Multi-material topology optimization of lattice structures using geometry projection

TOP Webinar

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Truss lattices

- Open cell
- Manufacturability
- Functional porosity
- Modeling with 1D elements
- High degree of redundancy
Multi-material lattices

- Materials with different modulus-to-density ratios → better stiffness for same weight –or– lighter for same stiffness
- One strut / one material → easier to manufacture
- Potential for multi-functionality
Graphical summary

Initial design

Optimal design

- Near-zero length bars made of mixture of all available materials
- Continuum mesh
- Specified material symmetries

Spatial layout of struts (including removal) and choice of best material for each strut

$E_1 = 6.5$
$\gamma_1 = 0.55$

$E_2 = 5.0$
$\gamma_2 = 0.45$

$E_3 = 4.5$
$\gamma_3 = 0.35$
Existing approaches to lattice design

- **Ground structure TO**
  - Modeling accuracy: Lower
  - Computational cost: Lower
  - Design freedom: Lower
  - Manufacturability: Lower

- **Density-based TO**
  - Modeling accuracy: Higher
  - Computational cost: Higher
  - Design freedom: Higher
  - Manufacturability: Lower

(Sigmund, 1994) (Gibianski and Sigmund, 2000)
Geometry projection: the idea

\[ \rho = \frac{|B_p^r \cup \omega|}{B_p^r} \]

Norato et al. (2004, 2015), Bell et al. (2012)

\[ C(z, p) = C_{void} + \sum_{i=1}^{N_m} (C_i - C_{void}) \rho_{eff}^i(z, p) \]

Ersatz material
Kazemi and Norato (2018)
Comparison

<table>
<thead>
<tr>
<th></th>
<th>Ground structure TO</th>
<th>Density-based TO</th>
<th>Geometry Projection</th>
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<tbody>
<tr>
<td>Modeling accuracy</td>
<td>Lower</td>
<td>Higher</td>
<td>Higher</td>
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<tr>
<td>Computational cost</td>
<td>Lower</td>
<td>Higher</td>
<td>Higher</td>
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<tr>
<td>Design freedom</td>
<td>Lower</td>
<td>Higher</td>
<td>Medium</td>
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<tr>
<td>Manufacturability</td>
<td>Higher</td>
<td>Lower</td>
<td>Higher</td>
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</tbody>
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Symmetry and No-cut constraint
Optimization problem

\[
\min_{\mathbf{z}} f(\mathbf{z})
\]
subject to
\[
a(\mathbf{u}^{(kl)}(\mathbf{z}), \mathbf{v}) = l(\mathbf{v}, e^{0(kl)}), \forall \mathbf{v} \in \mathcal{V}_0, \mathbf{u}^{(kl)} \in \mathcal{V}
\]
\[
g_w(\mathbf{z}) \leq w^*_f
\]
\[
g_d(\mathbf{z}) \leq \varepsilon_d^{(l)}
\]
\[
g_m(\mathbf{z}) \leq \varepsilon_m^{(l)}
\]
\[
g_n(\mathbf{z}) \leq \varepsilon_n
\]
\[
\mathbf{x}_{b_0}, \mathbf{x}_{b_f} \in \Omega
\]
\[
0.0 \leq \alpha_i^b \leq 1.0,
\]

**Bulk modulus**
\[
f(\mathbf{z}) \equiv -K(\mathbf{z})
\]
\[
K(\mathbf{z}) := \frac{1}{3} C_{1111} + \frac{2}{3} C_{1122}
\]

**Poisson’s ratio**
\[
f(\mathbf{z}) \equiv \nu(\mathbf{z})
\]
\[
\nu(\mathbf{z}) = \frac{C_{1122}}{2(C_{1122} + C_{2121})}
\]

Initial design

Cubic symmetry

\[
E_1 = 10 \\
\gamma_1 = 0.9
\]

\[
E_2 = 7.5 \\
\gamma_2 = 0.675
\]

\[
E_3 = 5 \\
\gamma_3 = 0.45
\]
Bulk modulus maximization / 2 materials

Color of struts indicates the choice of material.
Comparison to Hashin-Shtrikman-Walpole bounds (Gibianski and Sigmund, 2000)
Bulk modulus maximization / 3 materials

- $0.0667$  
- $0.0889$  
- $0.0944$  
- $0.094$  
- $0.12$  
- $0.169$  
- $0.1$  
- $0.155$

Color of struts indicates the choice of material.
Bulk modulus maximization / 3 materials

Bulk modulus maximization design for $w_f = 0.0667$
Poisson’s ratio min. / 2 and 3 materials

iso

side

$w_f$ 0.0333 0.0389 0.0444 0.05 0.0667 0.0389
$\nu$ -0.057 -0.118 -0.204 -0.639 -1.573 -0.426

Color of struts indicates the choice of material.
Poisson’s ratio min. / 3 materials

Poisson’s ratio minimization design for $w_f = 0.0389$
Epilogue

- In practice, polymeric materials for multi-material printers have similar physical densities → optimizer uses only stiffer material.

- Possibility 1: impose minimum angle constraints to prevent overlaps among struts that make manufacturing more difficult.

- Possibility 2: struts that are hollow or fiber-reinforced (work in progress)
Thank you!