu}) = \frac{1}{|Y|}\int_Y {}_2\boldsymbol{D}^{\mathrm{MI}}(\boldsymbol{I}-\boldsymbol{bu})\mathrm{d}Y \qquad \eta_2^{\mathrm{H}}(\boldsymbol{\mu}) = \frac{1}{|Y|}\int_Y {}_2\boldsymbol{\eta}^{\mathrm{MI}}\mathrm{d}Y $$

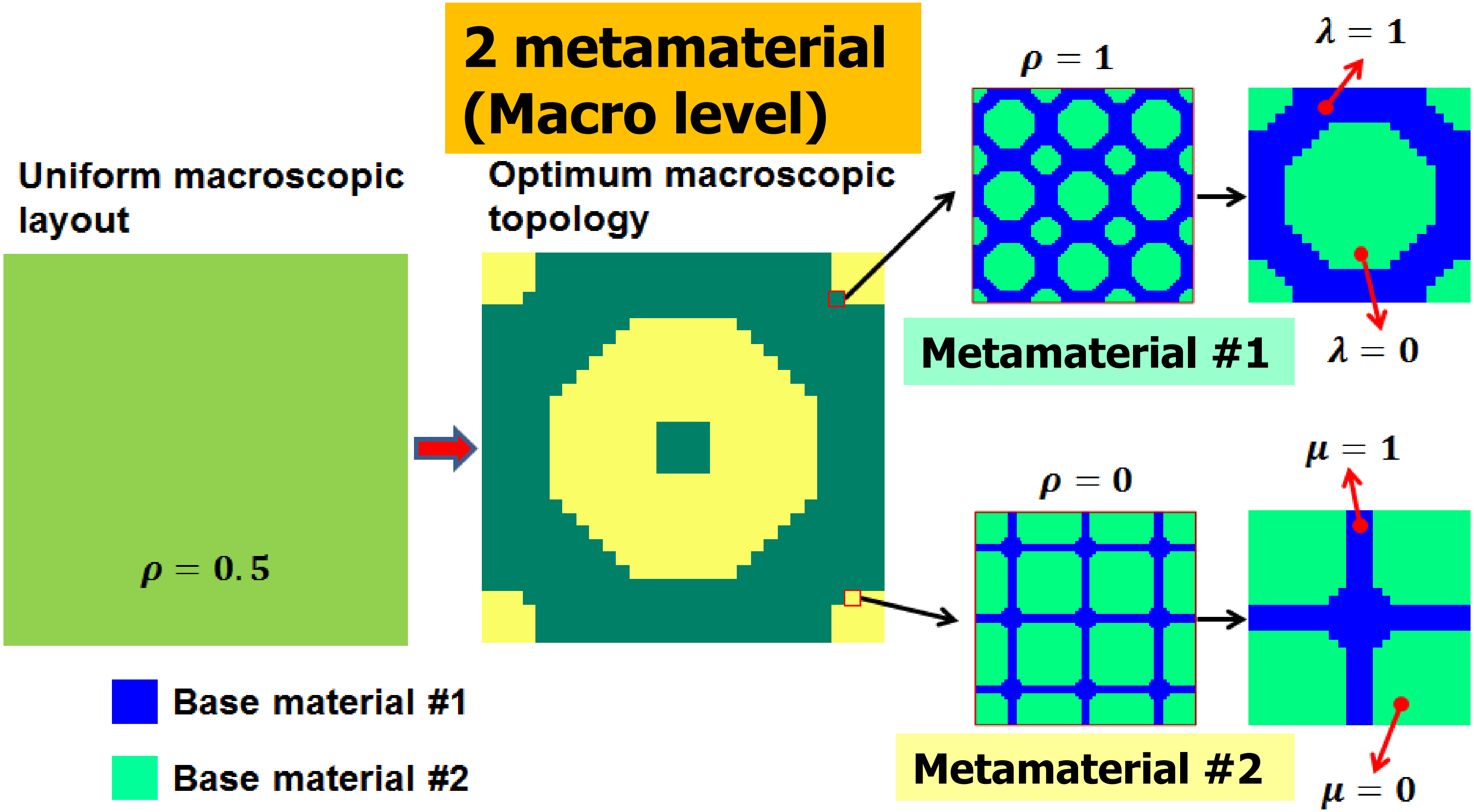
Macroscale: 2 metamaterial interpolation

Macrostructure design variable: ρ

$$ \boldsymbol{D}^{\mathrm{MA}}(\rho) = \rho^p \boldsymbol{D}_1^{\mathrm{H}}(\boldsymbol{\lambda}) + (1-\rho^p)\boldsymbol{D}_2^{\mathrm{H}}(\boldsymbol{\mu}) \qquad \eta^{\mathrm{MA}}(\rho) = \rho^q \eta_1^{\mathrm{H}}(\boldsymbol{\lambda}) + (1-\rho^q)\eta_2^{\mathrm{H}}(\boldsymbol{\mu}) $$

Modeling and Methods

Example of 2 base material and 2 metamaterial



2 metamaterial (Macro level)

