Successive Iteration of Analysis and Design (SIAD) for large-scale eigenvalue topology optimization

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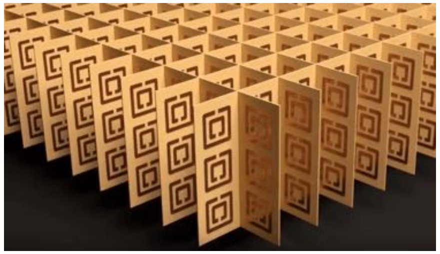
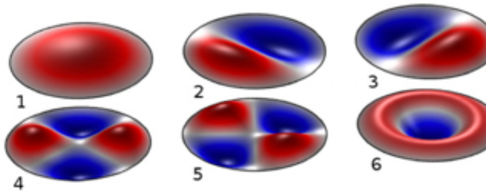
Background: Large scale eigenvalue-related TO

- Natural frequency/ forbidden interval/ frequency gap
- PnC and PtC metamaterials
- Linear buckling

Accurate analysis requires fine FE discretization.





Example: frequency maximization

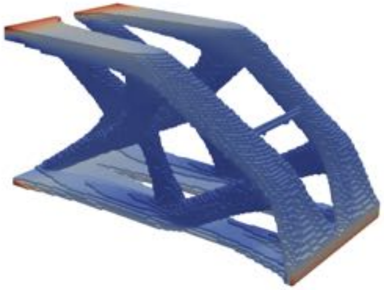
$$\max: \quad \omega_j$$

$$\text{s.t.} \quad \mathbf{K}\boldsymbol{\varphi}_j = \omega_j^2 \mathbf{M}\boldsymbol{\varphi}_j$$

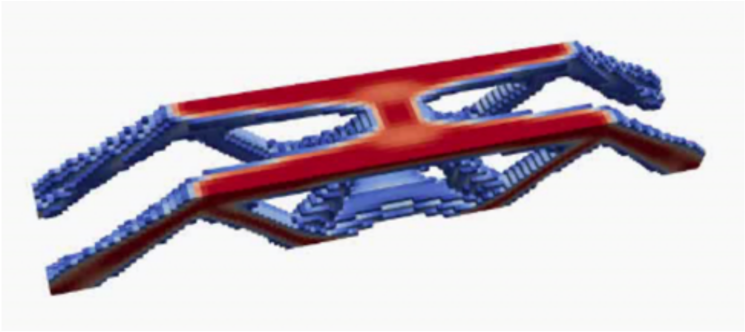
$$\sum_{e=1}^{N_\mathrm{e}} x_e V_e - f_\mathrm{v} V_0 \le 0$$

