Multi-material topology optimization of lattice structures using geometry projection

TOP Webinar

Hesaneh Kazemi and Julián Norato

Department of Mechanical Engineering, University of Connecticut

June 2nd, 2020

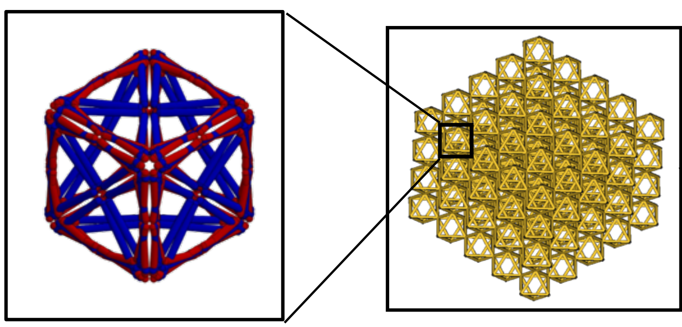
STRUCTURAL OPTIMIZATION LABORATORY

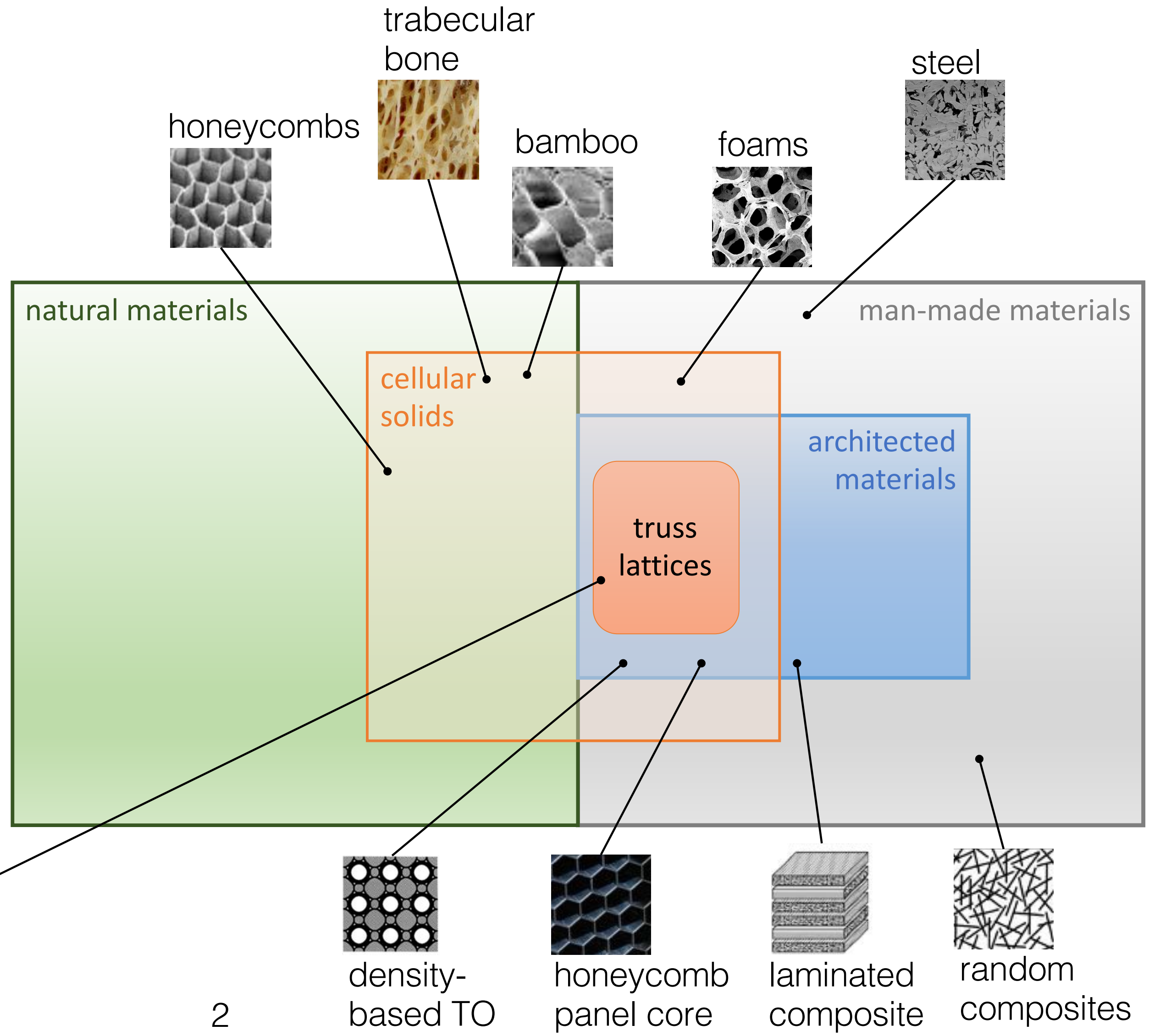
NSF

UCONN
UNIVERSITY OF CONNECTICUT

Truss lattices

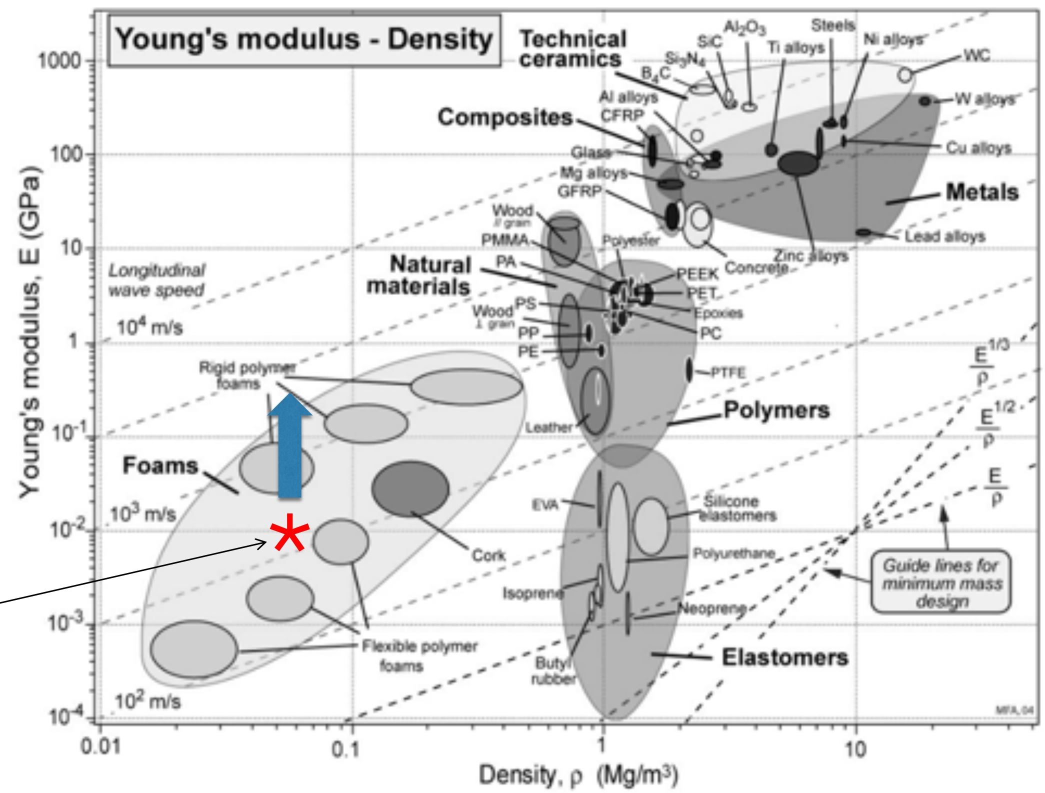
- Open cell
  - Manufacturability
  - Functional porosity
- Modeling with 1D elements
- High degree of redundancy