Uniform macroscopic layout

$\rho = 0.5$

Optimum macroscopic topology

$\rho = 1$

$\lambda = 1$

$\lambda = 0$

Metamaterial #1

$\rho = 0$

$\mu = 1$

$\mu = 0$

Metamaterial #2

🟦 Base material #1

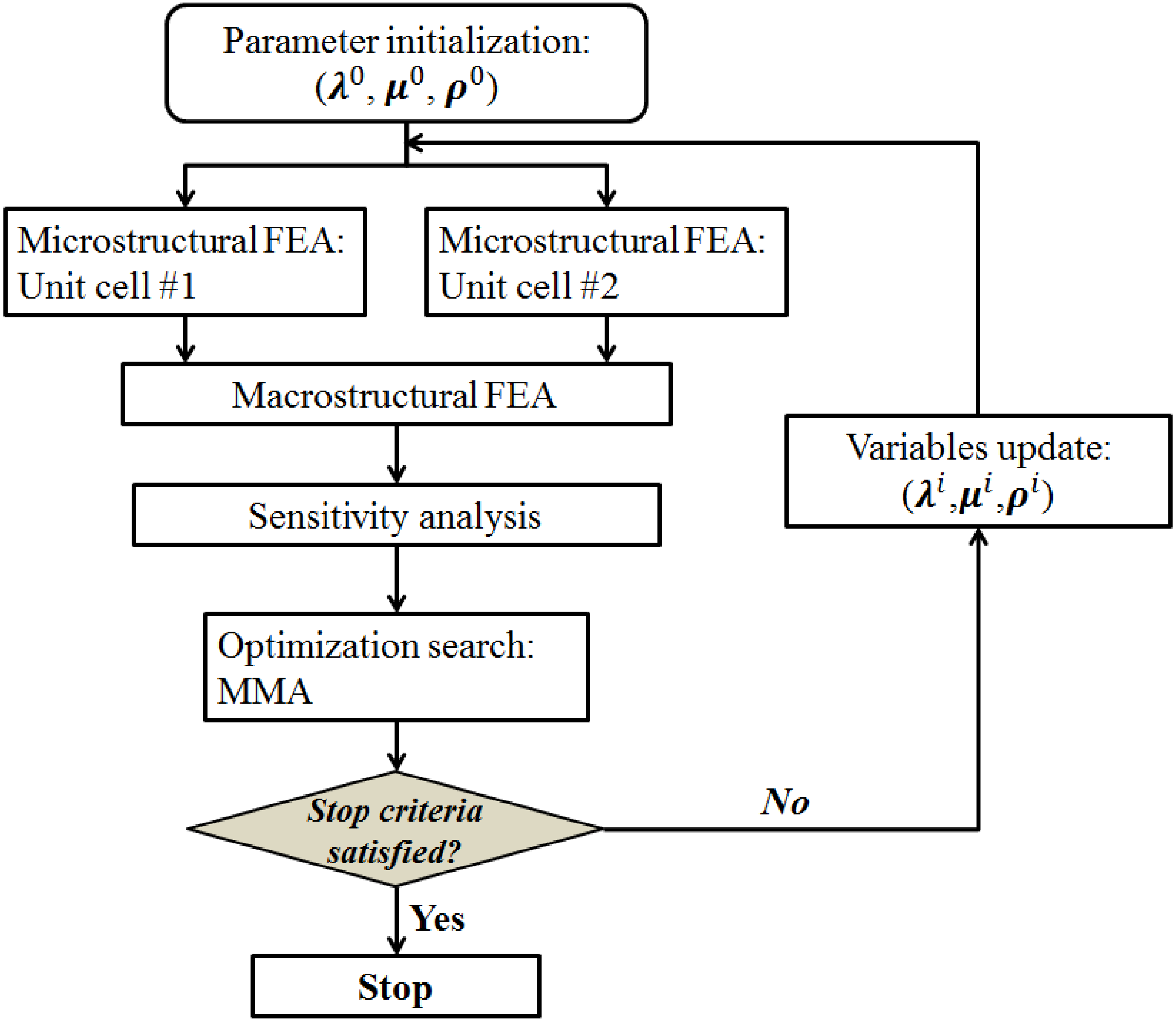
🟩 Base material #2

2 base material

2 metamaterial (Macro level)

Modeling and Methods

Flow chart of 2-scale topology optimization for band-gap tuning



Parameter initialization: $(\boldsymbol{\lambda}^0, \boldsymbol{\mu}^0, \boldsymbol{\rho}^0)$

Microstructural FEA: Unit cell #1

Microstructural FEA: Unit cell #2

Macrostructural FEA

Sensitivity analysis

Optimization search: MMA

Stop criteria satisfied?

No

Variables update: $(\boldsymbol{\lambda}^i, \boldsymbol{\mu}^i, \boldsymbol{\rho}^i)$

Yes

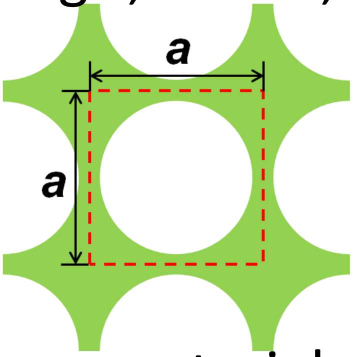
Stop

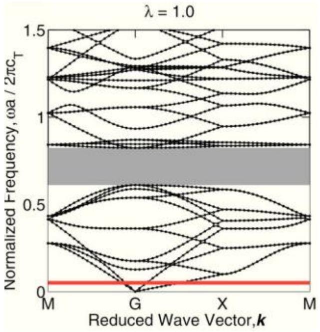
Numerical examples

Example 1 – Verification of band gap

Micro-level (unit cell)

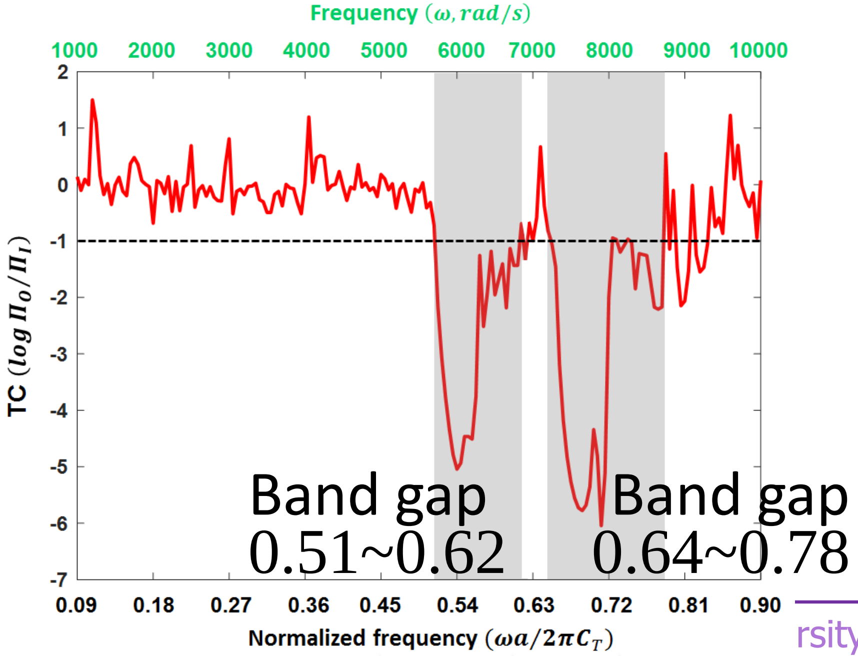
Wang P, Shim J, Bertoldi K (2013)