$$0 < \underline{x} \le x_e \le 1, \qquad e = 1, \cdots, N_\mathrm{e}$$

RAMP model $E_e = \dfrac{x_e}{1 + p(1 - x_e)} E_0, \quad \rho_e = x_e \rho_0$



Major issue: computational cost for analysis

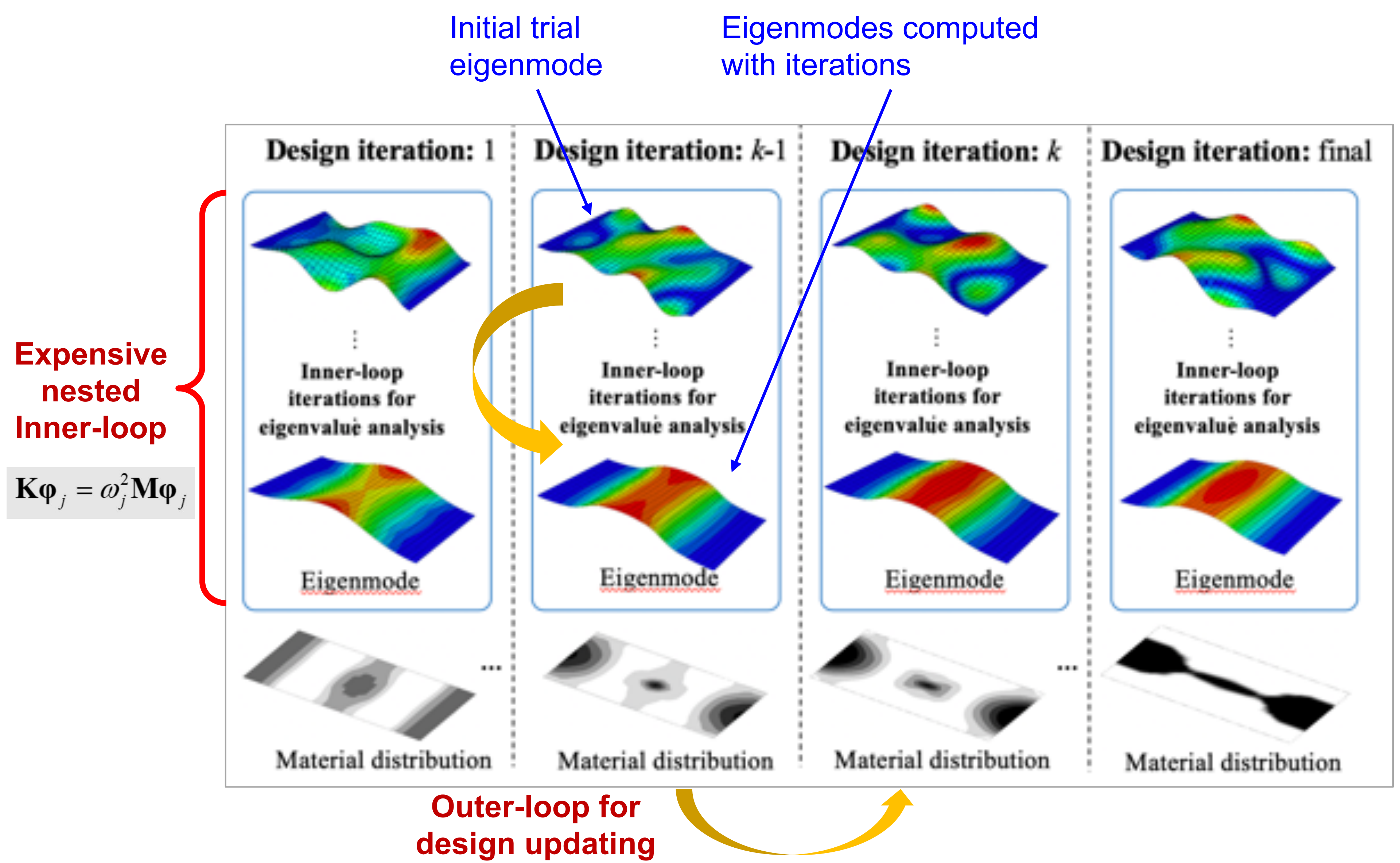


Cost for static compliance minimization of a 12.83 Million DOF model

Procedures	Computing time per iteration
Assembling global stiffness matrices	33 sec. (3.1%)
Finite element solution $\mathrm{Ku} = \mathrm{f}$	1005 sec. (95.5%)
Sensitivity analysis & optimization	14 sec. (1.3%)

For eigenvalue-related problems, the difficulty is even worsened by higher-order eigenvalues, and repeated eigenvalues.