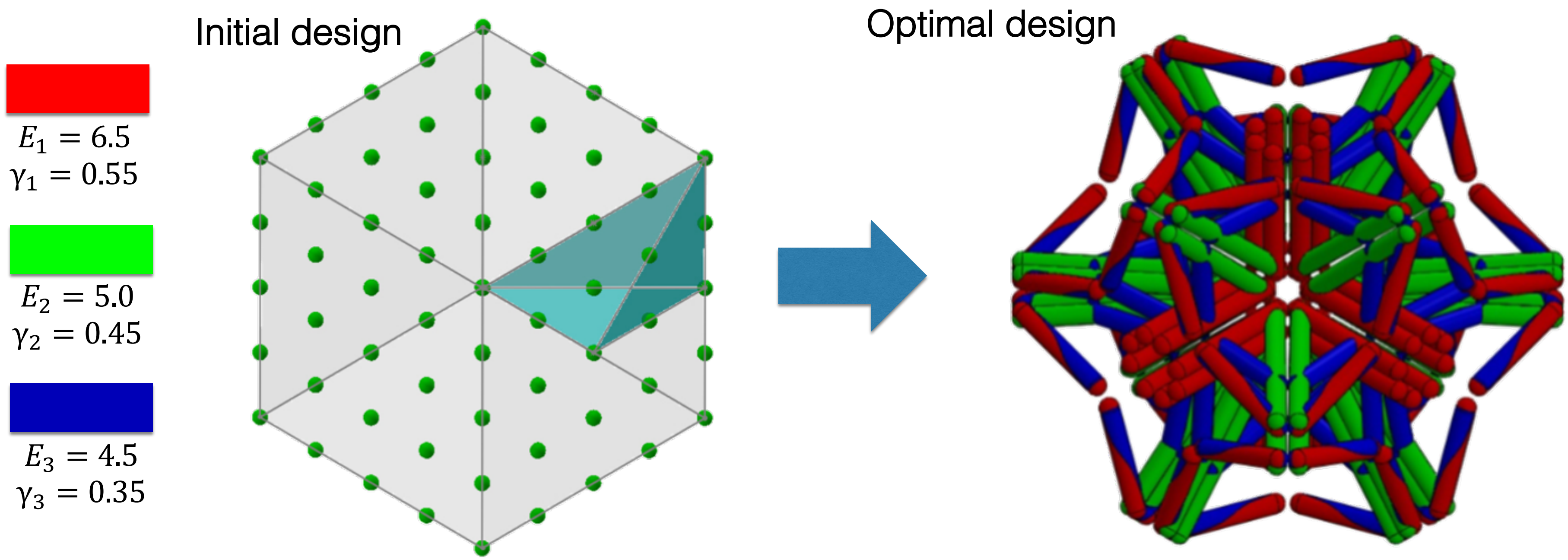


Multi-material lattices

- Materials with different modulus-to-density ratios → better stiffness for same weight –or– lighter for same stiffness
- One strut / one material → easier to manufacture
- Potential for multi-functionality



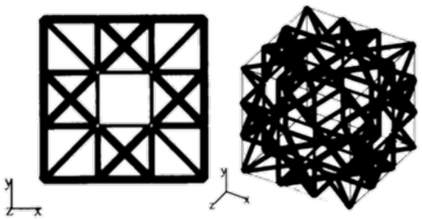
Graphical summary



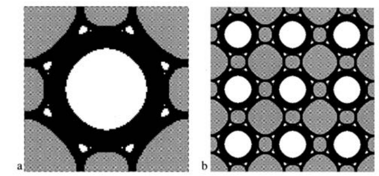
- Near-zero length bars made of mixture of all available materials
- Continuum mesh
- Specified material symmetries

Spatial layout of struts (including removal)
and
choice of best material for each strut

Existing approaches to lattice design



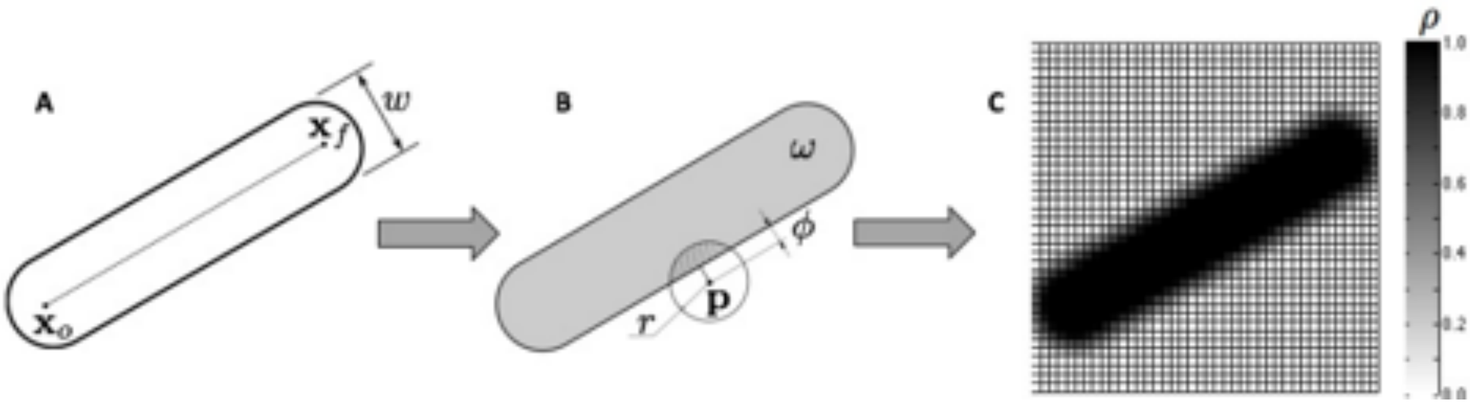
(Sigmund, 1994)



(Gibianski and Sigmund, 2000)

	Ground structure TO	Density-based TO
Modeling accuracy	Lower	Higher
Computational cost	Lower	Higher
Design freedom	Lower	Higher
Manufacturability	Higher	Lower