Base materials:
rubber and void

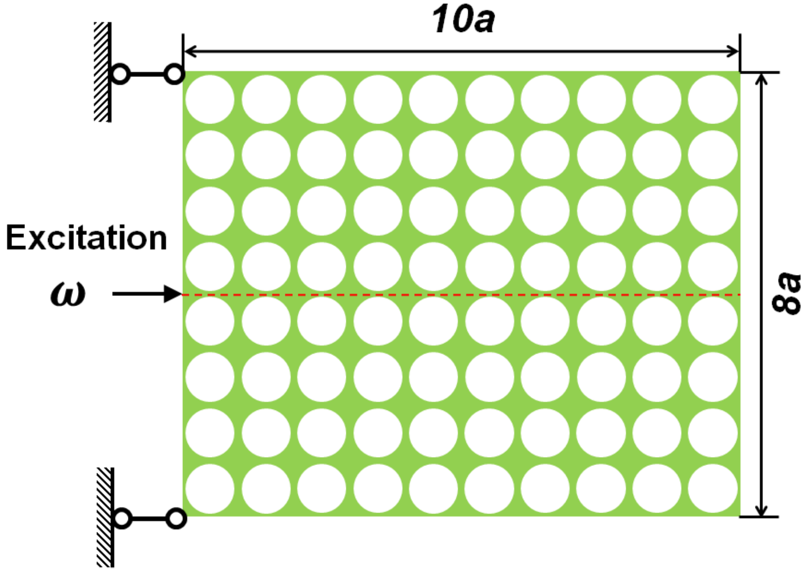
Band gap
0.61~0.82

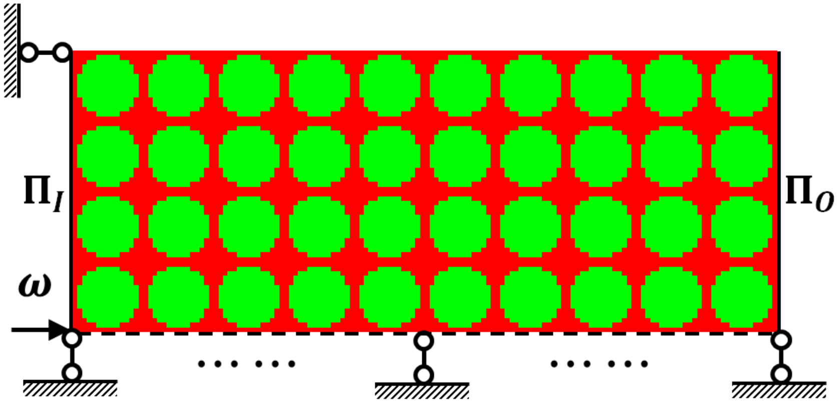


Band gap
0.51~0.62

Band gap
0.64~0.78

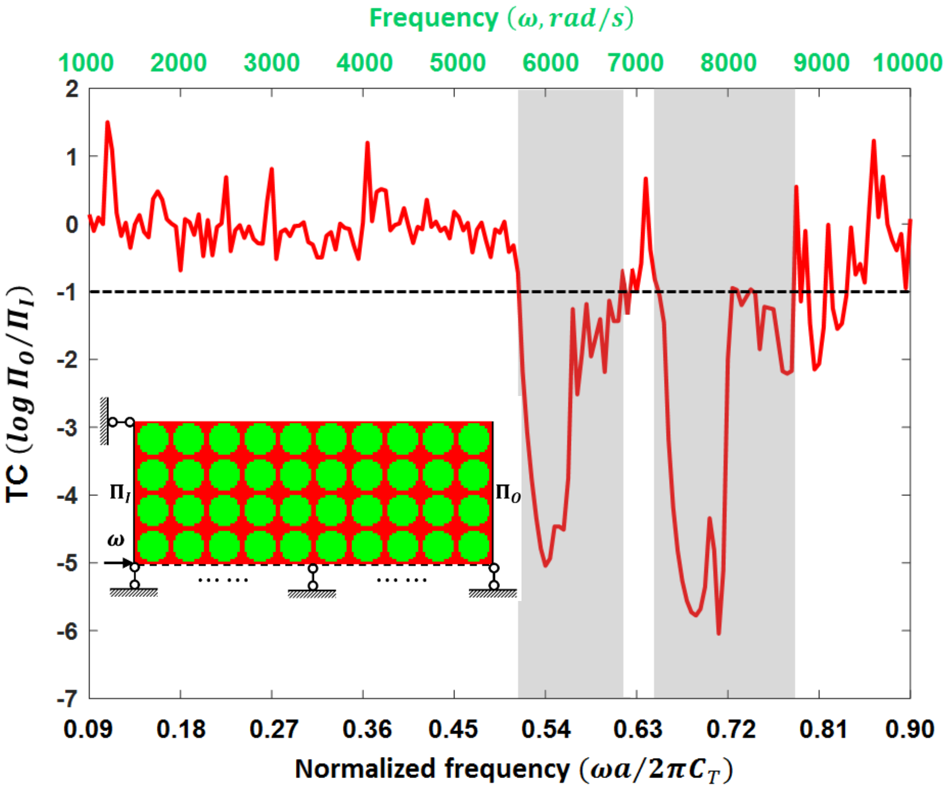
Macro-level (metamaterial structure)