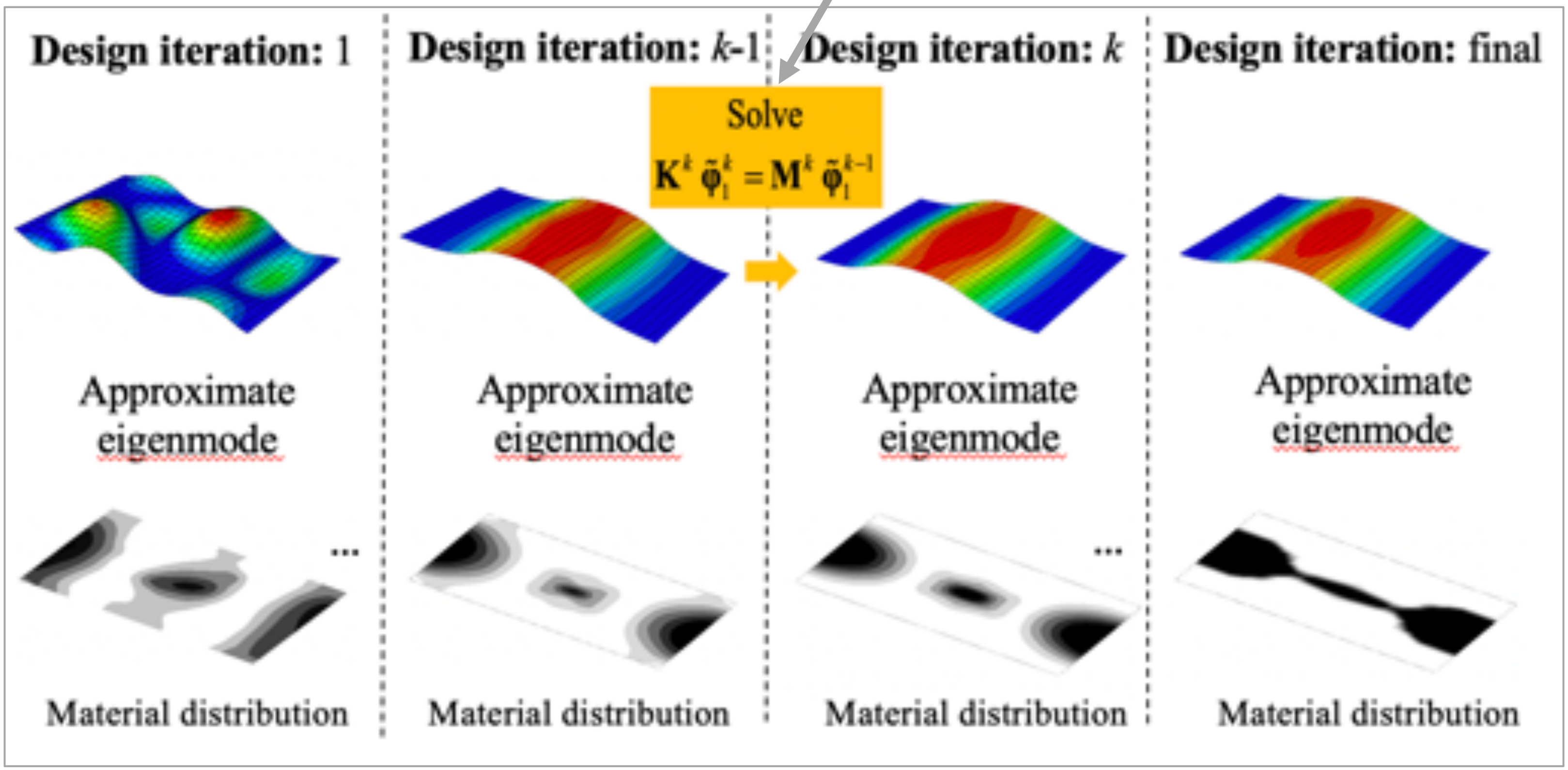
Conventional (nested double-loop) approach



Basic idea of SIAD method

Single loop iteration for eigenvalue analysis and design

Approximate eigenmodes improved through a linear equation



Kang et al. *CMAME* 2020; 362: 112847

Procedures of SIAD

☐ Case of fundamental frequency

Iteration k-1 → $\mathbf{K}^k \tilde{\boldsymbol{\varphi}}_1^k = \mathbf{M}^k \tilde{\boldsymbol{\varphi}}_1^{k-1}$ → **Iteration k**

- "Inverse iteration" (single step)
- Similar cost as in static FEA

Approx. mode $\tilde{\boldsymbol{\varphi}}_1^{k-1}$

Improved approx. mode $\tilde{\boldsymbol{\varphi}}_1^k$

$k=k+1$

Approx. eigenfrequency (Rayleigh quotient)

$$\tilde{\omega}_1^k = \sqrt{\left(\tilde{\boldsymbol{\varphi}}_1^k\right)^{\mathrm{T}} \mathbf{K}^k \tilde{\boldsymbol{\varphi}}_1^k}$$

Approx. Sensitivity

$$\frac{\partial \tilde{\omega}_1^k}{\partial x_e} = \frac{\left(\tilde{\boldsymbol{\varphi}}_1^k\right)^{\mathrm{T}} \left(\frac{\partial \mathbf{K}^k}{\partial x_e} - \left(\tilde{\omega}_1^k\right)^2 \frac{\partial \mathbf{M}^k}{\partial x_e}\right) \tilde{\boldsymbol{\varphi}}_1^k}{2\tilde{\omega}_1^k}$$

Design updating

Discussion on convergence (case of distinct eigenvalue)