Geometry projection: the idea



$$\rho = \frac{|\mathbf{B}^r_{\mathbf{p}} \cup \omega|}{\mathbf{B}^r_{\mathbf{p}}}$$

Norato et al. (2004, 2015), Bell et al. (2012)

$$\mathbb{C}(\mathbf{z},\mathbf{p}) = \mathbb{C}_{void} + \sum_{i=1}^{N_m} (\mathbb{C}_i - \mathbb{C}_{void})\, \rho^i_{eff}(\mathbf{z},\mathbf{p})$$

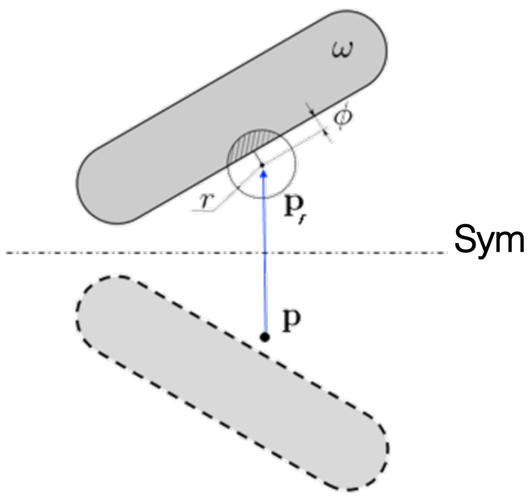
Ersatz material

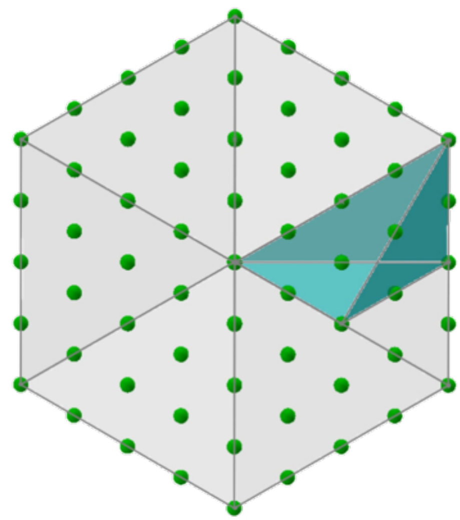
Kazemi and Norato (2018)

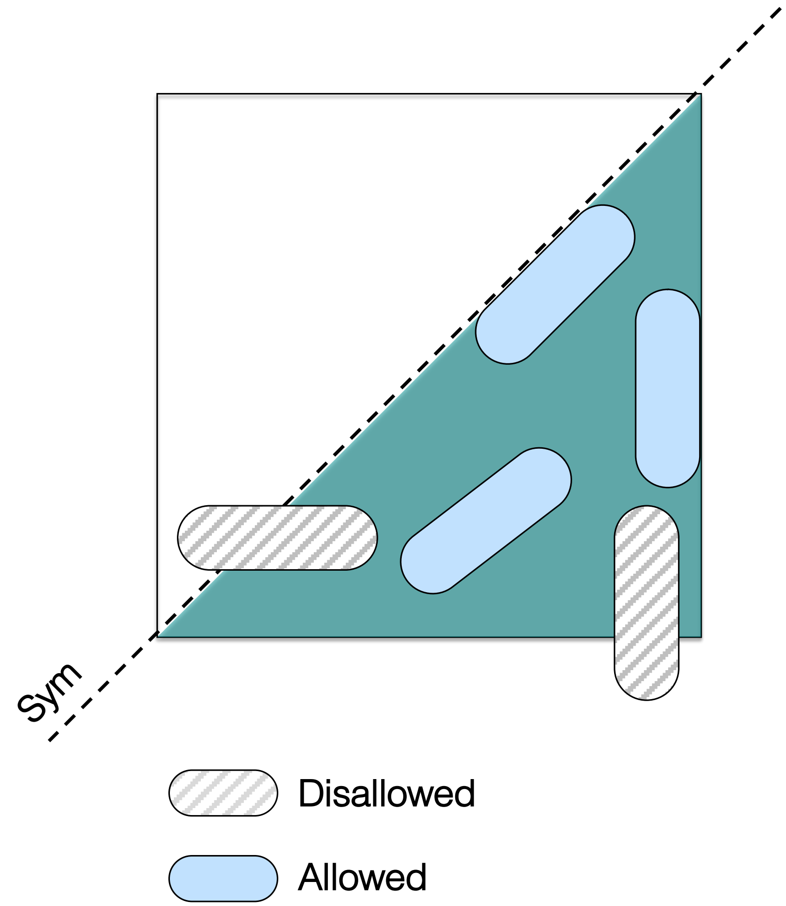
Comparison

	Ground structure TO	Density-based TO	Geometry Projection
Modeling accuracy	Lower	Higher	Higher
Computational cost	Lower	Higher	Higher
Design freedom	Lower	Higher	Medium
Manufacturability	Higher	Lower	Higher