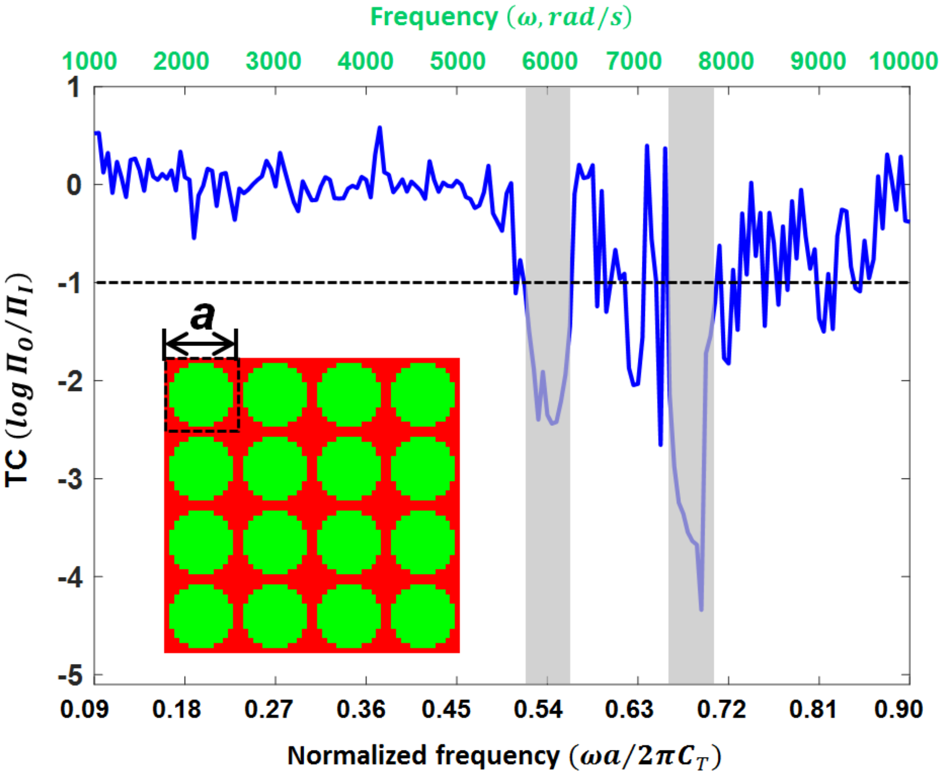




Example 1 – Verification of band gap

Size effect of macrostructure

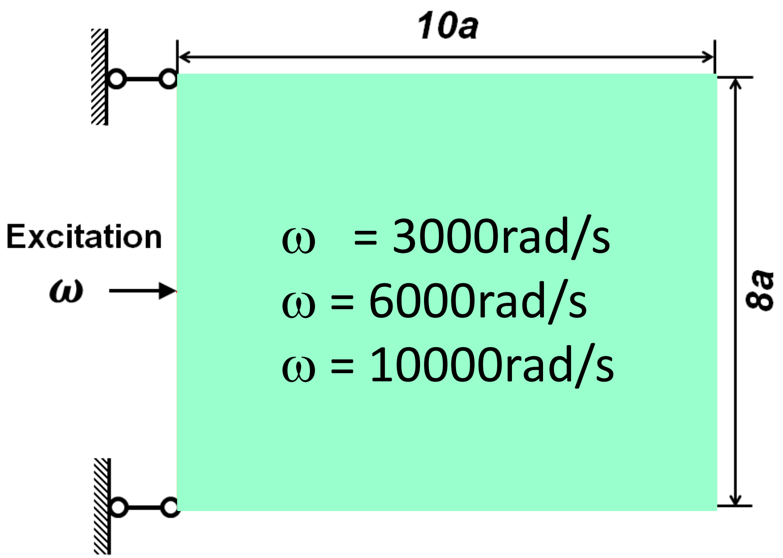




Band gap 1	Band gap 2	Band gap 1	Band gap 2
0.51~0.62	0.64~0.78	0.51~0.57	0.65~0.71

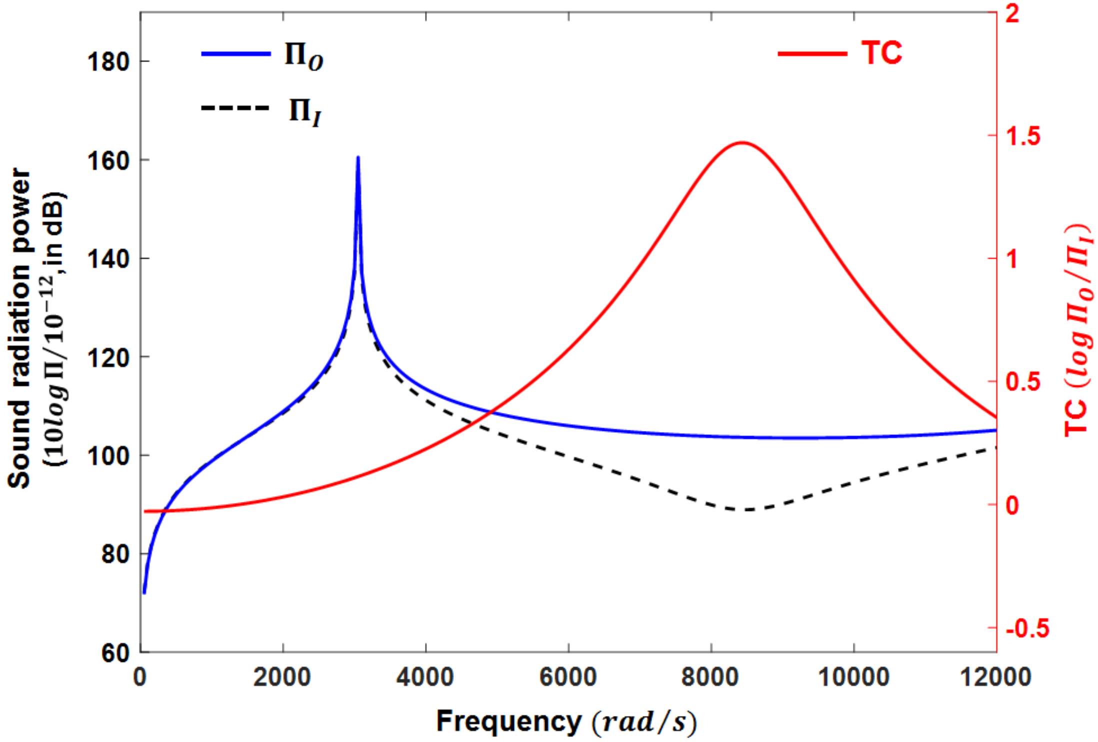
Example 2 – 2 scale optimization for band gap tuning

Base materials: rubber and Aluminum (Al)



ω = 3000rad/s
ω = 6000rad/s
ω = 10000rad/s

Initial uniform design of macro-structure

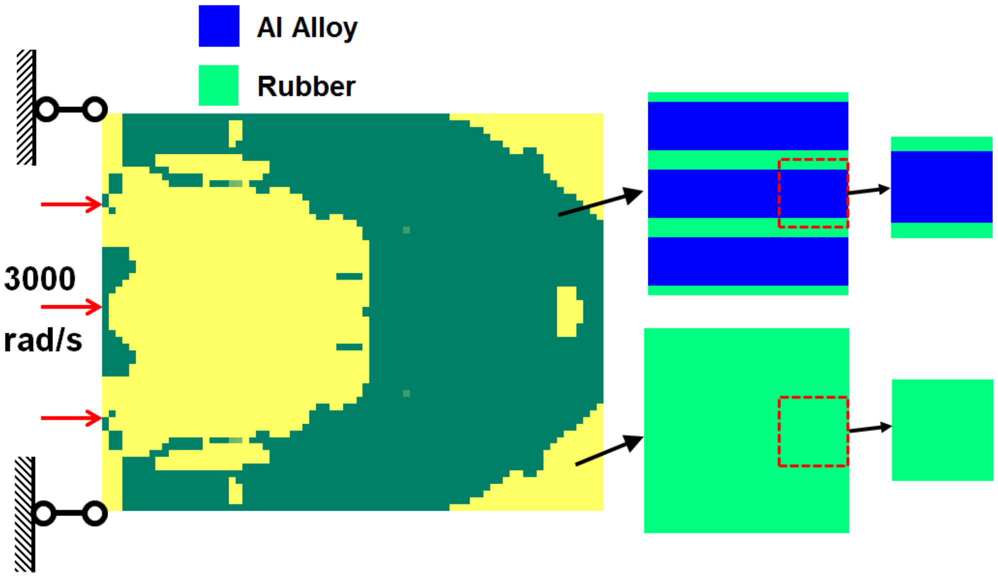


Sound radiation power in the input and output surface and transmission coefficient (TC) of the uniform initial design.