Exact eigenpairs for (k-1)th design: $\lambda_1^{k-1}, \lambda_2^{k-1}, \ldots, \lambda_n^{k-1}$ $\quad$ $\boldsymbol{\varphi}_1^{k-1}, \boldsymbol{\varphi}_2^{k-1}, \ldots, \boldsymbol{\varphi}_n^{k-1}$

for kth design: $\lambda_1^{k}, \lambda_2^{k}, \ldots, \lambda_n^{k}$ $\quad$ $\boldsymbol{\varphi}_1^{k}, \boldsymbol{\varphi}_2^{k}, \ldots, \boldsymbol{\varphi}_n^{k}$

$$\Rightarrow \begin{cases} \Delta\lambda_j^k = \left(\boldsymbol{\varphi}_j^{k-1}\right)^{\mathrm{T}} \left(\Delta\mathbf{K}^k - \lambda_j^{k-1}\Delta\mathbf{M}^k\right)\boldsymbol{\varphi}_j^{k-1} \\ 2\left(\boldsymbol{\varphi}_j^{k-1}\right)^{\mathrm{T}} \mathbf{M}^{k-1}\Delta\boldsymbol{\varphi}_j^k = -\left(\boldsymbol{\varphi}_j^{k-1}\right)^{\mathrm{T}} \Delta\mathbf{M}^k\boldsymbol{\varphi}_j^{k-1} \end{cases}$$

Approximate eigenpairs for (k-1)th and kth designs: $\tilde{\boldsymbol{\varphi}}^{k-1} = \sum_{j=1}^{n} a_j^k \boldsymbol{\varphi}_j^k$ $\quad$ $\tilde{\boldsymbol{\varphi}}^{k-1} = \sum_{j=1}^{n} a_j^{k-1} \boldsymbol{\varphi}_j^{k-1}$

Define: $\eta_j^{k-1} = \dfrac{a_j^{k-1}}{a_1^{k-1}}, j = 1, 2, \ldots, n$ $\qquad$ $\eta_j^{k} = \dfrac{\lambda_1^k a_j^k}{\lambda_j^k a_1^k}, j = 1, 2, \ldots, n$

We prove that convergence can be ensured on the condition that design change in each iteration is sufficiently small:

$$\eta_j^k \approx \frac{\lambda_1^{k-1} a_j^{k-1}}{\lambda_j^{k-1} a_1^{k-1}} < \eta_j^{k-1}, \quad j = 2, 3, \ldots, n$$

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Case of higher-order frequencies

Subspace iteration-like procedures:

$$\mathbf{K}^k \tilde{\tilde{\boldsymbol{\Psi}}}^k = \mathbf{M}^k \tilde{\boldsymbol{\Psi}}^{k-1}$$

Inverse iteration (one step) of subspace

$$\left(\mathbf{K}^*\right)^k \tilde{\mathbf{A}}^k = \left(\mathbf{M}^*\right)^k \tilde{\mathbf{A}}^k \tilde{\boldsymbol{\Lambda}}^k$$

Eigenpairs in subspace (low-dimensional)

$$\left(\mathbf{M}^*\right)^k = \left(\tilde{\tilde{\boldsymbol{\Psi}}}^k\right)^{\mathrm{T}} \mathbf{M}^k \tilde{\tilde{\boldsymbol{\Psi}}}^k$$

$$\left(\mathbf{K}^*\right)^k = \left(\tilde{\tilde{\boldsymbol{\Psi}}}^k\right)^{\mathrm{T}} \mathbf{K}^k \tilde{\tilde{\boldsymbol{\Psi}}}^k$$

$$\tilde{\boldsymbol{\Psi}}^k = \tilde{\tilde{\boldsymbol{\Psi}}}^k \tilde{\mathbf{A}}^k$$

Mode updating

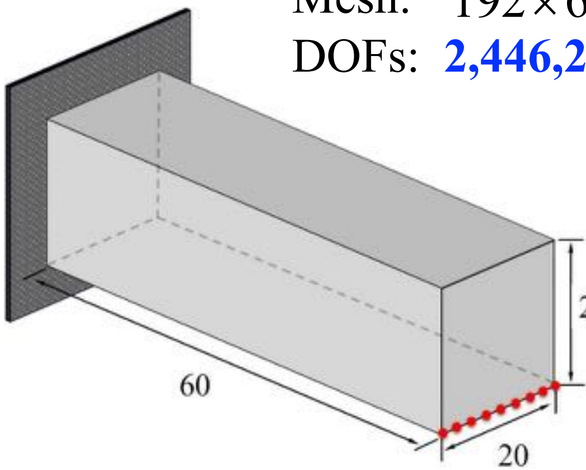
* For repeated eigenvalues, a sub-problem of eigenvalue analysis needs to be solved to find Gateaux differentiable eigenmodes:

$$\mathbf{F}\mathbf{c}_i = \lambda_i' \mathbf{c}_i \qquad \text{where} \quad \mathbf{F} = \left(\tilde{\boldsymbol{\Phi}}^k\right)^{\mathrm{T}} \left(\partial \mathbf{K}^k / \partial x_e - \left(\tilde{\omega}_r^k\right)^2 \partial \mathbf{M}^k / \partial x_e\right) \tilde{\boldsymbol{\Phi}}^k$$

