Layout optimization of simplified trusses

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Why do we need simplified trusses?

Basic

Optimized (-56%)

Optimized & simplified (-51%)

Fairclough et al, The Structural Engineer, 2019
Layout Optimization & Geometry Optimization

≈ Ground Structure Method (GSM)
≈ Truss topology optimization

\[
\begin{align*}
\min & \quad V = l^T a \\
\text{subject to} & \quad Bq = f \\
& \quad |q| - \sigma a < 0
\end{align*}
\]

- Linear programming – very fast and globally optimal
- Geometry optimization - adds node positions as variables. Non-convex, but uses layout optimization as starting point

Educational Python script available:
He et al, SMO, 2019

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Manual simplification

- Can identify structures that are reasonably simple and have low volume
- Geometry optimization can be used to improve a manually interpreted solution
- But, time consuming, and not always easy

Result of layout & geometry optimization

Manual simplification & geometry optimization

Fairclough et al, Proc. R. Soc. A, 2018
Automatic simplification approaches

**Approach 1**

Adding complexity constraints from the outset

Rigorous, but often very slow

**Approach 2**

Automatically post-processing an existing solution

Fast, but results not rigorous
New complexity constraints

\[
\begin{align*}
\text{min} & \quad V = l^T a \quad \text{minimising volume} \\
\text{subject to} & \quad Bq = f \quad \text{equilibrium} \\
& \quad |q| - \sigma a < 0 \quad \text{limiting stress}
\end{align*}
\]

\[
\hat{M}v_j - \sum_{i \in J_j} a_i \geq 0 \\
v_j \in \{0,1\} \\
\Sigma v \leq \eta \quad \text{allow up to } \eta \text{ joints}
\]

Now a Mixed Integer Linear Programming (MILP) problem
Simple cantilever example

- Maximum of 5 joints permitted – using MILP approach
- Fully connected ground structure used:
  - 99 nodes
  - 4851 potential members = new integer variables
  - 11.8 million pairs of potential members (to check)
  - 2.8 million of which intersect = additional constraints

Max. 5 joints
Preventing crossovers

\[
\begin{align*}
\text{min} & \quad V = t^T a \quad \text{minimising volume} \\
\text{subject to} & \quad Bq = f \quad \text{equilibrium} \\
& \quad |q| - \sigma a < 0 \quad \text{limiting stress} \\
& \quad v_j - \sum_{i \in J_j} a_i \geq 0, \quad v_j \in \{0, 1\} \quad v_j = 1 \text{ if joint } j \text{ exists} \\
& \quad \sum v \leq \eta \quad \text{allow up to } \eta \text{ joints} \\
\end{align*}
\]

4851 new integer variables

2.8 million new constraints implemented as ‘lazy’ constraints

\[
\begin{align*}
Mw_i - a_i & \geq 0 \quad w_i = 1 \text{ if member } i \text{ has non-zero area (i.e. exists)} \\
w_i & \in \{0, 1\} \\
w_h + w_i & \leq 1 \quad \text{intersecting members } h \text{ and } i \text{ can’t both exist}
\end{align*}
\]
Michell cantilever example – with ‘lazy’ crossover constraints

- Add new constraint:
  \[ w_h + w_i \leq 1 \]
  where \( h \) and \( i \) are the intersecting bars

- Potential bound rejected, search continues...

- 3 lazy constraints added before optimal solution obtained (cf 2.8 million)

- Total solution time massively reduced to tractable levels
Further developments:

- Multiple load cases
- Allow (and count) crossovers
- Enforce symmetry
- Minimum angle between members
- Speed up of 20 times (for smaller problems)

For details see:


- CPU times for shown results in range 10 – 290 seconds
Automatic simplification approaches

**Approach 1**

Adding complexity constraints from the outset

Rigorous, but often very slow

**Approach 2**

Automatically post-processing an existing solution

Fast, but results not rigorous
Fast post-processing approach

- Modified geometry optimization formulation:
  - Minimize number of members
  - Subject to limit on increase in volume
- 0-1 member on-off variables approximated via smooth Heaviside function

Layout optimization result (0.1s) → Geometry optimization result (0.3s) → Simplification result (~0.5s) → Geometry optimization result (~0.2s)
More complex example

Cuboid domain with 2 load-cases

Benchmark

Simplified
0.2% volume increase

Conventional 3 cubic cell
32% volume increase

Peregrine

- Plugin for Rhino/Grasshopper parametric CAD software
- Specify complex 3D geometries and interact in real time
- Link to download and to register for upcoming webinars (first on 9th July 2020):

  www.buildopt.org
LayOpt

- Launches **today**!
- No download - simply open and experiment
- Usable on desktop, mobile or tablet

[www.layopt.com](http://www.layopt.com)