No band gap between 0~12000 rad/s

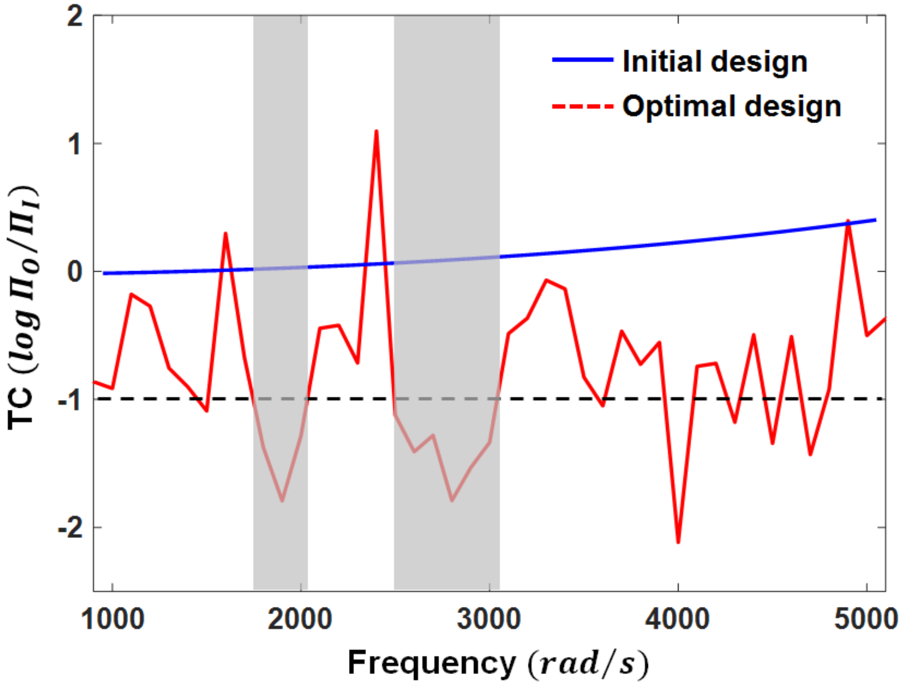
Example 2 – 2 scale optimization for band gap tuning

ω = 3000rad/s



Optimum bi-metamaterial **macro-structural topology**

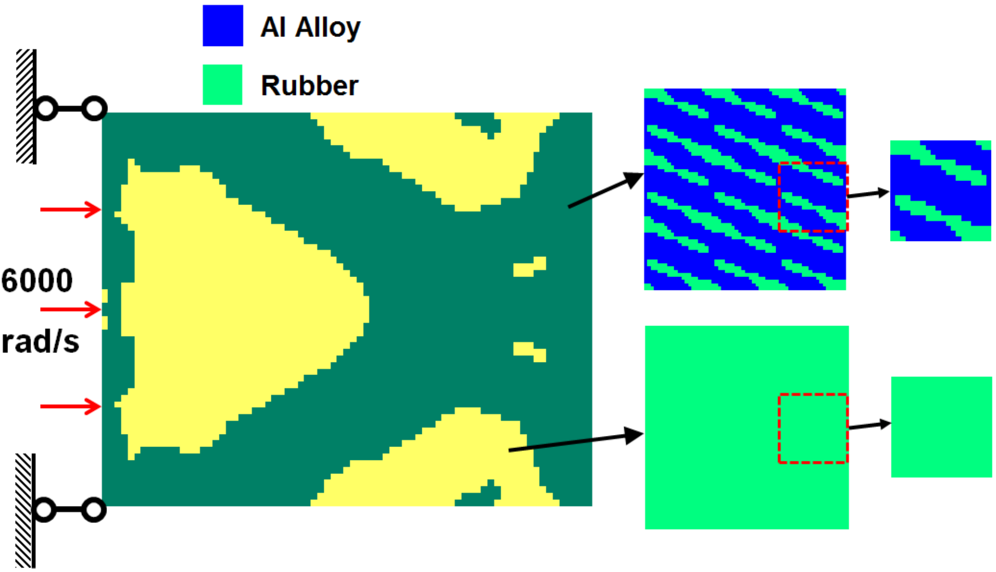
Optimum bi-basematerial **micro-structural topologies**