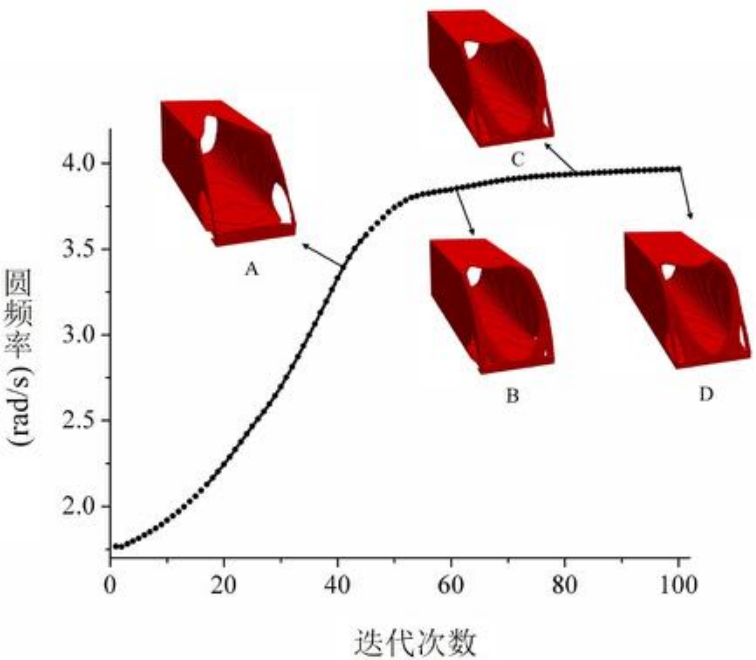
$$\bar{\tilde{\boldsymbol{\Phi}}}^k = \tilde{\boldsymbol{\Phi}}^k \mathbf{C}$$

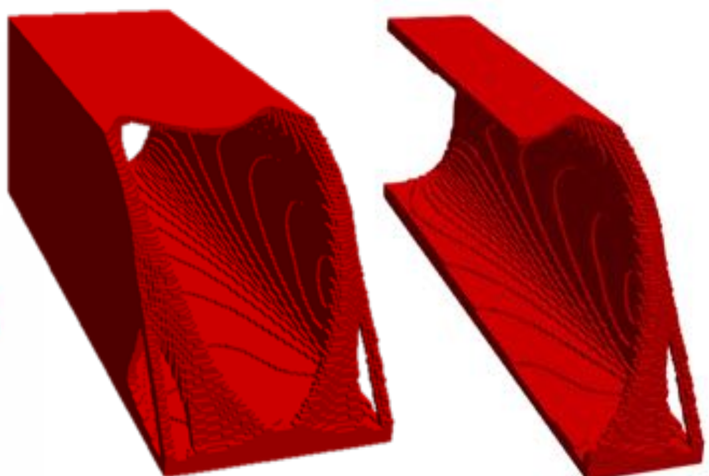
Example: Fundamental frequency maximization

Mesh: $192 \times 64 \times 64$
DOFs: **2,446,275**

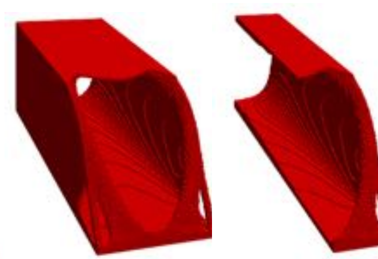


	Number of full-scale equation solutions	Computing time (sec.)
SIAD	100	**22,980**
Double-loop	1058	**247,680**