Symmetry and No-cut constraint







Optimization problem

$$\min_{\mathbf{z}} f(\mathbf{z})$$

subject to

$$\mathrm{a}(\mathbf{u}^{(kl)}(\mathbf{z}),\mathbf{v}) = \mathrm{l}(\mathbf{v},\boldsymbol{\epsilon}^{0(kl)}), \forall \mathbf{v} \in \mathscr{U}_0, \mathbf{u}^{(kl)} \in \mathscr{U}$$

$$g_w(\mathbf{z}) \leq w_f^*$$

$$g_d(\mathbf{z}) \leq \varepsilon_d^{(I)}$$

$$g_m(\mathbf{z}) \leq \varepsilon_m^{(I)}$$

$$g_n(\mathbf{z}) \leq \varepsilon_n$$

$$\mathbf{x}_{b_0}, \mathbf{x}_{b_f} \in \Omega$$

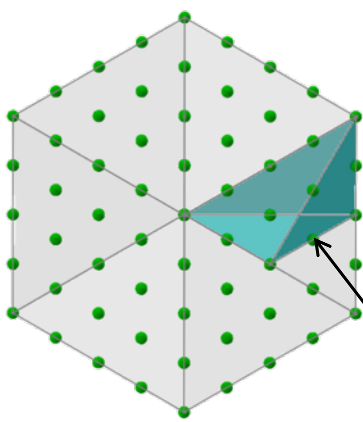
$$0.0 \leq \alpha_i^b \leq 1.0,$$

Bulk modulus $f(\mathbf{z}) \equiv -K(\mathbf{z})$

$$K(\mathbf{z}) := \frac{1}{3}C_{1111} + \frac{2}{3}C_{1122}$$

Poisson's ratio $f(\mathbf{z}) \equiv \nu(\mathbf{z})$

$$\nu(\mathbf{z}) = \frac{C_{1122}}{2(C_{1122} + C_{2121})}$$



Initial design

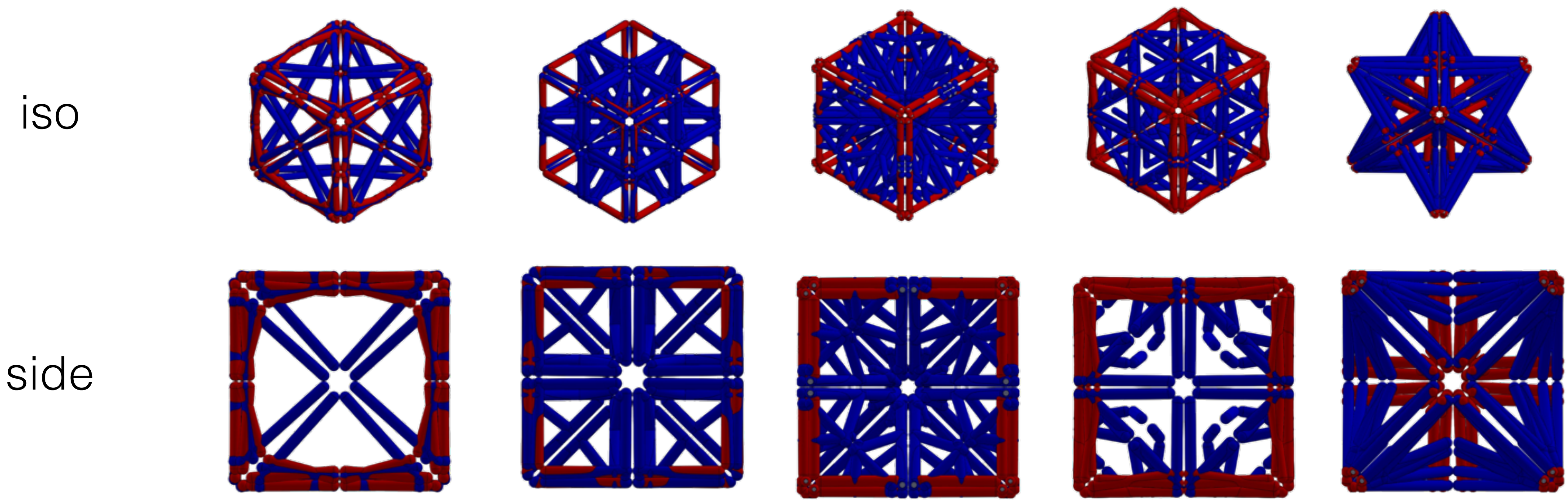
Cubic symmetry

$E_1 = 10$
$\gamma_1 = 0.9$

$E_2 = 7.5$
$\gamma_2 = 0.675$

$E_3 = 5$
$\gamma_3 = 0.45$

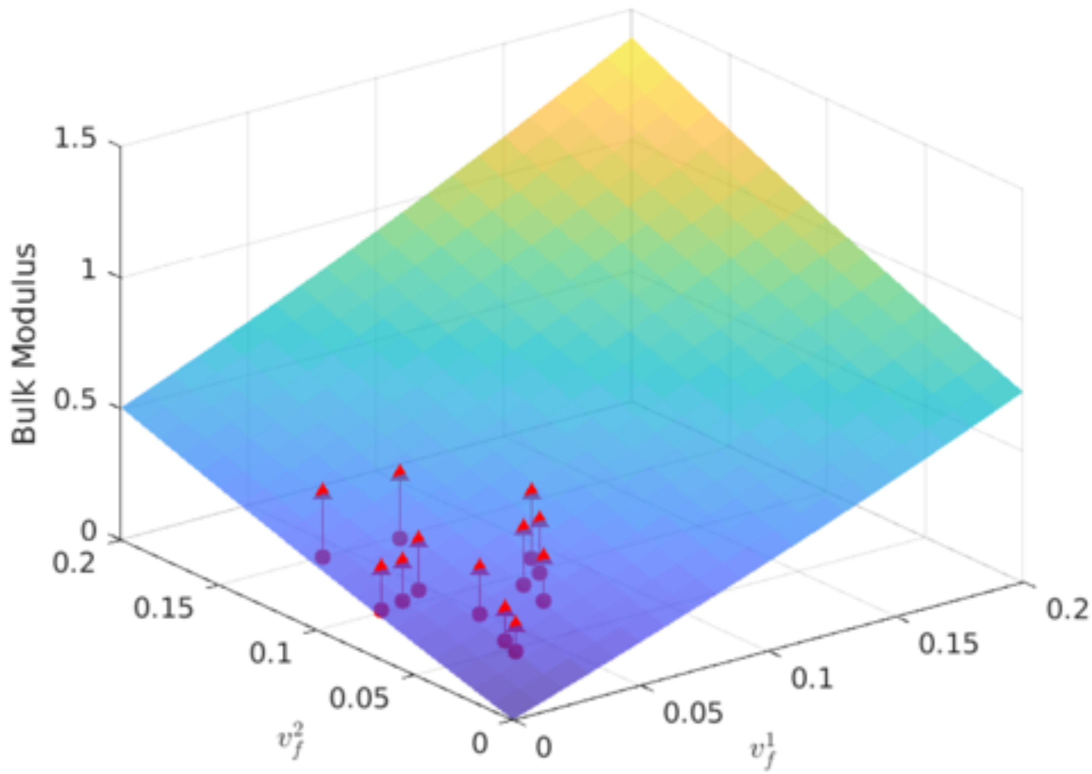
Bulk modulus maximization / 2 materials