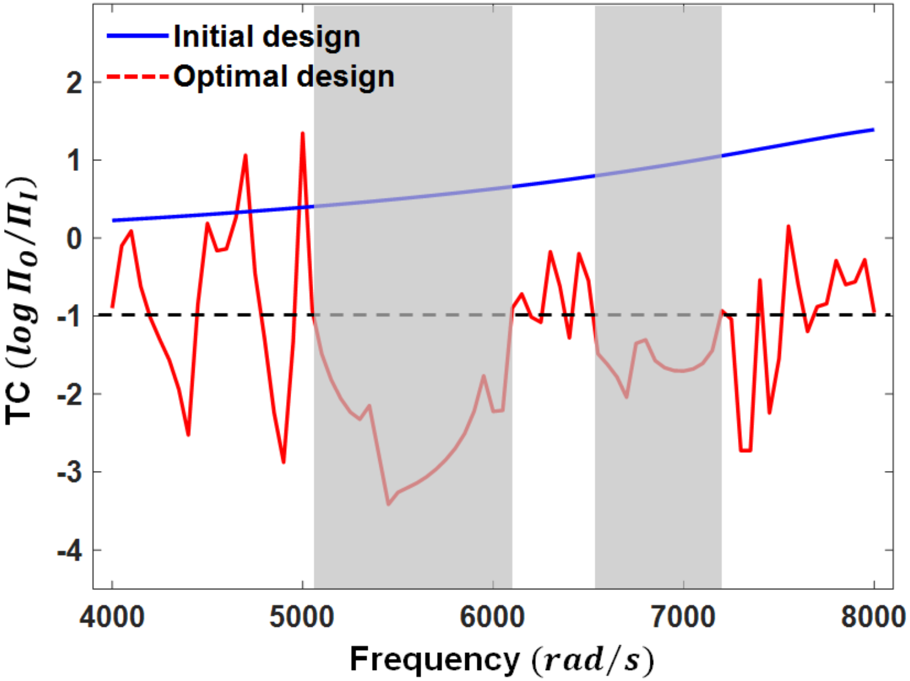


Band gap created by specified excitation frequency ω = 3000rad/s

Example 2 – 2 scale optimization for band gap tuning

ω = 6000rad/s





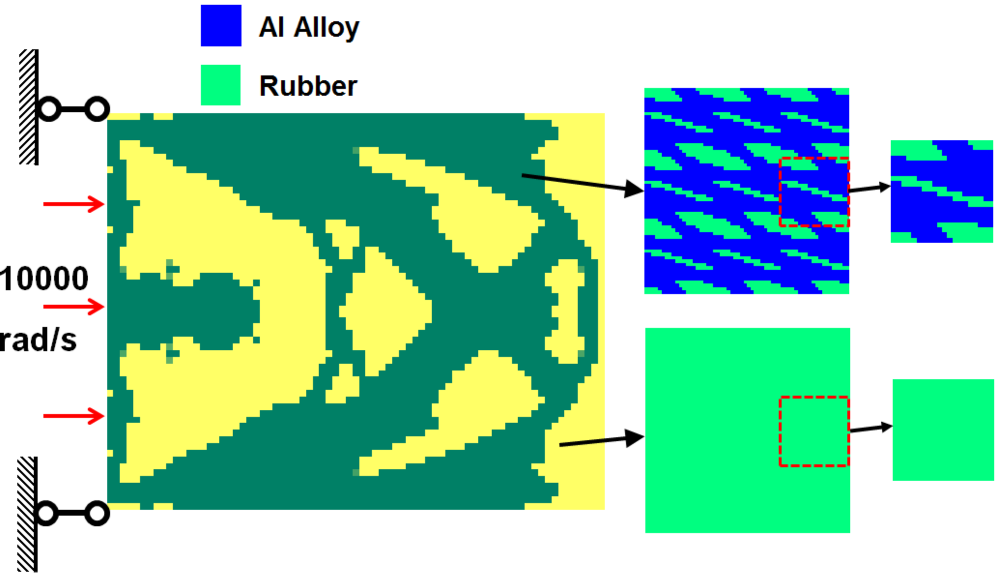
Optimum bi-metamaterial **macro-structural topology**

Optimum bi-basematerial **micro-structural topologies**

Band gap created by specified excitation frequency ω = 6000rad/s

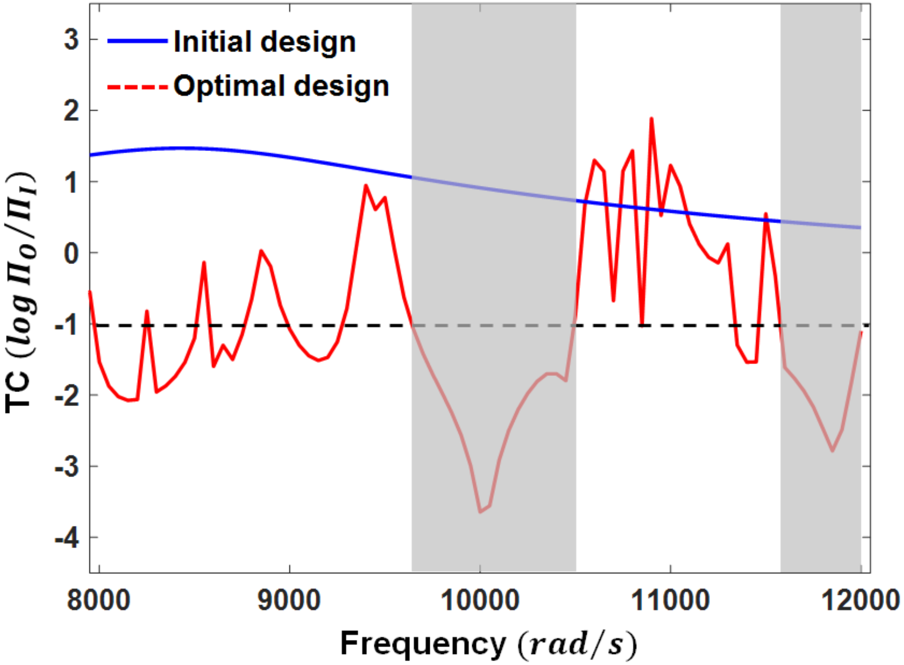
Example 2 – 2 scale optimization for band gap tuning

ω = 10000rad/s



Optimum bi-metamaterial **macro-structural topology**

Optimum bi-basematerial **micro-structural topologies**

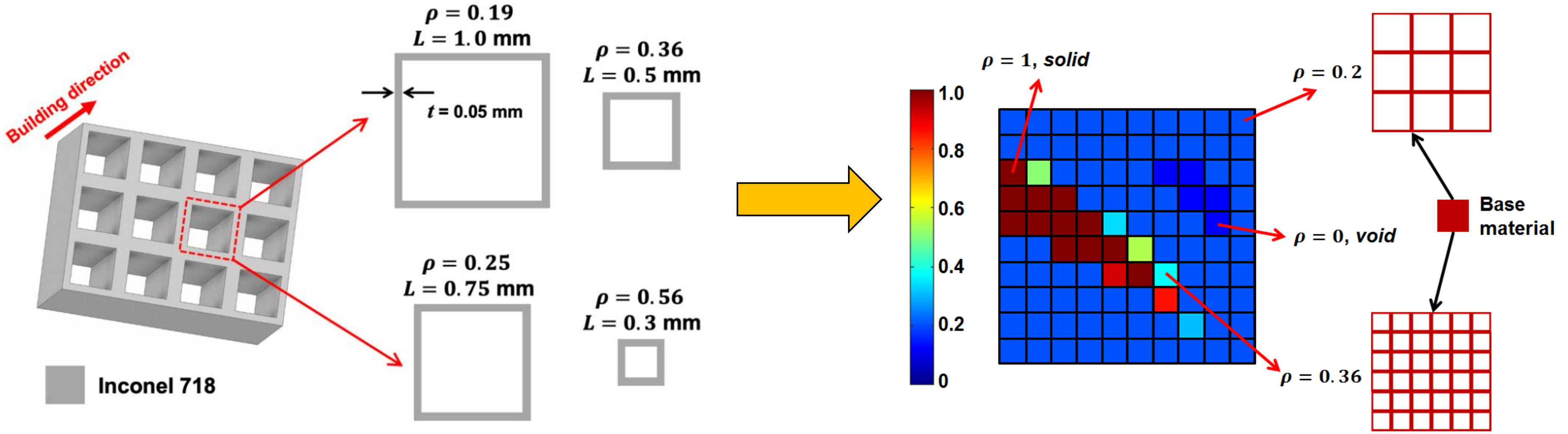


Band gap created by specified excitation frequency
ω = 3000rad/s

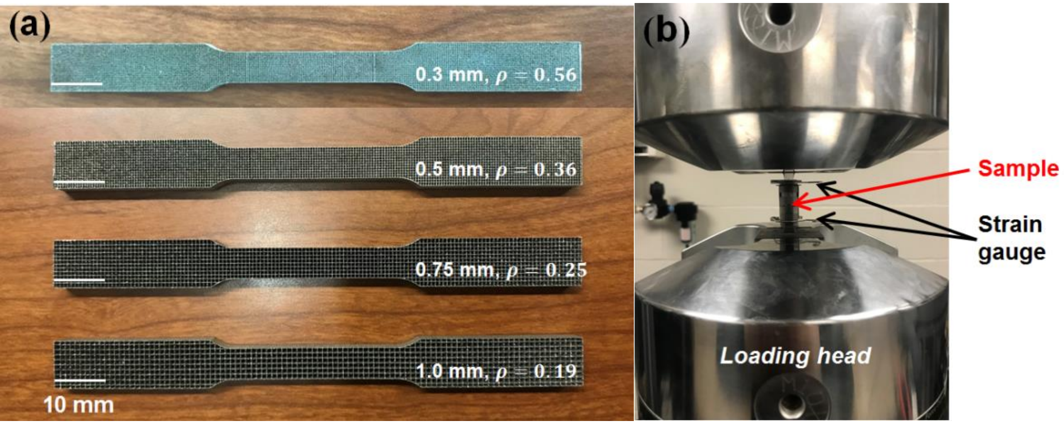
Extension of the work connecting additive manufactured lattice infills