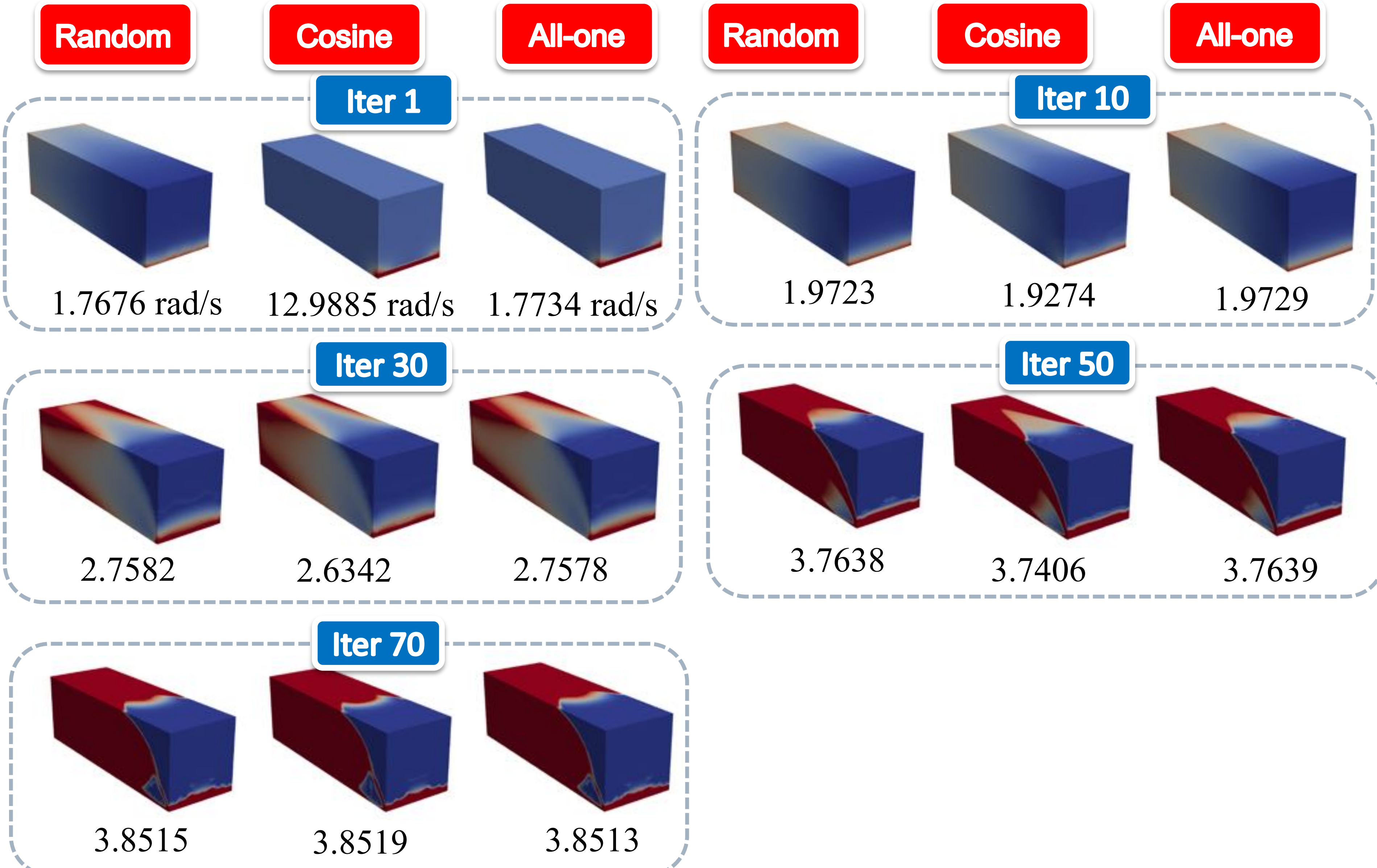


SIAD
3.97 rad/s



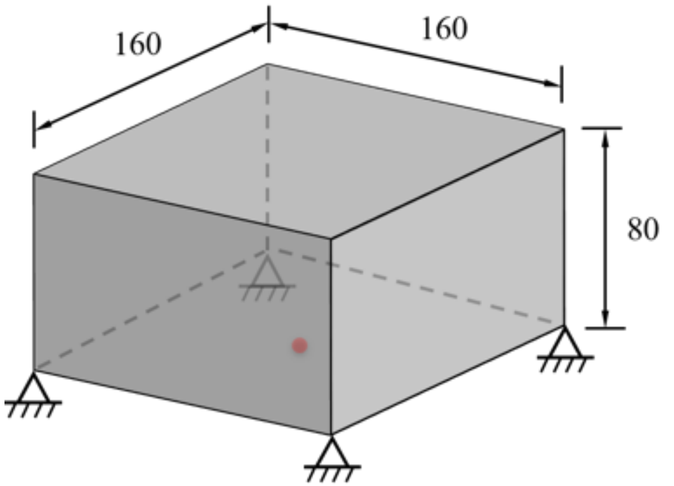
Double-loop
4.08 rad/s

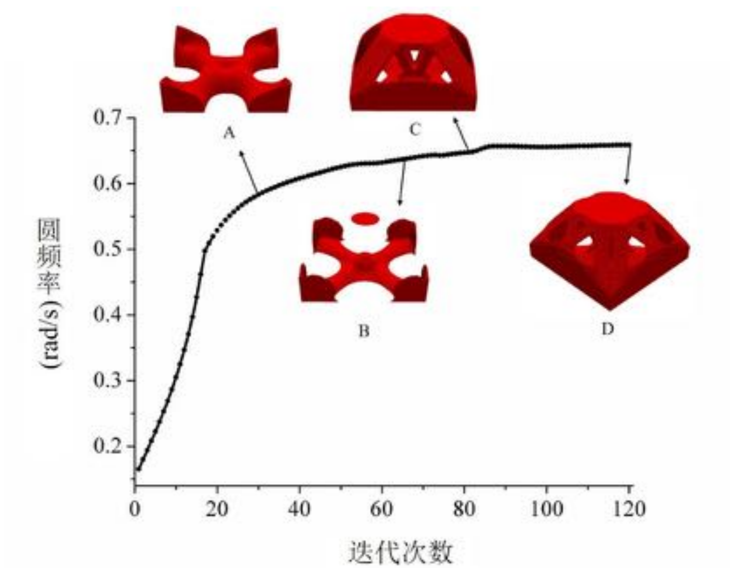