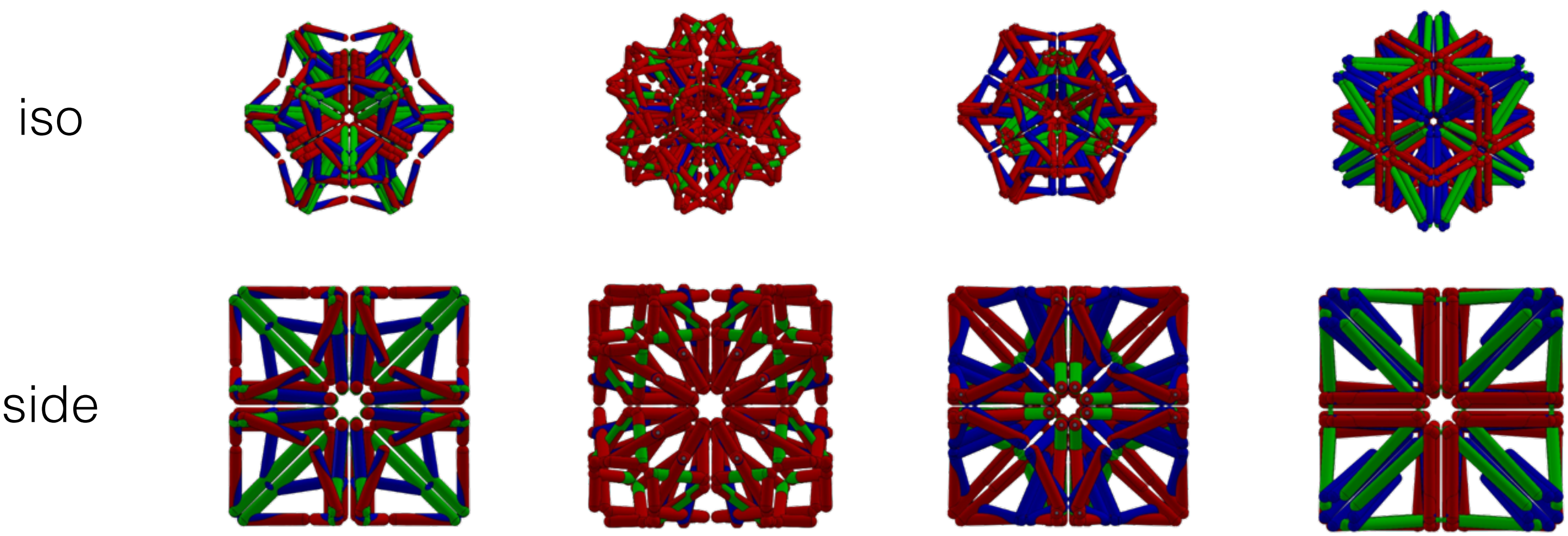
w_f	0.05	0.0556	0.0611	0.0667	0.0722
K	0.08	0.092	0.117	0.095	0.124

Color of struts indicates the choice of material.

Comparison to Hashin-Shtrikman-Walpole bounds (Gibianski and Sigmund, 2000)



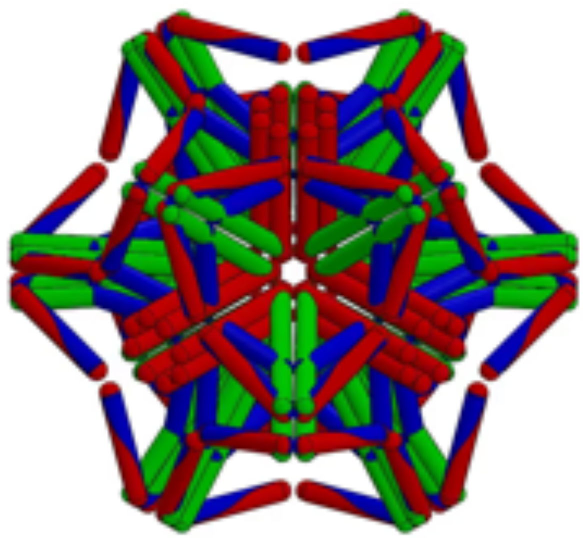
Bulk modulus maximization / 3 materials

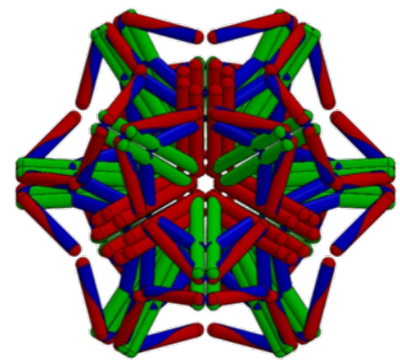


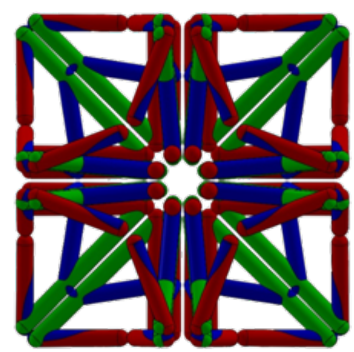
w_f	0.0667	0.0889	0.0944	0.1
K	0.094	0.12	0.169	0.155

Color of struts indicates the choice of material.