Consideration of additive manufacturing

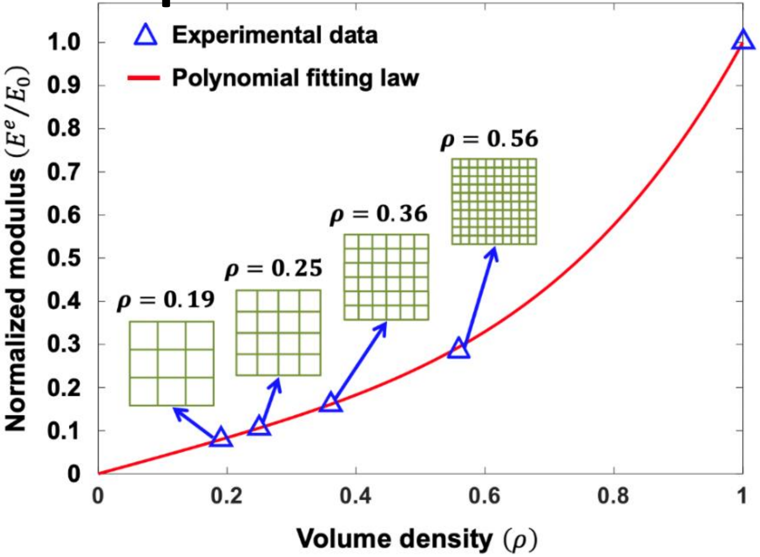
1、 Predefined microstructural pattern



2、 sample tensile test for obtaining equivalent material properties

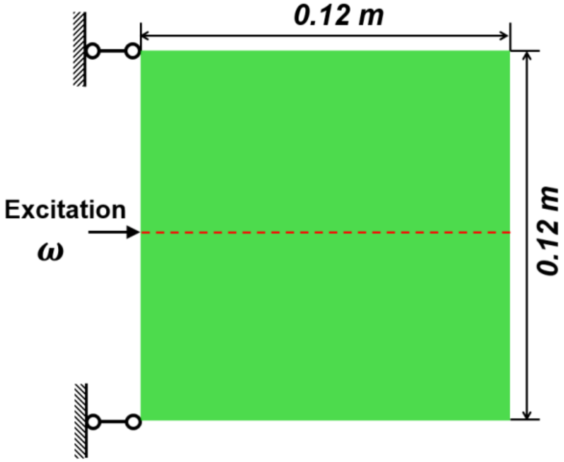


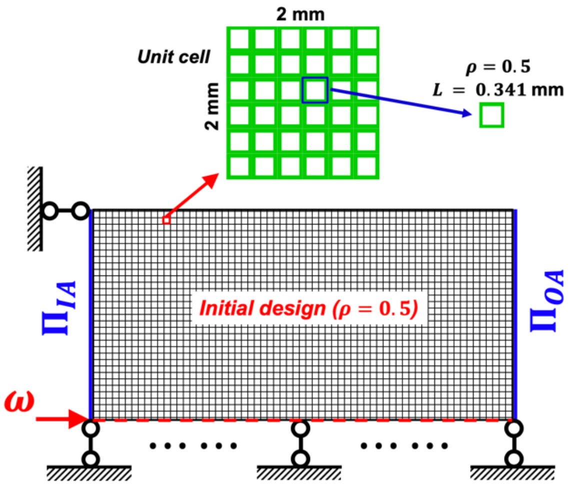
3、 LS fitting for obtaining interpolation formula

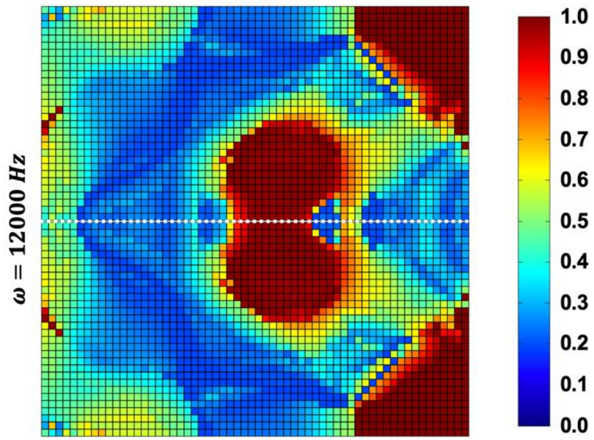


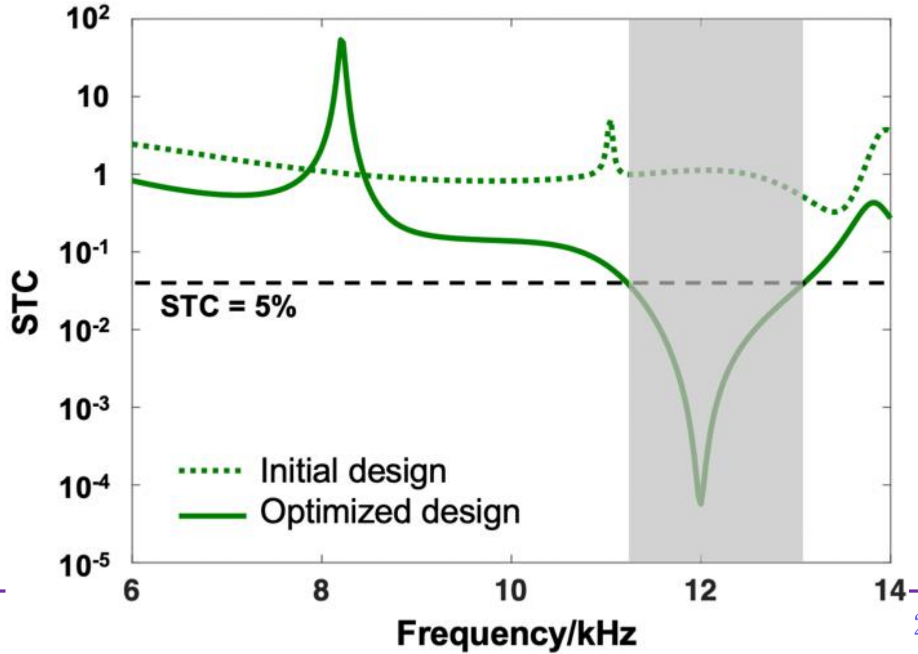
Consideration of additive manufacturing

4、Metamaterial topology optimization at macroscopic level for obtaining the desired band-gap