Insensitive to initial trial eigenvectors



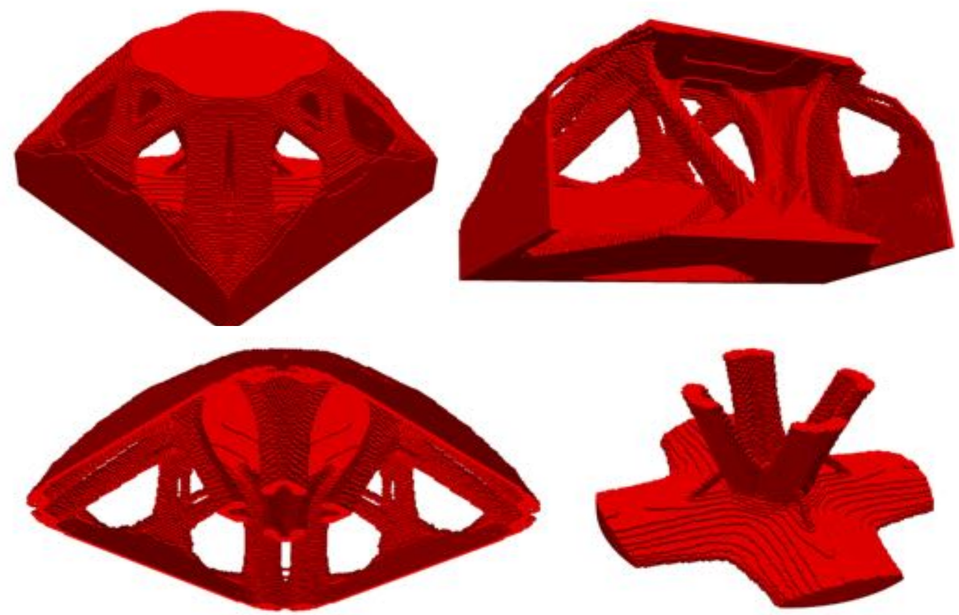
Example: Fundamental frequency maximization