Bulk modulus maximization / 3 materials

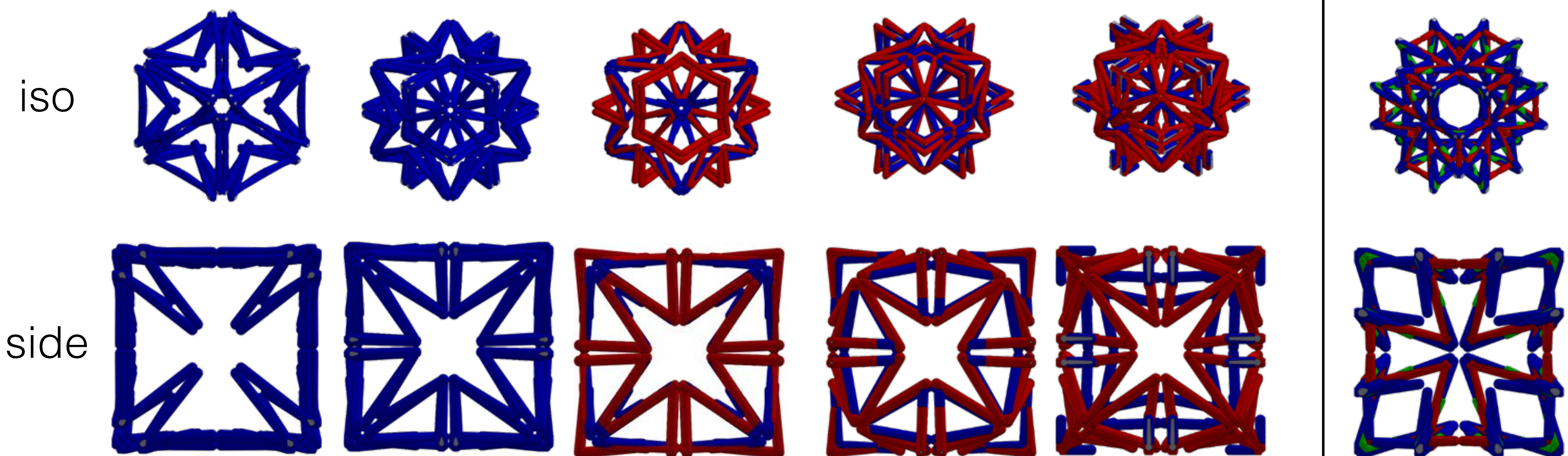






Bulk modulus maximization design
for w_f = 0.0667

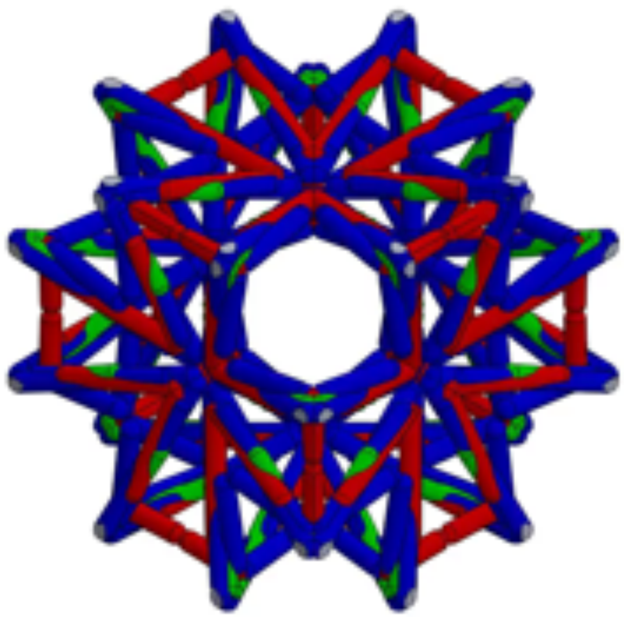
Poisson's ratio min. / 2 and 3 materials



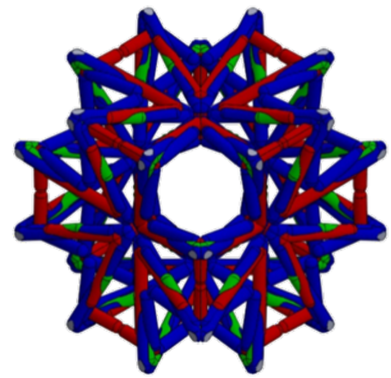
w_f	0.0333	0.0389	0.0444	0.05	0.0667	0.0389
ν	-0.057	-0.118	-0.204	-0.639	-1.573	-0.426

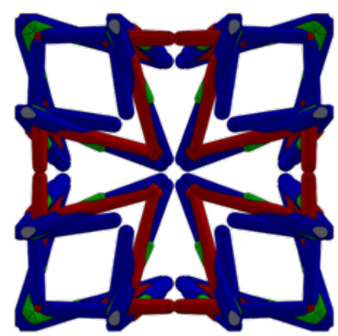
Color of struts indicates the choice of material.

Poisson's ratio min. / 3 materials



Poisson's ratio minimization
design for w_f = 0.0389





Epilogue

- In practice, polymeric materials for multi-material printers have similar physical densities → optimizer uses only stiffer material.
- Possibility 1: impose minimum angle constraints to prevent overlaps among struts that make manufacturing more difficult.
- Possibility 2: struts that are hollow or fiber-reinforced (work in progress)

Thank you!

Structural Optimization Laboratory

NSF

UCONN
University of Connecticut