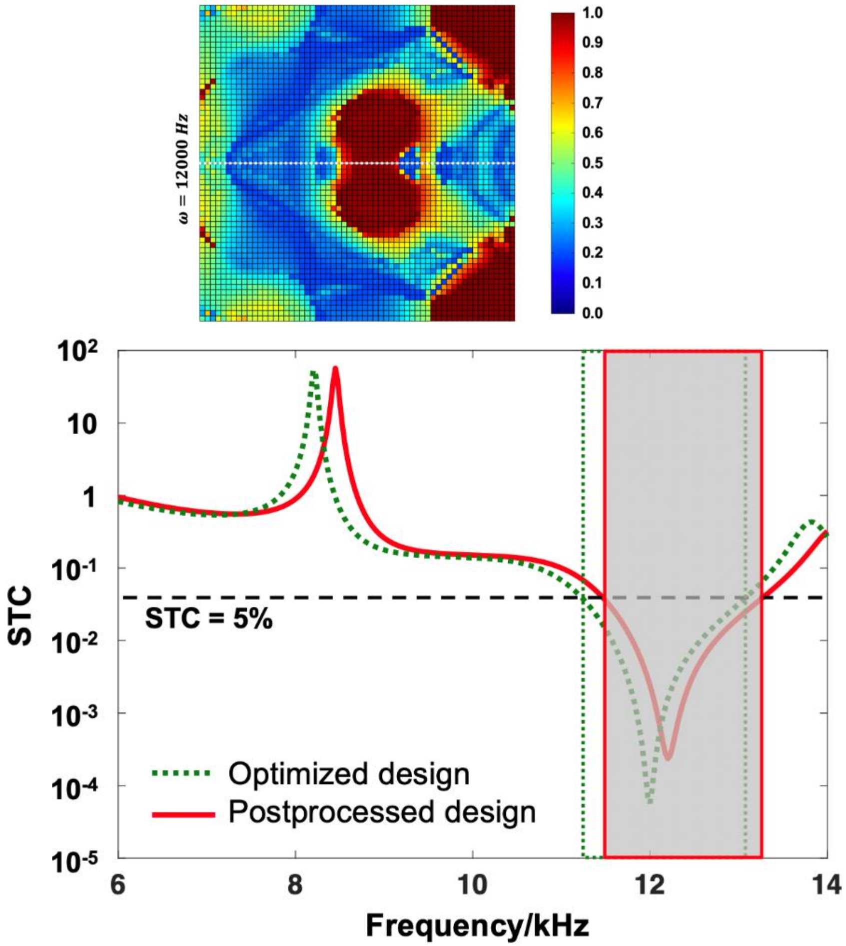


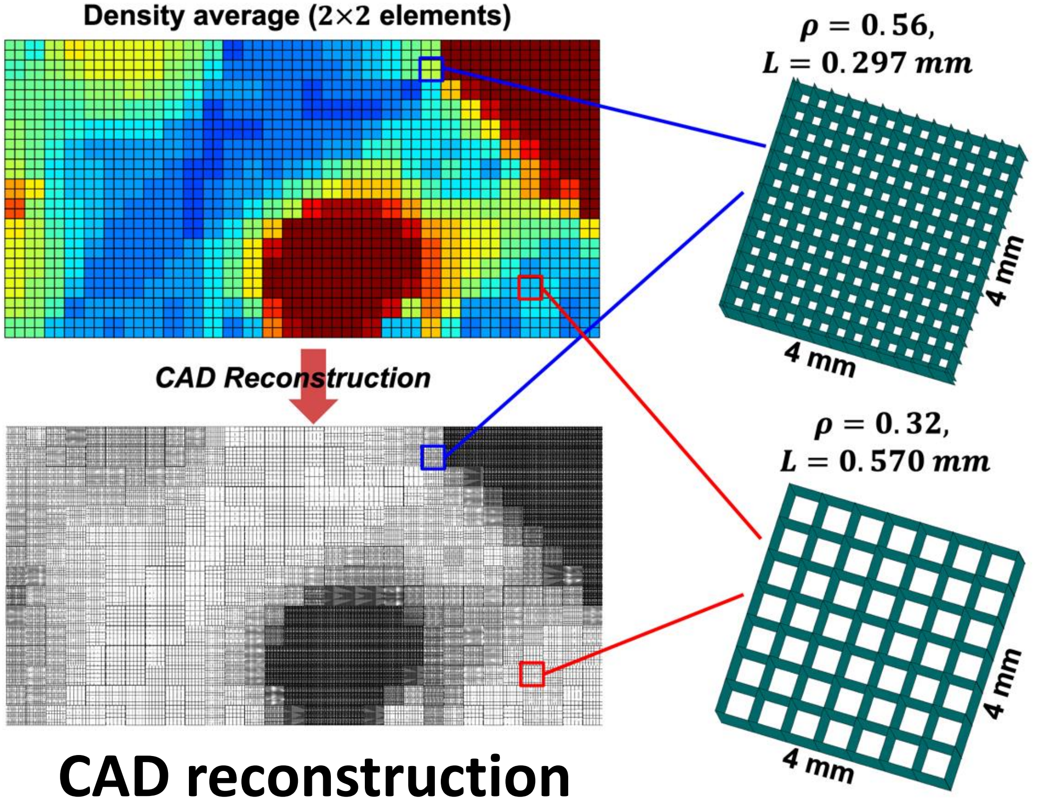




Consideration of additive manufacturing

5、Material density mapping and CAD reconstruction of the optimized structures using lattice infills





CAD reconstruction

lattice infills

Comparison of STC curves and band gap property of the original and postprocessed optimized material layout for frequency of 12000 Hz

Conclusions

- The accuracy of determining band-gap property based on transmission coefficients for elastic sound propagation of the macroscopic domain has been validated. Significant influence of the dimensional finitude of the macroscopic domain on the band-gap feature is also shown in this way .
- The proposed two-scale topology optimization methodology is able to create the desired band-gap properties at the targeted excitation frequency.
- It suggests that the proposed two-scale topology optimization can be a very promising tool to find the desired band-gap property for the macroscopic design domain with specified boundary conditions even though no reasonable initial guess of the material layout in the design domain is available.
- The method is easy extended for band-gap tuning design of the structure with additive manufactured lattice infills.

Thank you for your attention !