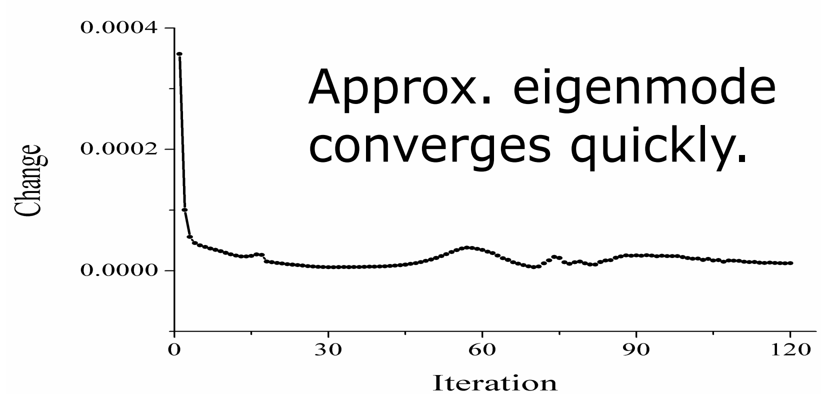
Mesh: 160×160×80
DOFs: **6,298,803**





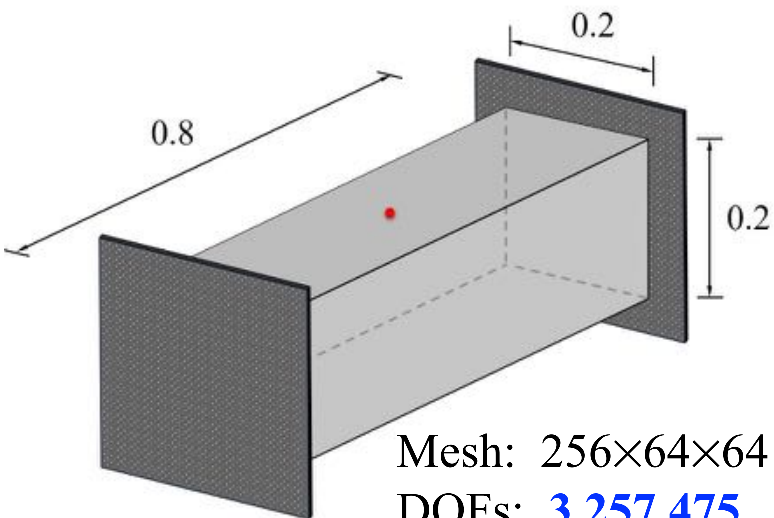
Optimized design obtained with
random initial trial eigenmodes





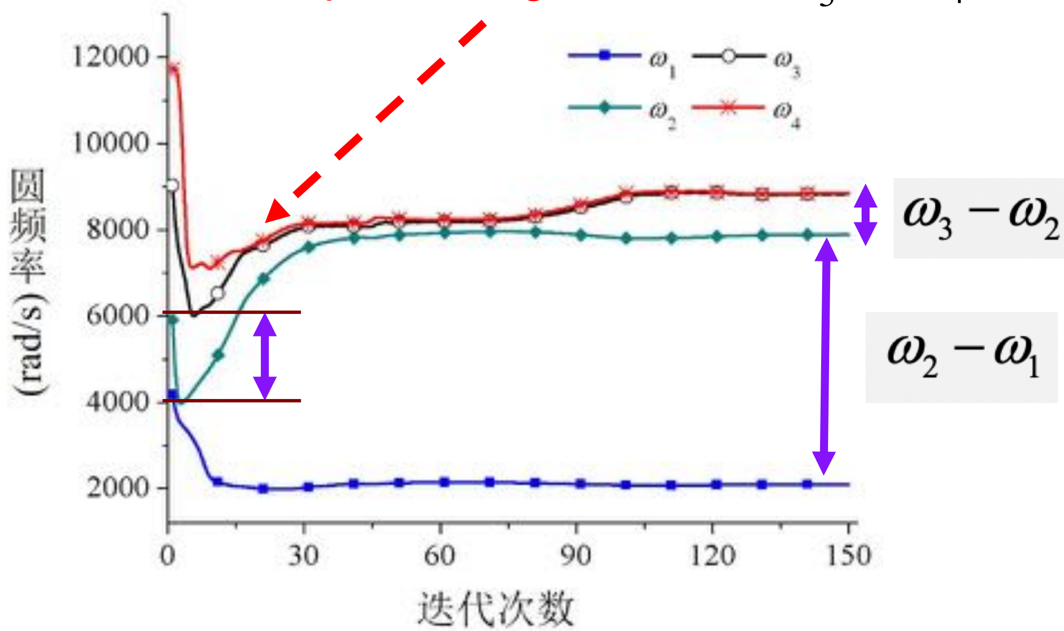
Approx. eigenmode converges quickly.

Example: frequency gap maximization



Mesh: 256×64×64
DOFs: **3,257,475**

Repeated eigenvalues: $\omega_3 = \omega_4$



$$\max: \ \omega_2 - \omega_1$$

$$\text{s.t.} \quad \omega_3 - \omega_2 \geq \beta \qquad \beta = 1000$$

$$\sum_{e=1}^{N_e} x_e V_e - f_v V_0 \leq 0$$

$$0 < \underline{x} \leq x_e \leq 1, \quad e = 1, \cdots, N_e$$

Conclusions

- SIAD implements **concurrent convergence** of eigenvalue analysis and design optimization.
- Affordable solution of eigenvalue optimization with **millions of DOFs** on a desktop computer.

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THANK YOU !